Transmission, Reflections, Eigenfrequencies, Eigenmodes Tranversal and Bending waves (VTAF05)

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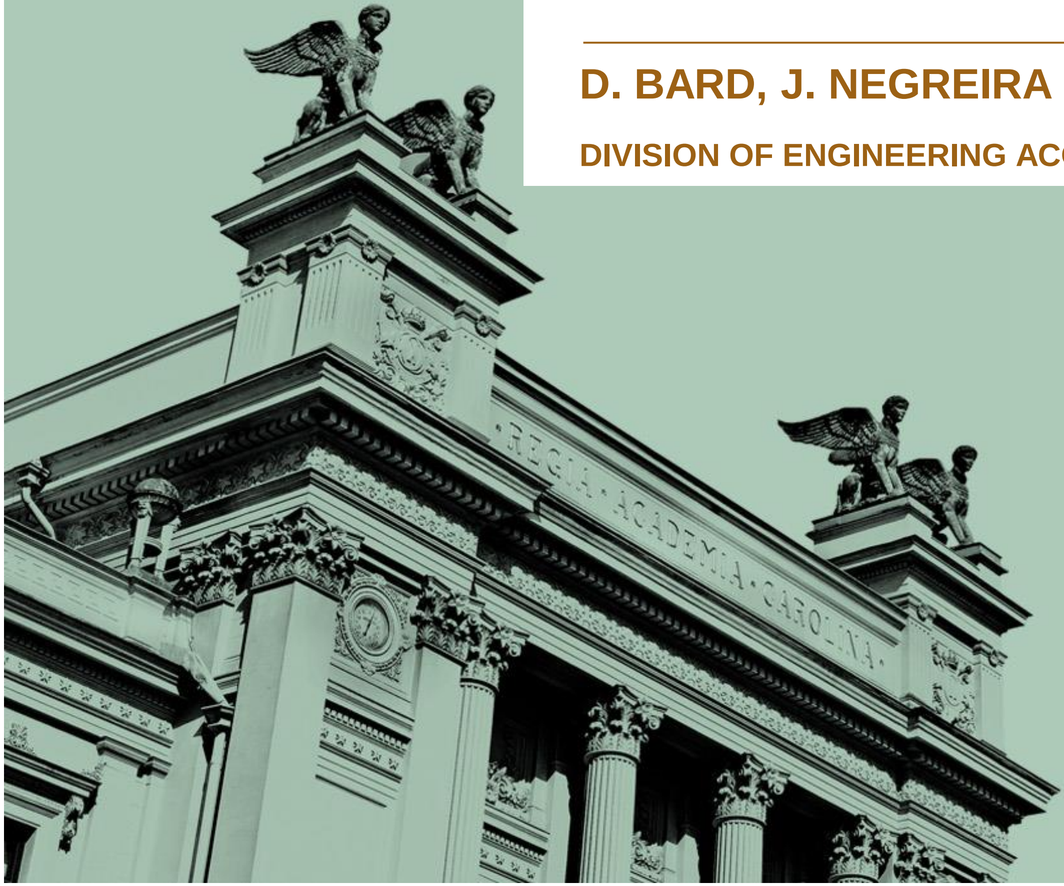


Outline

Introduction

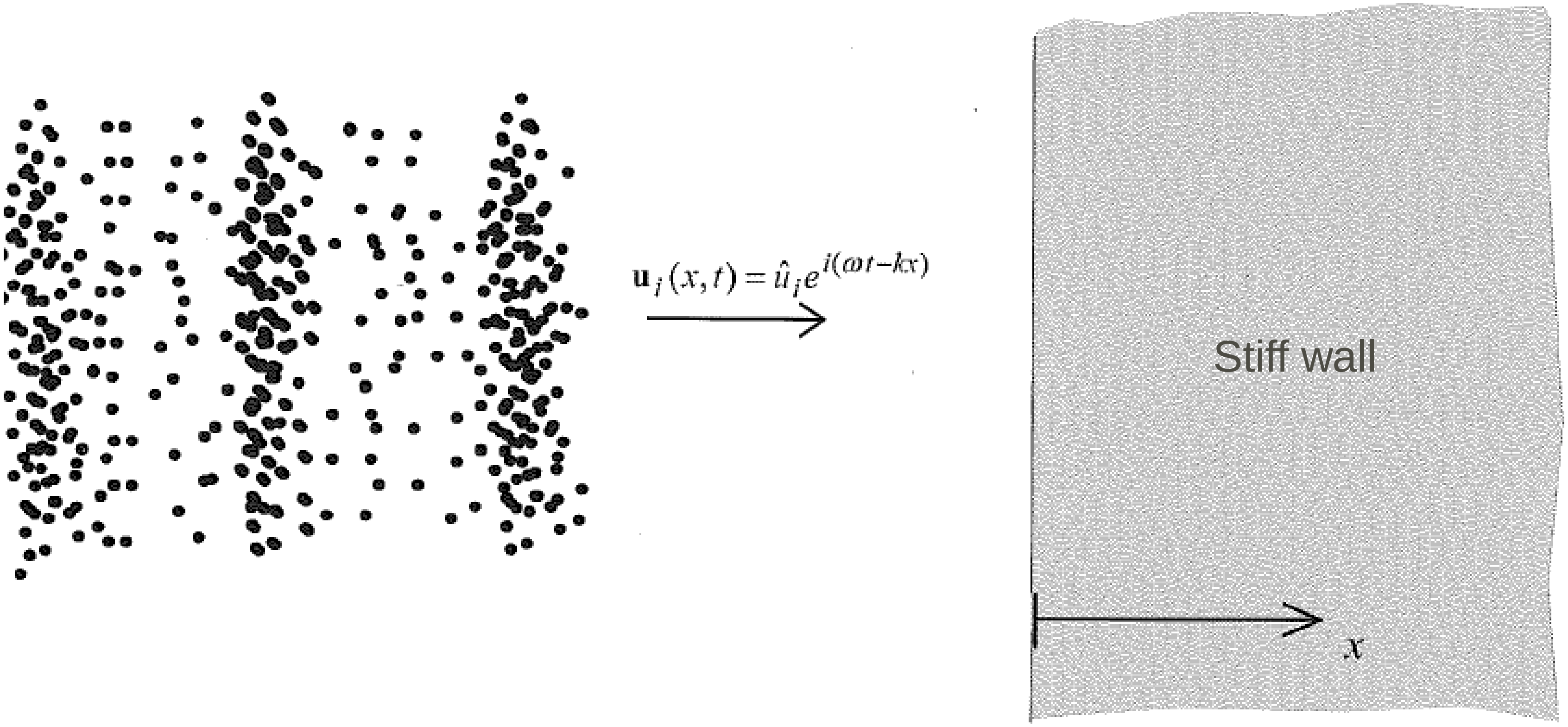
Types of waves

Eigenfrequencies & Eigenmodes

Acoustic Impedance & Acoustic Power

Reflection & Transmission of sound waves

Plane wave incident to a stiff wall



$$\mathbf{u}_i(x,t)=\hat{u}_i e^{i(\omega t-kx)}$$

Figure: Sound and vibrations - Bodén

Standing waves – velocity function

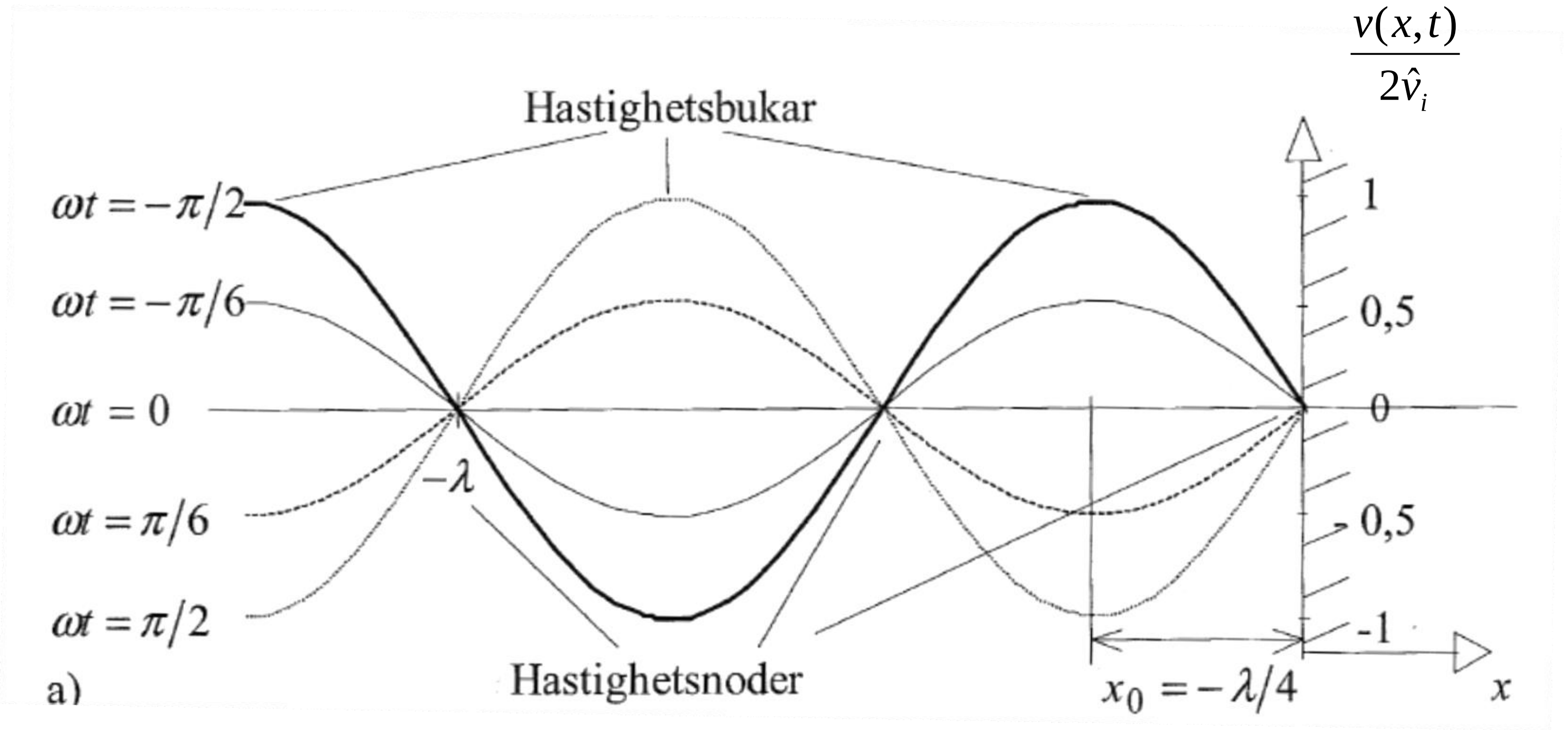


Figure: Ljud och vibrationer - Bodén

Standing wave – pressure function

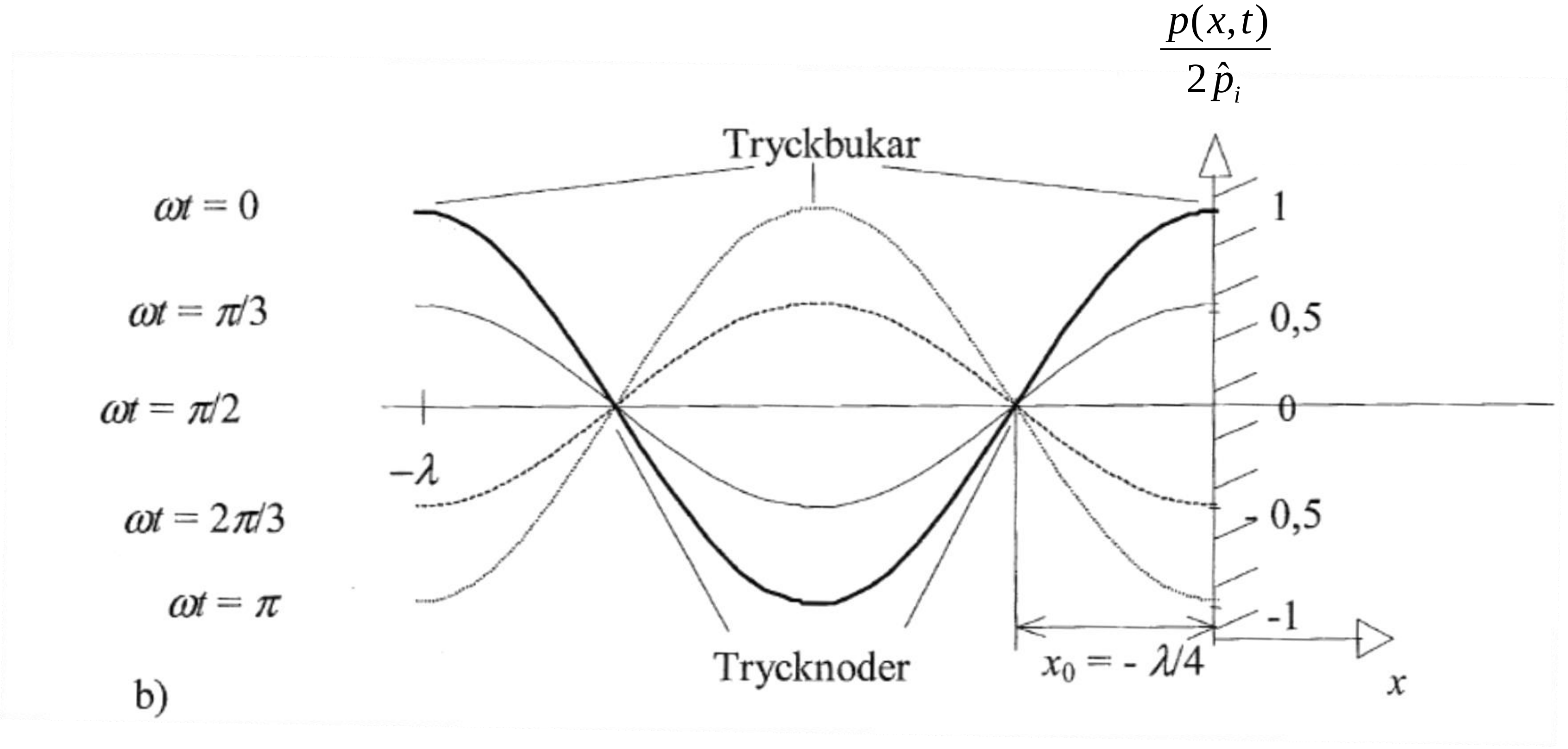
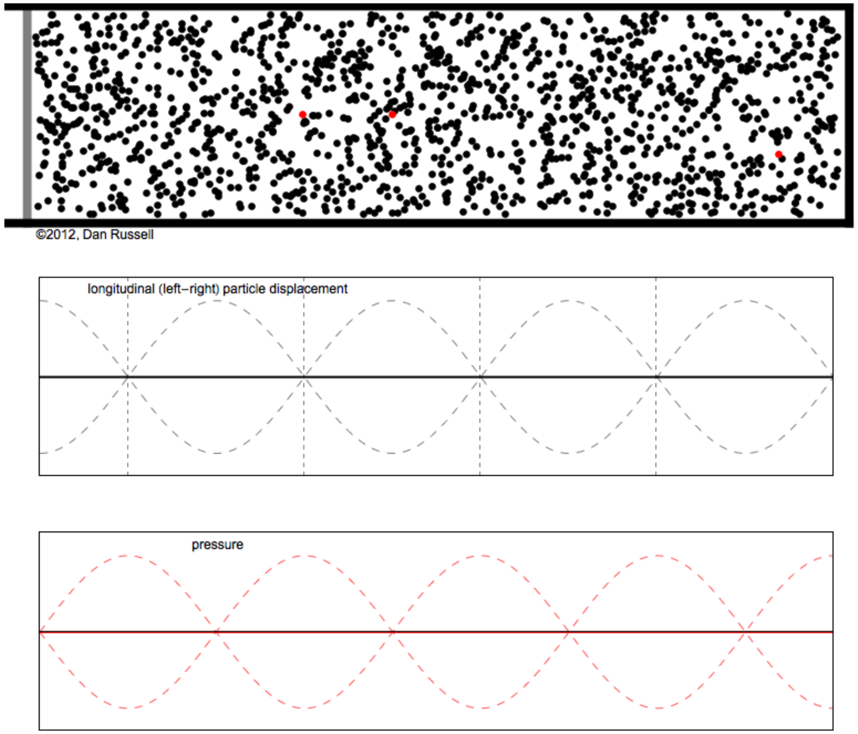


Figure: Ljud och vibrationer - Bodén

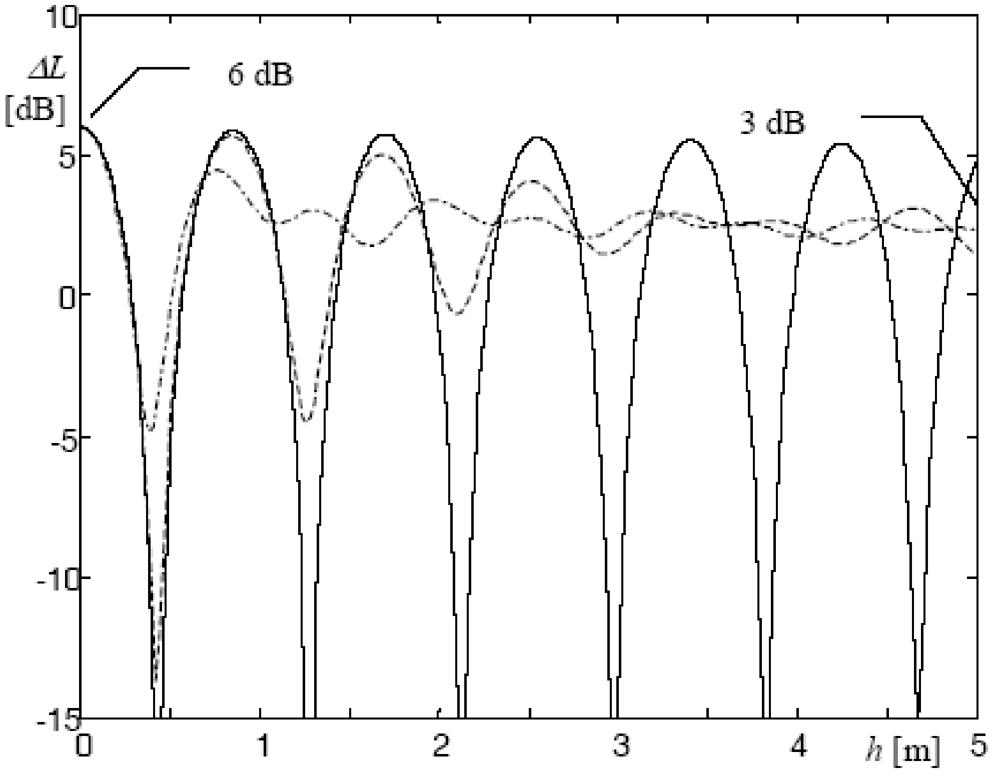
Standing wave particle motion



One of the red particles does not move at all – it is located at a **displacement node**, a location where the amplitude of the displacement always zero

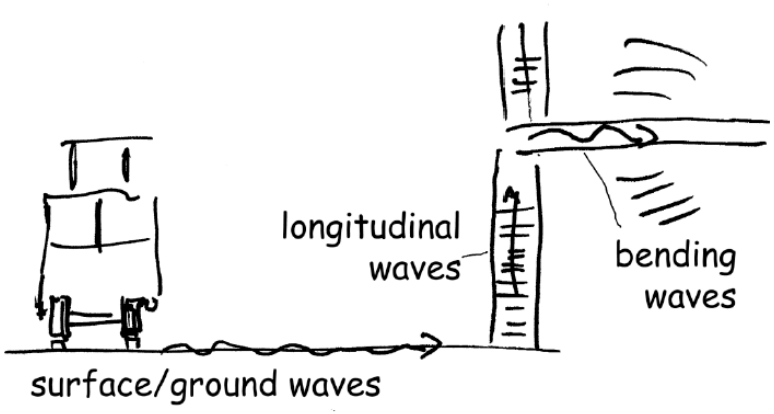
Sound measurement at hard facade surface

- The sound interferes with its own reflection



Types of waves

Noise for instance from traffic is transmitted through façades, windows and doors into houses.

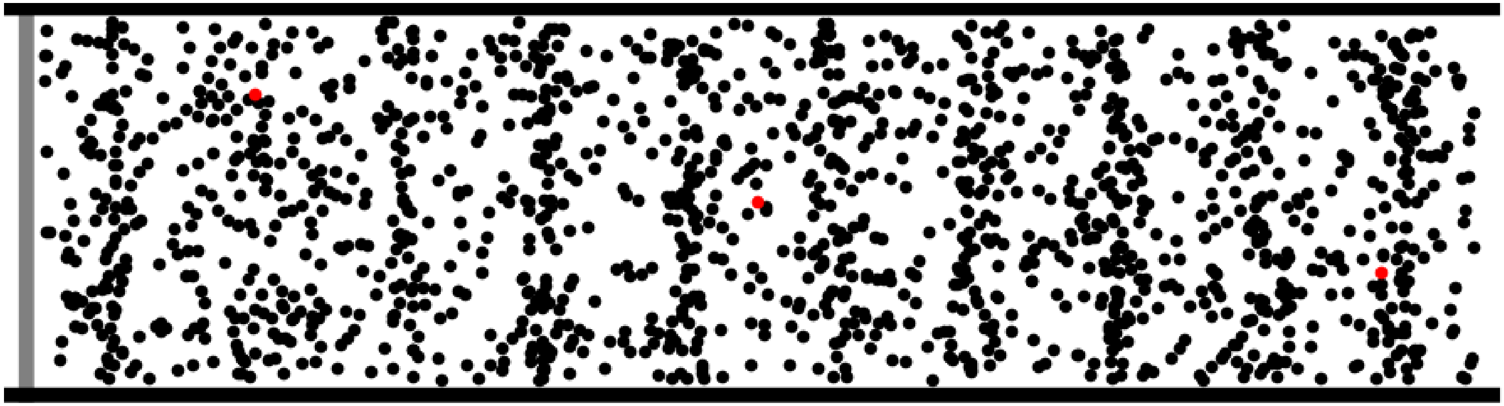


Waves in Structures - Wolfgang Kropp

Vibration for instance form trains or heavy vehicles is propagating trough the ground into the fundament of houses. From there it is propagating trough the building structure and radiated into room where people are living.

- In all these processes vibration of structures are involved. There are two main types of waves important:
  - **Longitudinal waves** and **bending waves**.

Longitudinal waves (I)



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In a longitudinal wave the particle displacement is parallel to the direction of wave propagation.

The particles simply oscillate back and forth about their individual equilibrium positions. Pick a single particle and watch its motion. The wave is seen as the motion of the compressed region (i.e, it is a pressure wave), which moves from left to right.

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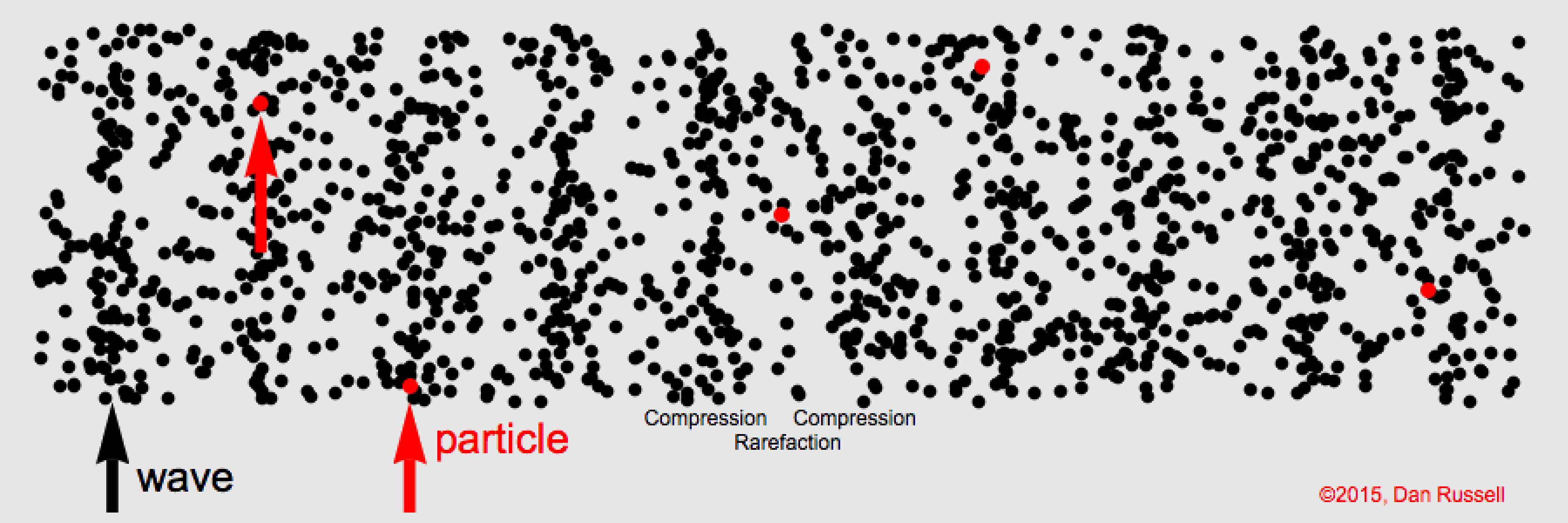
Longitudinal waves (II)



- The animation shows the difference between the oscillatory motion of individual particles and the propagation of the wave through the medium. It also identifies the regions of compression and rarefaction.

Longitudinal waves (III)

For longitudinal waves in beams: $c_{L,Beam} = \sqrt{\frac{E}{\rho}}$

Where E is the Young's modulus, ρ the density of the material.

When we have longitudinal waves on a beam the cross section changes during propagation (although very little). You can demonstrate this behaviour of change in cross section demonstrate with a soft piece of rubber.

Longitudinal waves (IV)

The situation becomes different when considering a plate of an infinite elastic space (e.g. inside the earth). In this case each element has neighbours, which also want to move. Only at the borders of the material, a cross contraction of the particles is possible.

As a consequence the material is softer when cross contraction is possible than when it is not and the speed of sound will be higher in solids than in beams

$$c_{L,solid} = \sqrt{\frac{1}{\rho}\frac{E(1-\mu)}{(1+\mu)(1-2\mu)}}.$$

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Longitudinal waves (V)

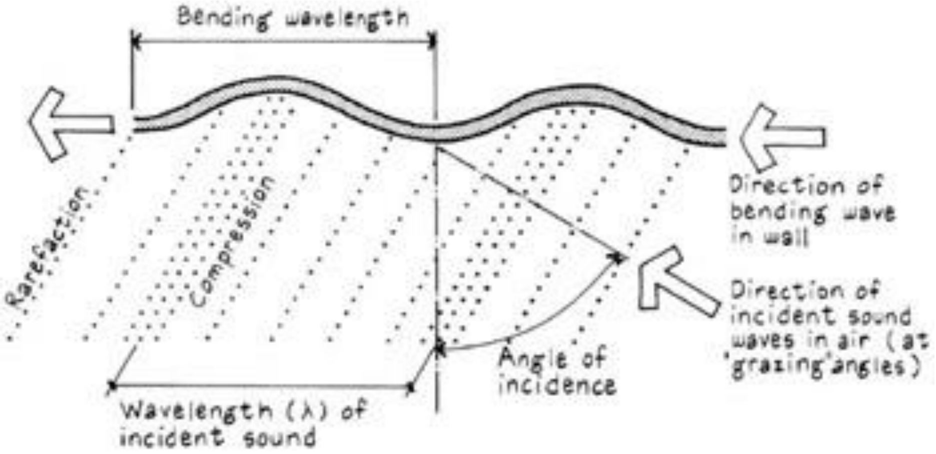
In plates the speed of sound is between both values

$$c_{L,plate} = \sqrt{\frac{1}{\rho}\frac{E}{(1+\mu^2)}}.$$

where μ is the Poisson's number.

Bending waves

First of all they are mainly responsible for the radiation of sound from vibrating structures since they have a displacement component in the normal direction to the surface of the structure. Secondly, it is the most common wave type when dealing with structure borne sound.



Natural frequencies shapes – velocity function

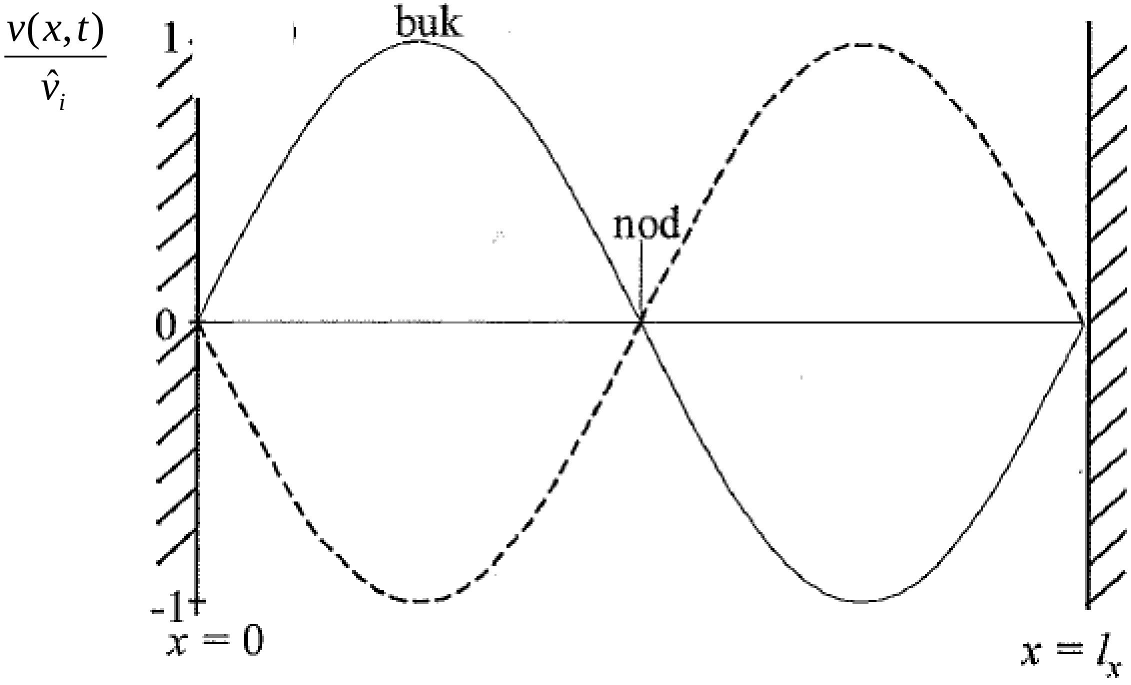


Figure: Ljud och vibrationer - Bodén

Natural frequencies shapes – pressure function

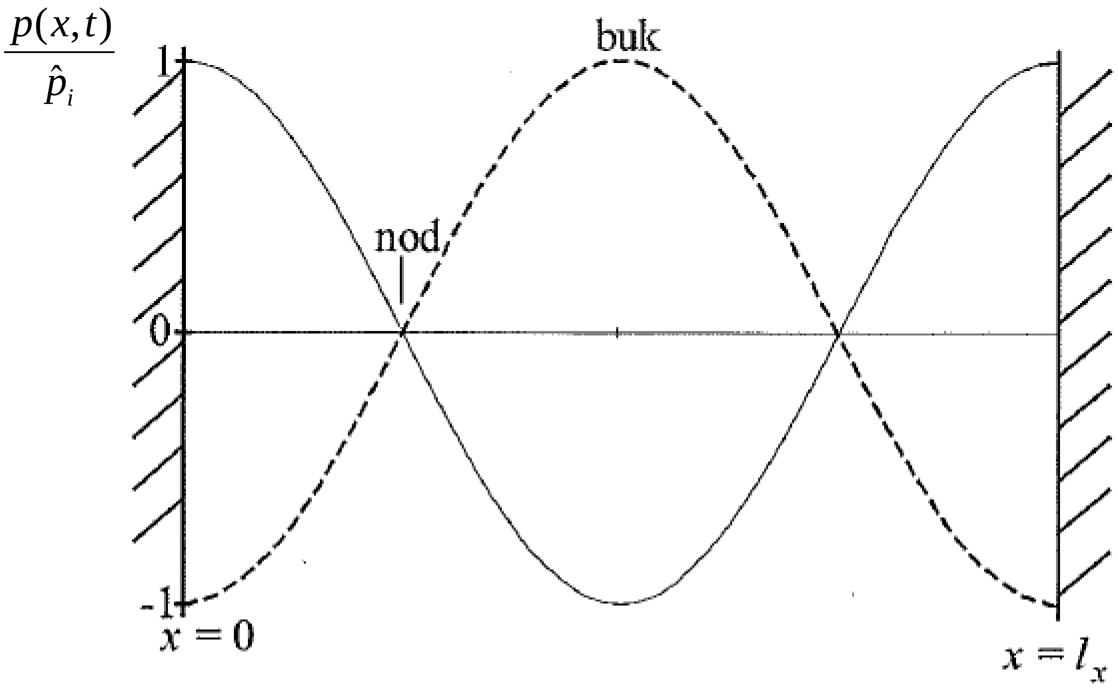
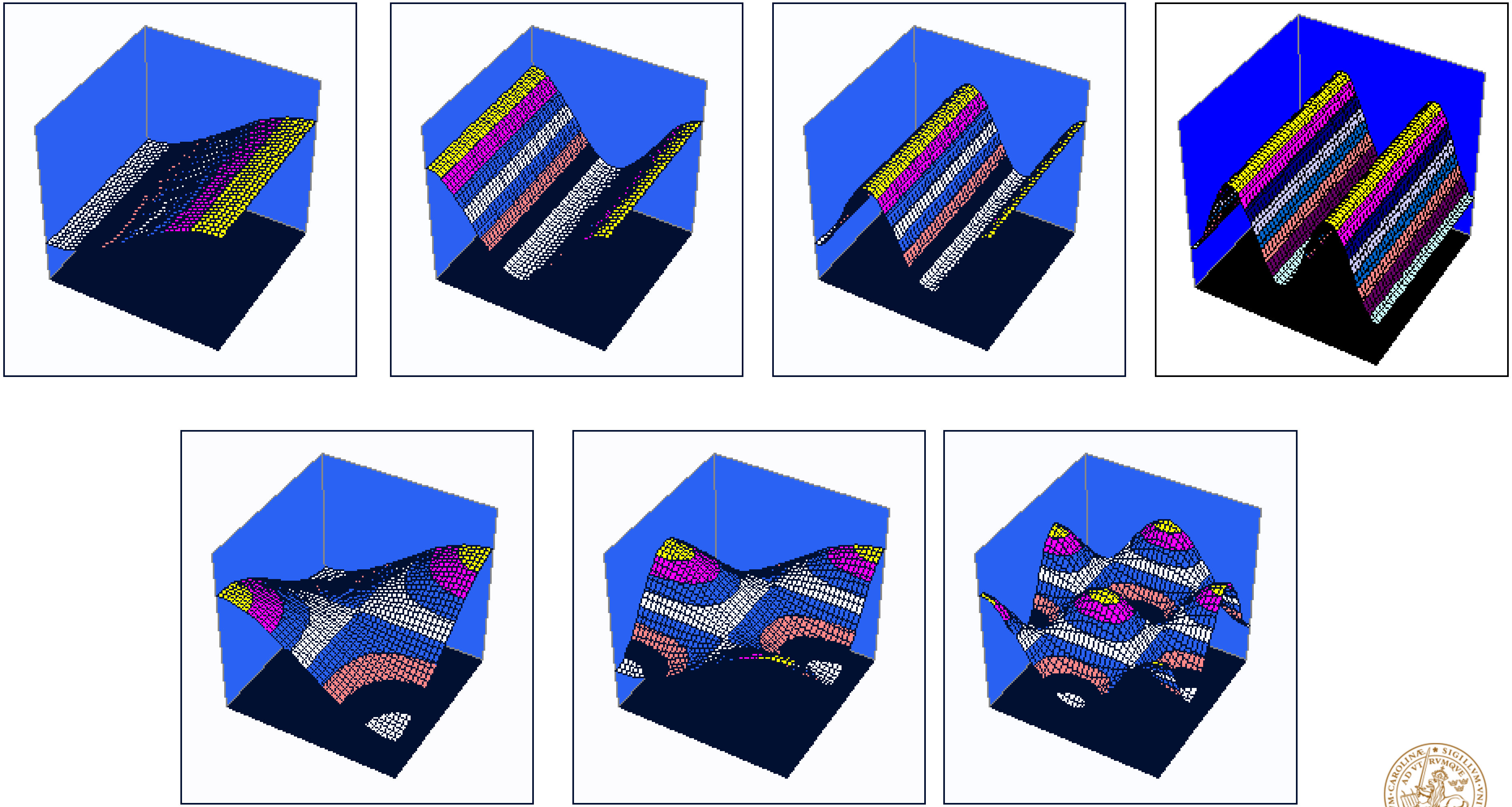


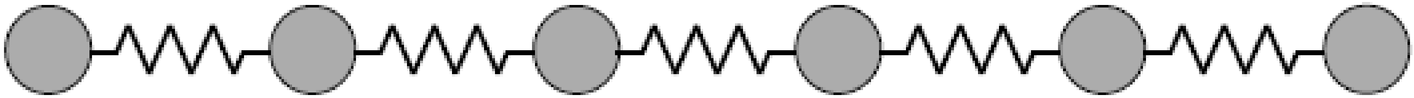
Figure: Ljud och vibrationer - Bodén

Eigenmodes



Acoustic impedance

- Pressure, p, is applied to a molecule it will exert pressure the adjacent molecule, which exerts pressure on its adjacent molecule.
- It is this sequence that causes pressure to propagate through medium.



Acoustic impedance (II)

- Acoustic pressure increases with particle velocity, v, but also depends upon properties of the medium
- Relationship between acoustic pressure and particle velocity is characterised by the **acoustic impedance** of the medium

$$Z = \frac{p}{v} \quad (\mathrm{kgm^{-2}s^{-1}} \text{ or a } \textbf{rayl})$$

- Acoustic impedance is also related to the elasticity of the medium
- Stiffer bonds between molecules increase the pressure exerted by a molecule moving with velocity v.

Acoustic impedance (III)

- A springy material will have high molecular motion and absorb sound energy in the bonds
  - » Less energy will be transferred between molecules

$$Z = \frac{B}{v}$$

- Wave propagation speed depends upon elasticity of medium and density

$$Z = \rho c = \sqrt{B\rho}$$

Acoustic power

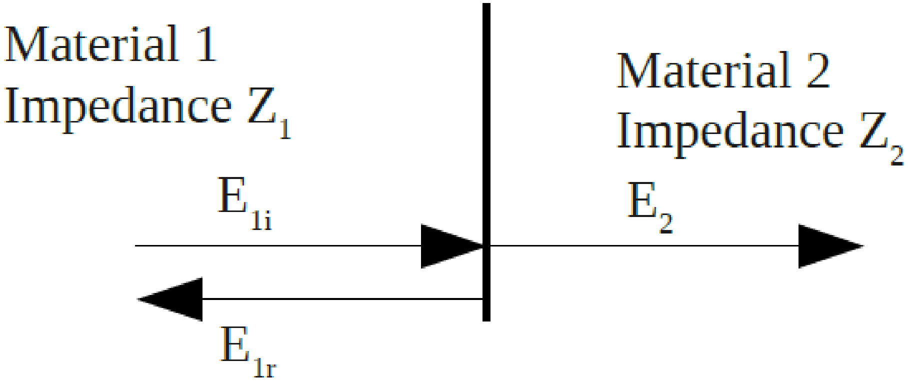
- Sound energy measures in Joules [J]
- Sound power [Watts]

$$\mathrm{P} = \mathrm{pv} = \mathrm{v}^2\mathrm{Z}$$

Reflection & transmission of sound waves (I)

A pulse of sound incident on an interface between media with different mechanical properties can undergo two processes

Transmission or Reflection



Reflection & transmission of sound waves (II)

Amount of reflected

and transmitted light

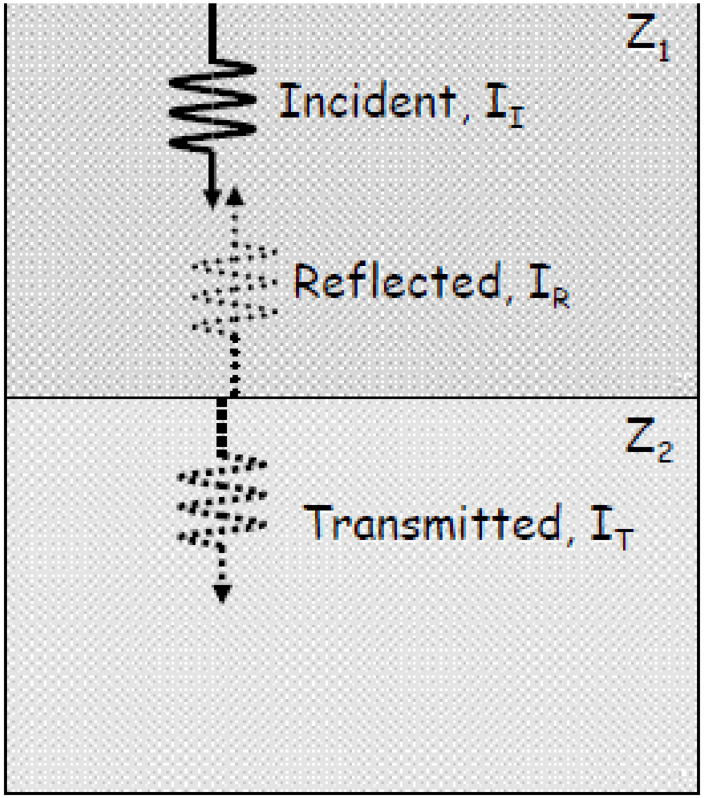
depends upon impedance difference

$$I_T = I_T + I_R$$

Reflected Intensity

$$I_T = I_I \times R$$

$$R = \frac{I_R}{I_I} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$



Reflection & transmission of sound waves (III)

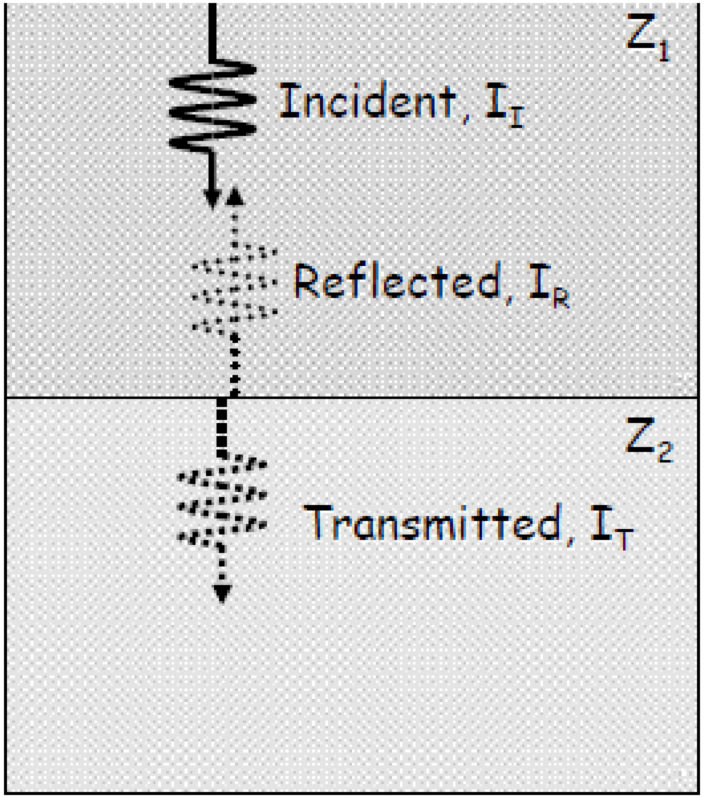
Transmitted Intensity

$$I_T = I_I \times T$$

$$T = \frac{I_T}{I_I} = \frac{4Z_1Z_2}{(Z_1 + Z_2)^2}$$

If no energy is lost to medium

T+R=1



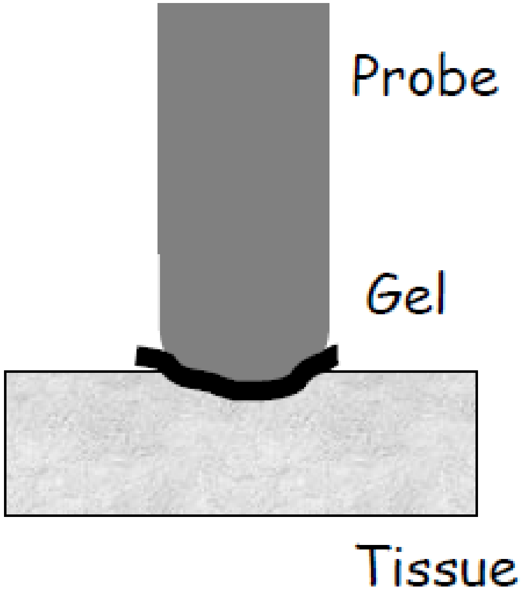
Reflection & transmission of sound waves (IV)

Reflection and transmission of sound waves forms the basis of ultrasound imaging

Impedance matching

To optimise transmission of US into patient from probe an impedance matching medium is used

$$Z_M = \sqrt{Z_T \times Z_P}$$



Example

If a transducer and tissue have acoustic impedances of 30×10^6 & 1.5×10^6 $kgm^{-2}s^{-1}$ respectively, what acoustic impedance should a matching medium have to minimise reflection?

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Practice questions

1. A sound wave propagates at 300 ms^{-1} through a medium with an acoustic pressure of 10 Pa. Calculate the acoustic impedance of the medium
2. A sound wave propagates at 4080 ms^{-1} through a medium with a density of 1700 kgm^{-3}. Calculate the acoustic impedance of the medium

Thank you for your attention!

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