2.11.5 Bending (or flexural) wave

- most important wave type for acoustical building analysis
  - ↳ most ~~generally~~ easily generated by airbone sound and most readily radiated into air
- combination of longitudinal and transverse waves
- can be converted from other type of waves
- important difference: speed of sound is different for frequencies and always slower than the other wave types
- Mostly used: Thin beam theory for analysis of transverse behaviour of a bending beam

$$\frac{\partial^4 v_y}{\partial t^4} = -c^2 \frac{\partial^4 v_y}{\partial x^4} \quad , \quad c = \sqrt{\frac{B}{\rho \cdot c}}$$

(Bernoulli-Euler-Equation for bending vibration)

→ valid only if bending wave length is large compared with dimension of object (for low frenquencies)

→ for short waves (high frequencies) → correction factor, because: take finite thickness of plate into account
↳ correction of pure bending waves result in a bending propagation velocity (is lower than expected)

- practical limit for validity of formula: 10% error in phase velocity when $\lambda_B = 6 \cdot h$, h = thickness

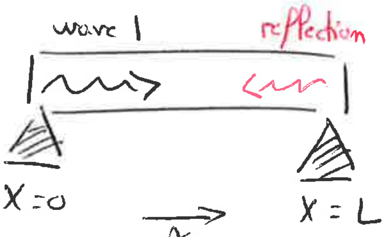
near field is decaying with distance

- Higher-frequency waves travel faster than lower-frequency waves

Bending waves in structure:

- Used for noise control measures
- We assume that:
  - beams have uniform cross section
  - homogenous material / isotropic
  - thin beam theory assumed (shear deformation and rotary = 0)

1. Standing waves in a beam:



- Measure the eigenfrequency (resonance frequency)

$$f_m = \frac{m^2 \pi}{2L^2} \sqrt{\frac{B}{M}} \qquad m = 1, 2, 3 \ldots$$

$B = E \cdot I$

M = mass density

wave 1 : $V_1 = V_+ e^{-jk_B x}$

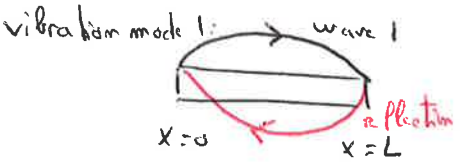
reflexion : $V_{ref} = V_- e^{jk_B x}$ $\Rightarrow V = V_1 + V_{ref}$

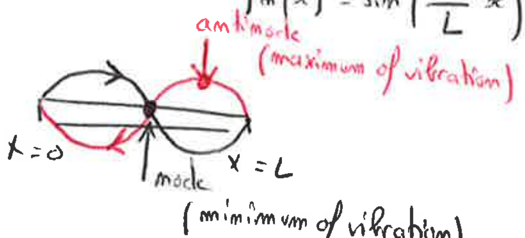
Boundary conditions: at $x = 0 \Rightarrow M = 0$
$x = 0 \Rightarrow V = 0$
$x = L \Rightarrow V = 0$

equation to solve : $-\frac{B}{j\omega} \frac{\partial^2 V}{\partial^2 x} = 0 \Rightarrow V = -2j \sin(k_B x)$

with $k_B L = m\pi$

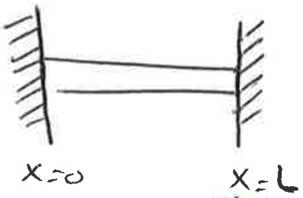
The resonance frequencies are associated with modes: $\varphi_m(x) = \sin\left(\frac{m\pi}{L} x\right)$, the eigenfunction.





2. Clamped beam at two ends:

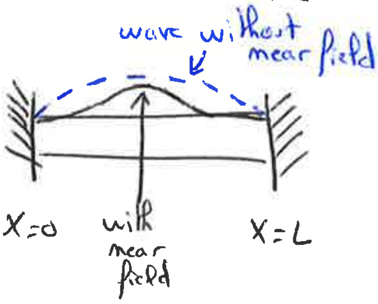
This time, the central structure is constrained



We [redacted] add a near field to take this new constraints in account:

$$V_{+j} = V_{+j} e^{k_B x}$$

$$V_{-j} = V_{-j} e^{-k_B x}$$



This will change the value of the eigenfrequency to:

$$f_m = \frac{\pi (2m+1)^2}{8L^2} \sqrt{\frac{B}{M}} \qquad m = 1, 2, 3 \ldots$$

2.12.2 Damping in structures

Energy losses

- Material damping
- Coupling or boundary losses
- Sound radiation

Complex E-modulus, $\bar{E}$ $\bar{E} = E(1 + j\eta)$

$v = v_+ e^{-jk_B x}$ flexural wave number $\bar{k}_B = k_B\left(1 - \frac{j\eta}{4}\right)$

Amplitude decrease with a factor $e^{-k_B^{II} x}$

Attenuation at distance x

$\Delta L = 20 \log\left|\frac{v(0)}{v(x)}\right| = 20 k_B^{II} x \log(e)$

Standing wave - Large attenuation
Propagating wave - small —//—

Δf = difference from resonance freq.

$\eta = \frac{2\Delta f}{f_n}$

Reverberation

$10^{-6} = e^{-\eta\omega T} \Rightarrow \eta = \frac{\ln(10^6)}{\omega T} = \frac{2{,}2}{fT}$

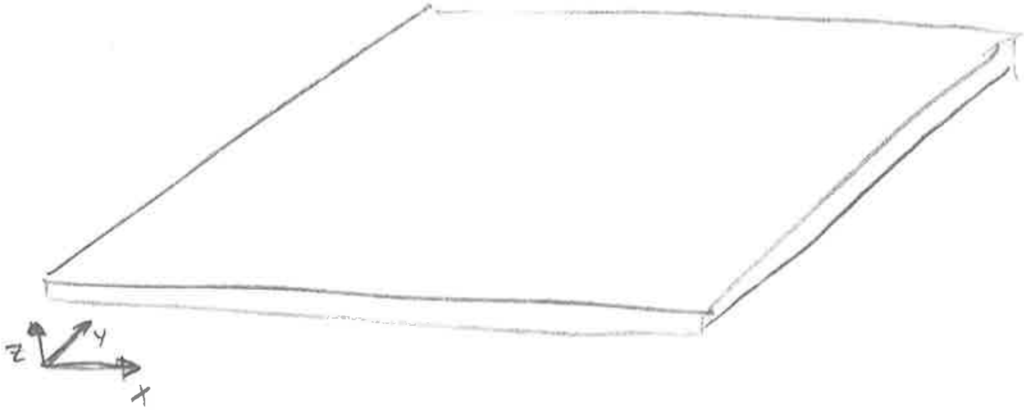
All waves
Not too much dampening
A number of resonance freq.

Dampening layers
metal plates
viscoelastic

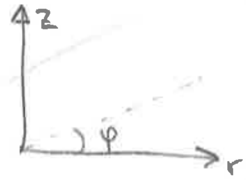
Loss factor for longitudinal waves and bending waves differ

quasi-long. - $\eta_L = \eta_d \frac{E_d d_d}{E_1 d_1 + E_d d_d}$ bending - $\eta_B = \eta_d \frac{E_d I_d}{E_d I_d + E_1 I_1}$ $I_d = I_{od} + a^2 S_d$

2.12.3 Bending wave equation for plate.



Cylindrical coordinate system
- to explain wave propagation



Vibration velocity $\left|\frac{V}{V_+}\right|^2 = \frac{2}{\pi k_B r}$

k_B - wave number of standing wave.

Point Impedance for infinite plate

Plate excited perpendicular to its plane
→ velocity v_0 proportional to Force F

Impedance $Z = \frac{F}{V}$

$\rightarrow Z = 8\sqrt{M, B}$

Power injected in plate

$W = \frac{|\tilde{F}|^2}{Z}$

(Analogous with Ohm's law $W = \frac{V^2}{R}$)

Bending vibrations of finite plates,

"Knowledge of a panel's or beam's natural frequencies and the associated mode shapes is partically useful, because one can avoid having excitation frequncies."

$$f_n = \frac{\pi}{2}\sqrt{\frac{B}{M_s}}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2\right]$$ Natural freq.

B is a stiffness parameter
M_s is the plate's area-density $(\rho \cdot h_{plate})$

$$\varphi_n(x,y) = \sin\left(\frac{n_x \pi x}{L_x}\right)\sin\left(\frac{n_y \pi y}{L_y}\right)$$ Mode shape (2D)

2.12.4 Input force and moment impedances

When a wall or a floor is excited directly by a source such as footsteps or collision,

· Power input depends on

- Property of <u>structure</u>
- Property of <u>source</u>

The power input depends on the applied force and the mobility of the structure: (related to impedance)

Definition of the mobility of the structure

$$Y = \frac{1}{Z} = \frac{v_0}{F} \qquad [\text{Eqn } 2.227]$$

v_0 ← velocity of the plate

F ← exciting point force

Mobility and impedance can be determined by the power input:

(1) structure excited by force

$$W = F^2 \operatorname{Re}\{Y\} \qquad [\text{eqn } 2.228]$$

(2) structure excited by velocity

$$W = v^2 \operatorname{Re}\{1/Y\} \qquad [\text{eqn } 2.229]$$

The power flow can also be injected into a structure by a moment excitation

$$J = \frac{M}{\omega} \qquad [\text{eqn } 2.230]$$

J = Moment impedance

M - exciting moment

ω = angular velocity

☆ Y is assumed to be undamped here. If plate is damped, use damping factor (η) to determine E. [$\bar{E} = E(1 + j\eta)$ (eqn 2.194)]

☆ Expressions are for case of <u>thin</u>, <u>Isentropic</u> plates only.

BENDING VIBRATIONS OF PLATES/BEAMS

Point force F excites a thin plate at (x, y).

Generated bending velocity $v(x,y)\,e^{j\omega t}$ is calculated using modal expansion:

$$v(x,y) = \sum_{n=1}^{\infty} \frac{j\omega \varphi_n(x,y)}{M_s(\omega_n^2 - \omega^2)} F_n$$

where

φ_n : eigenmode $\left(\varphi_n(x) = \sin\left(\frac{n\pi}{L} x\right) \right)$

ω_n : eigenfrequency ; ω : angular frequency

M_s : surface density

Reduces to

$$v(x) = \sum_{n=1}^{\infty} \frac{j\omega F_n \varphi_n(x)}{M_s(\omega_n^2 - \omega^2)}$$

for beams