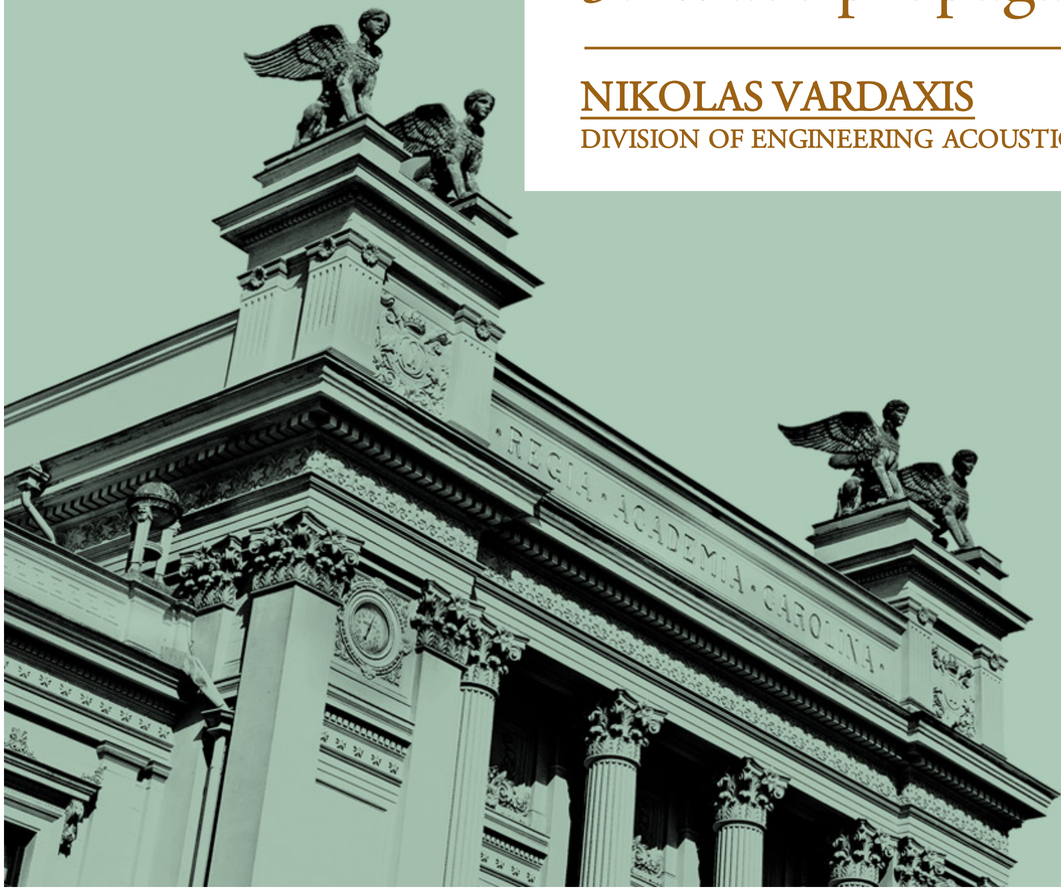

Acoustics (VTAN01)

3. Wave propagation

NIKOLAS VARDAXIS
DIVISION OF ENGINEERING ACOUSTICS, LUND UNIVERSITY

Outline

Introduction

Wave propagation in solid media

Wave equation solution

Wave propagation in fluids

Wave phenomena

Musical instruments

Summary

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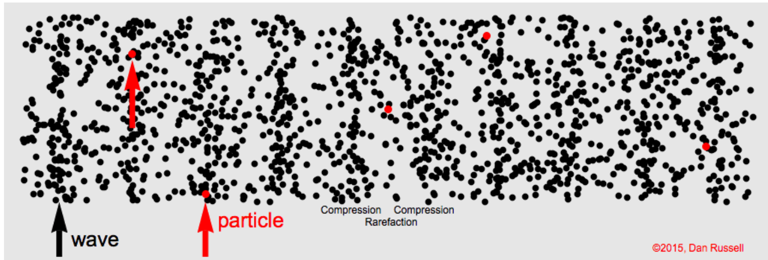
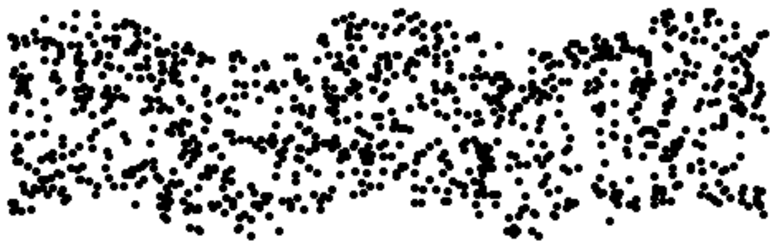
Learning outcomes

- Wave propagation in solid media
  - Longitudinal/quasi-longitudinal waves
  - Shear waves
  - Bending waves
- Wave equation solution
- Wave propagation in fluids
- Wave phenomena
- Musical acoustics

Types of waves – Classification

- Depending on propagation media
  - Mechanical waves (solids and fluids)
  - Electromagnetical waves (vacuum)
- Propagation direction
  - 1D, 2D and 3D
- Based on periodicity
  - Periodics and non-periodics
- Based on particles' movement in relation with propagation direction:
  - Longitudinal waves (solids and fluids)
  - Transverse waves (solids)
- …





<u>NOTE:</u> waves do not transport mass, just energy

Outline

- Introduction
- Wave propagation in solid media
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Types of waves in solid media

- Longitudinal waves (∞ medium ≈ beams)
  - Quasi-longitudunal waves (finite ≈ plates)

$$\frac{\partial^2 u_x}{\partial x^2} - \frac{\rho}{E'}\frac{\partial^2 u_x}{\partial t^2} = 0$$

$$c_L = \sqrt{\frac{E}{\rho}}$$

$$c_{qL} = \sqrt{\frac{E'}{\rho}} = \sqrt{\frac{E}{\rho(1-\upsilon^2)}}$$

- Shear waves

$$\frac{\partial^2 u_y}{\partial x^2} - \frac{\rho}{G}\frac{\partial^2 u_y}{\partial t^2} = 0$$

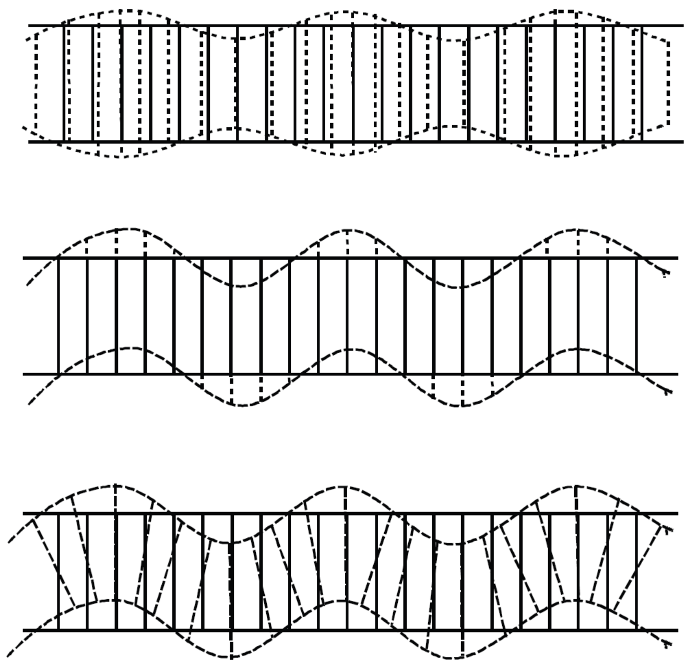
$$c_{sh} = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{2(1+\upsilon)\rho}}$$

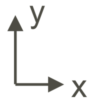
- Bending waves (dispersive)

$$B\frac{\partial^4 u_y}{\partial x^4} + m\frac{\partial^2 u_y}{\partial t^2} = 0$$

$$c_{B(\omega)} = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$$

Plate: E, G, ρ, υ, h





$$m = \rho h \qquad B_{beam} = E\frac{bh^3}{12}$$

$$B_{plate} = \frac{Eh^3}{12(1-\upsilon^2)}$$

<u>NOTE:</u> torsional waves (beams and columns) are not addressed here

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Wave equation solution (I)

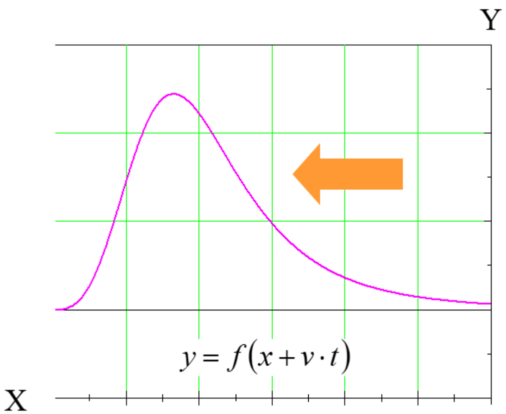
- Travelling waves: $y = f(x \pm vt)$

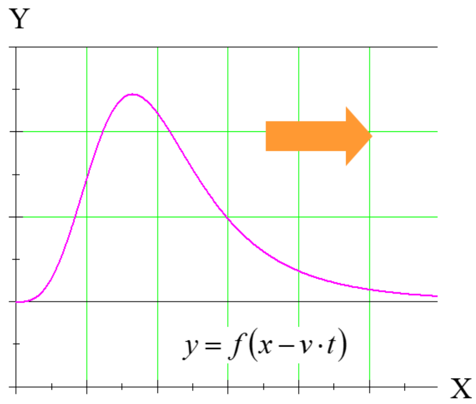
Space

Sign

Propagation speed

Time

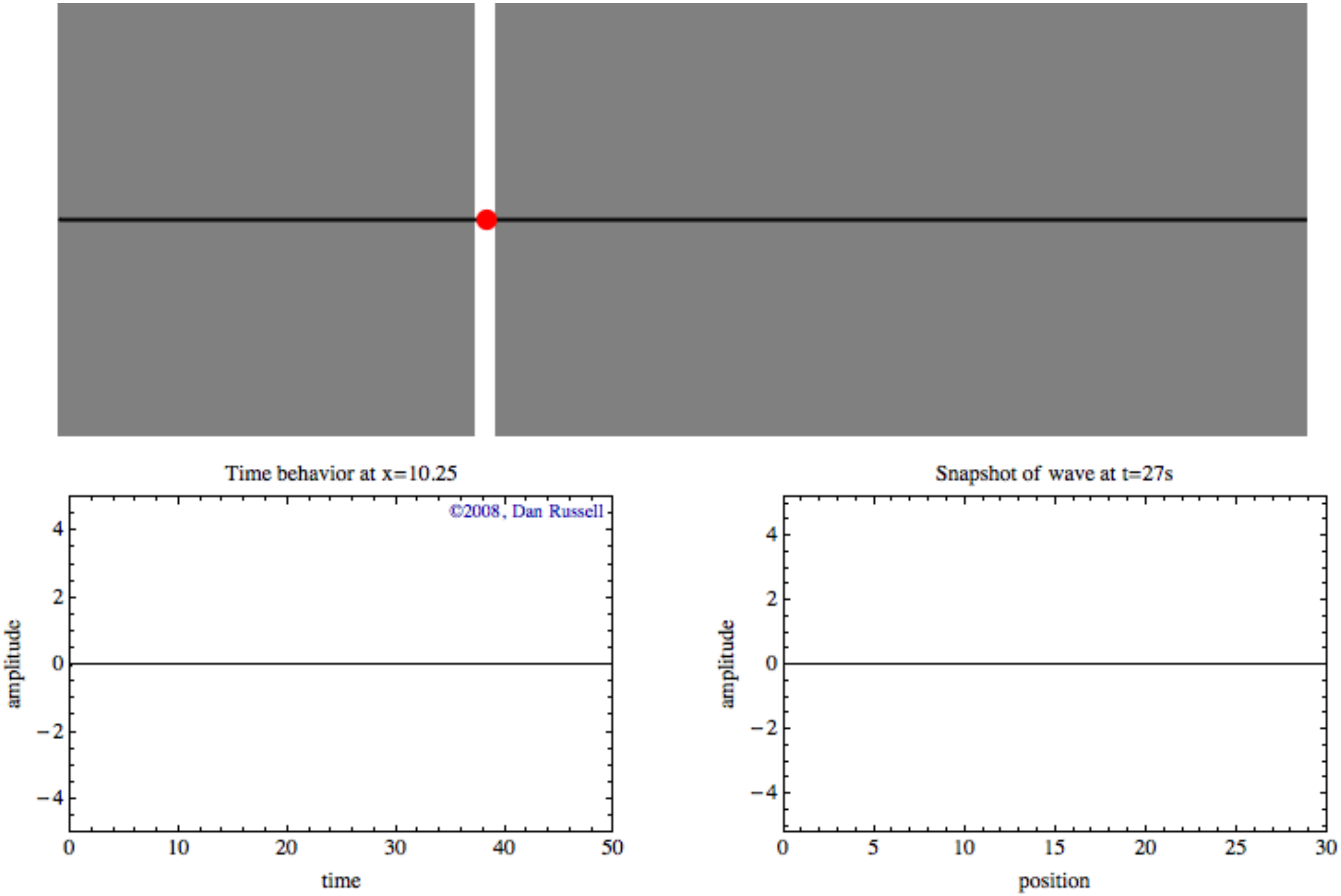




- Alternative forms: $y = f\left(x \pm \frac{\omega}{k} t\right) = f\left(\frac{kx \pm \omega t}{k}\right) = f(kx \pm \omega t)$
- Note:
  - Periodic functions: $f(x \pm vt) = f(x \pm vt + T)$
  - Harmonic functions: f is a sinus or cosinus

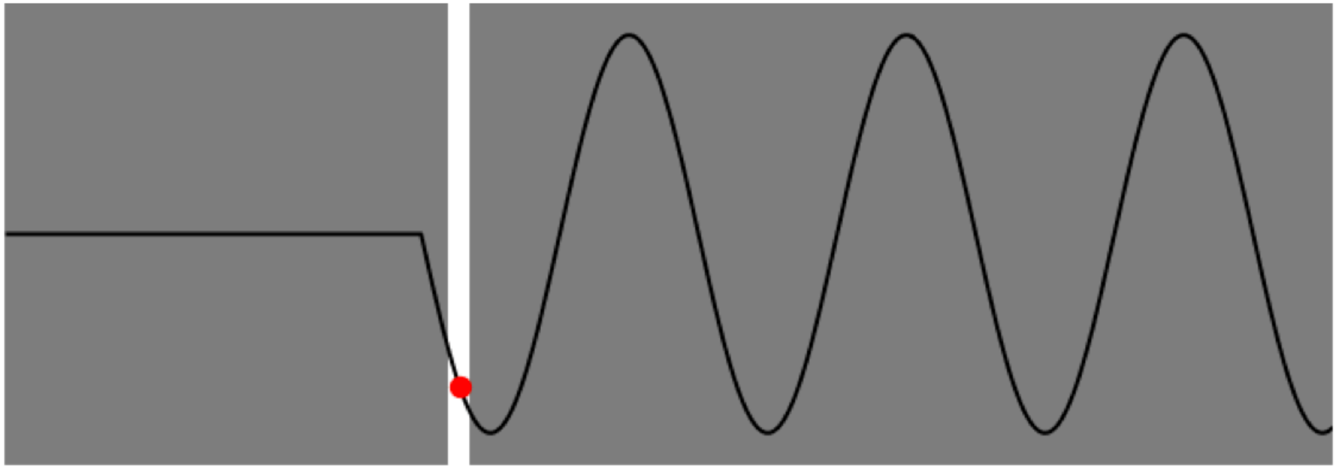
Wave equation solution (II)

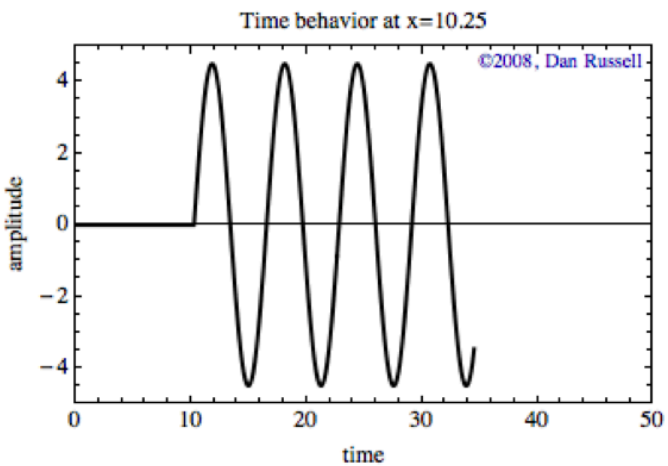
Time and position dependency: $u(x,t) = \widehat{u_+}\cos(\omega t - kx) = \widehat{u_+}e^{-i(\omega t - kx)}$

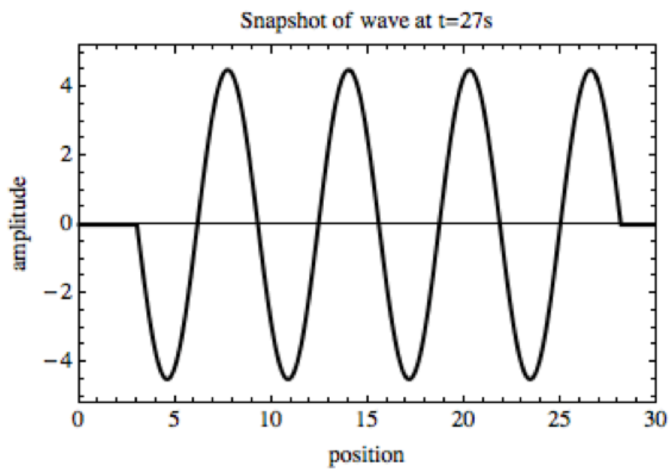


Wave equation solution (II)

[Still frame] Time and position: $u(x,t) = \widehat{u_+}\cos(\omega t - kx) = \widehat{u_+}e^{-i(\omega t - kx)}$

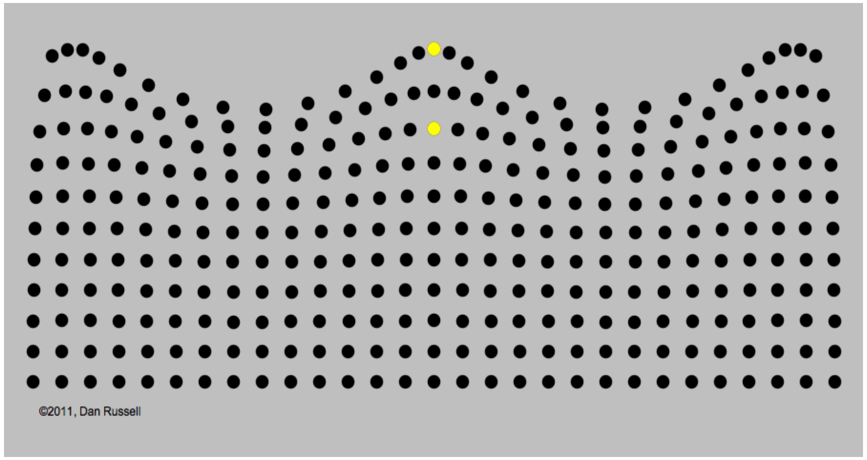


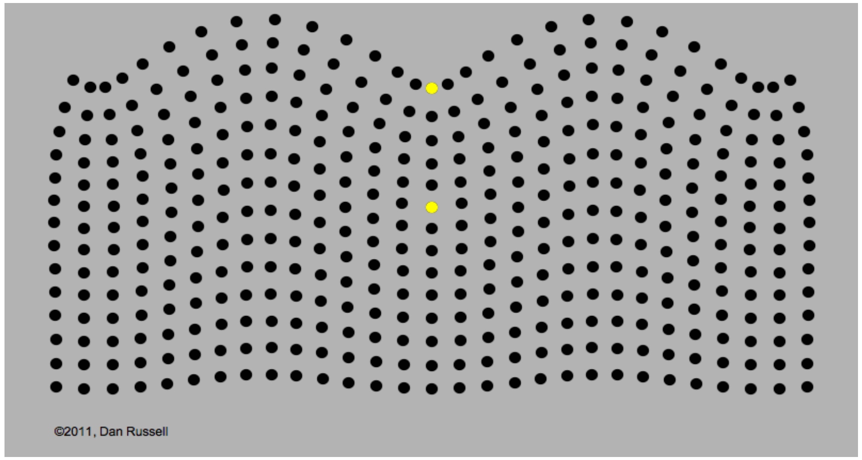




Other types…

- In reality, combinations of aforementioned waves can exist, e.g.





- Surface waves

Water waves

(long+transverse waves)

Particles in *clockwise circles*. The radius of the circles decreases increasing depth

Pure shear waves don't exist in fluids

- Body waves

Rayleigh waves

(long+transverse waves)

Particles in elliptical *paths*. Ellipses width decreases with increasing depth

Change from depth>1/5 of λ

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Waves in fluid media

- Sound waves: longitudinal waves
  - Pressure as field variable

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \longrightarrow p(x,t) = \widehat{p_\pm}\cos(\omega t \pm kx) = \widehat{p_\pm} e^{-i(\omega t \pm kx)}$$

  - Velocity as field variable

$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2} \longrightarrow v(x,t) = \frac{1}{\rho c}\widehat{p_\pm}\, e^{-i(\omega t \pm kx)}$$

Comparing both equations: $Z \equiv \frac{p_\pm}{v_\pm} = \pm\rho c$ (acoustic impedance)

$$c_{medium} = \sqrt{\frac{D}{\rho}}, \quad c_{air} = \sqrt{\frac{\gamma P_0}{\rho(T = 0°C)}\left(1 + \frac{T}{2 \cdot 273}\right)} = 331.4\left(1 + \frac{T}{2 \cdot 273}\right), \quad k = \frac{2\pi}{\lambda}$$

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SPL & SIL & SWL

- Sound pressure level (SPL / L_p)

$$L_p = 10\log\left(\frac{\tilde{p}^2}{p_{ref}^2}\right) = 20\log\left(\frac{\tilde{p}}{p_{ref}}\right)$$

$\tilde{p} = \tilde{p}(f) \equiv$ RMS pressure
$p_{ref} = 2{\cdot}10^{-5}$ Pa = 20 μPa
$p_{atm} = 101\,300$ Pa
$p_{tot}(t) = p_{atm} \pm p(t)$

- Sound intensity
  - Sound power (i.e. rate of energy) per unit area [W/m^2]
    - » Instantaneous value: $\vec{I}(t) = p(t)\vec{v}(t)$
    - » Vector quantity: energy flow and direction: $\bar{\vec{I}} = \langle pv \rangle = \frac{1}{T}\int_0^T p(t)\vec{v}(t)dt$
    - » In a free field: $\bar{I} = \frac{\widetilde{p^2}}{\rho c}$
  - In decibels (SIL)… $L_I = 10\log\left(\frac{\bar{I}}{I_{ref}}\right)$; $I_{ref} = 10^{-12}\,{}^{W}/_{m^2}$
- Sound power
  - Rate of energy transported through a surface [W=J/s]: $W(t) = \int_S I_n(\vec{x},t)dS$
  - In decibels (SWL / L_W / L_Π)… $L_W = 10\log\left(\frac{\overline{W}}{W_{ref}}\right)$; $W_{ref} = 10^{-12}$W

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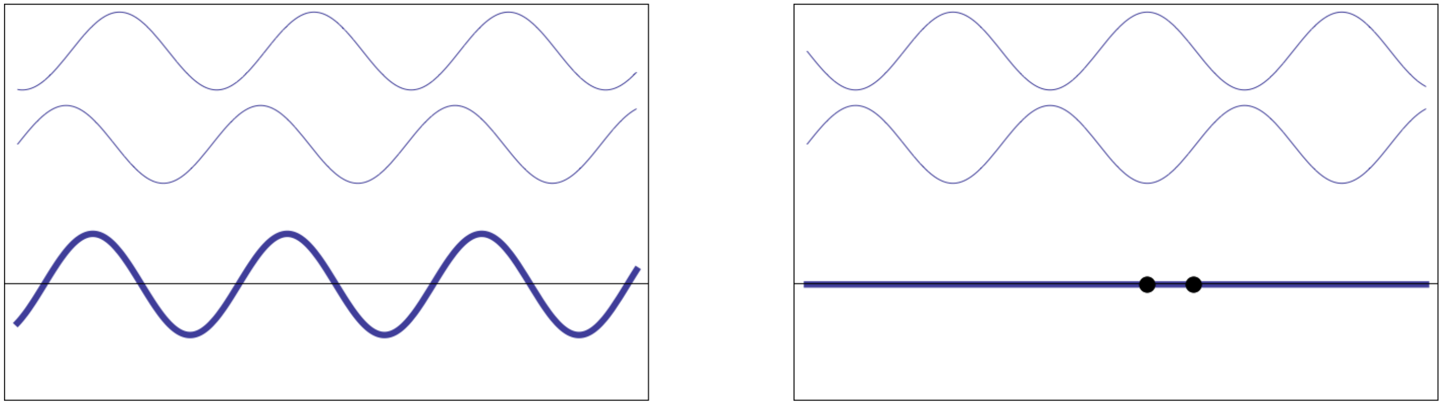
Wave phenomena

- Interferences: constructive / destructive

Constructive/destructive depending on Φ

$$\left.\begin{aligned} y_1(x,t) &= \hat{y}\cos(\omega t - kx) \\ y_2(x,t) &= \hat{y}\cos(\omega t - kx + \theta) \end{aligned}\right| \; y(x,t) = y_1(x,t) + y_2(x,t) = 2\hat{y}\boxed{\cos\left(\frac{\theta}{2}\right)}\sin(\omega t - kx + \theta)$$



Source: Dan Russell

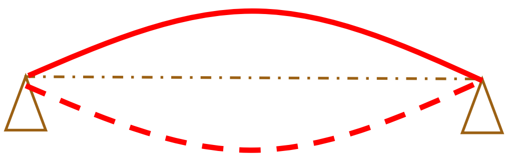
- Standing waves (coherent source)

$$\boxed{\begin{aligned} y_-(x,t) &= \hat{y}\cos(\omega t - kx) \\ y_+(x,t) &= \hat{y}\cos(\omega t + kx) \end{aligned}} \; \Bigg| \; y(x,t) = y_-(x,t) + y_+(x,t) = \boxed{2\hat{y}\sin(kx)}\cos(\omega t)$$

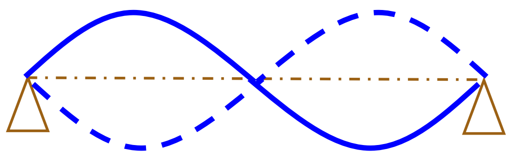
Two travelling waves of same frequency, type and fixed phase relation propagating in opposite directions

Position-dependent amplitude oscillating according to cos(ωt)

Standing waves in a string: resonances & eigenmodes



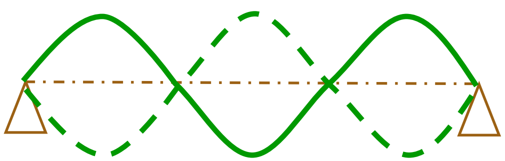
$\lambda=2L$
$f_1=v/2L$
Fundamental eigenfrequency / 1st harmonic



$\lambda=L$
$f_2=2f_1$
Second eigenfrequency / 2nd harmonic

In general:

$\lambda=2L/n$
$f_n=n\cdot v/2L$



$\lambda=(2/3)L$
$f_3=3f_1$
Third eigenfrequency / 3rd harmonic

Eigenmode: different ways a string (structure in general) can vibrate generating standing waves

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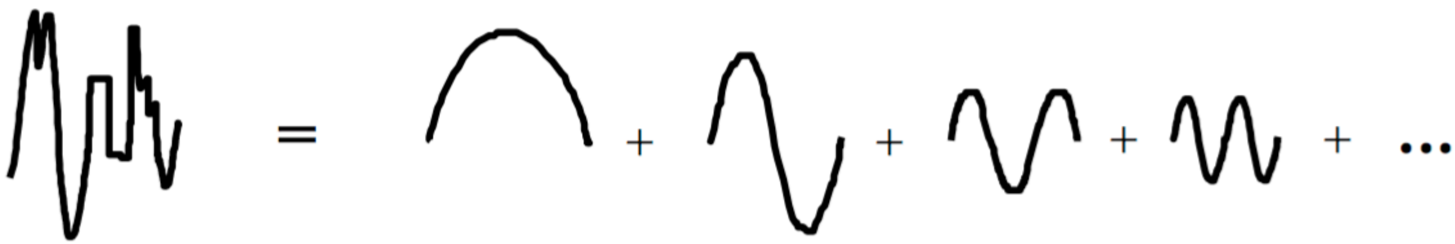
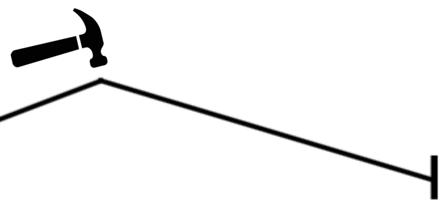
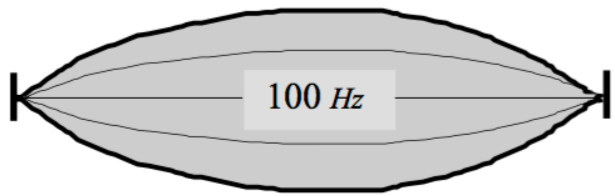
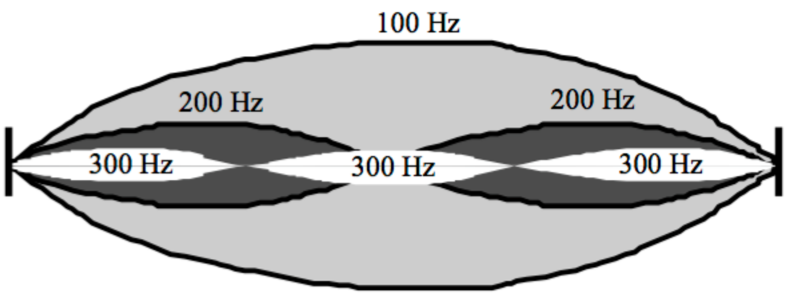
Standing waves and higher harmonics (I)

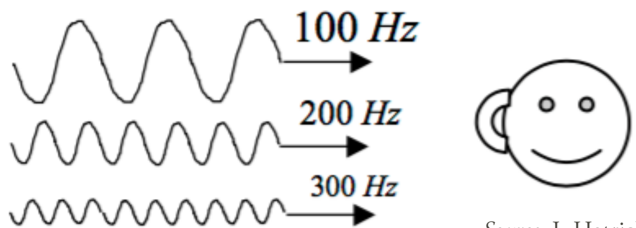


Any motion = sum of motion of all the harmonics









Source: J. Hetricks

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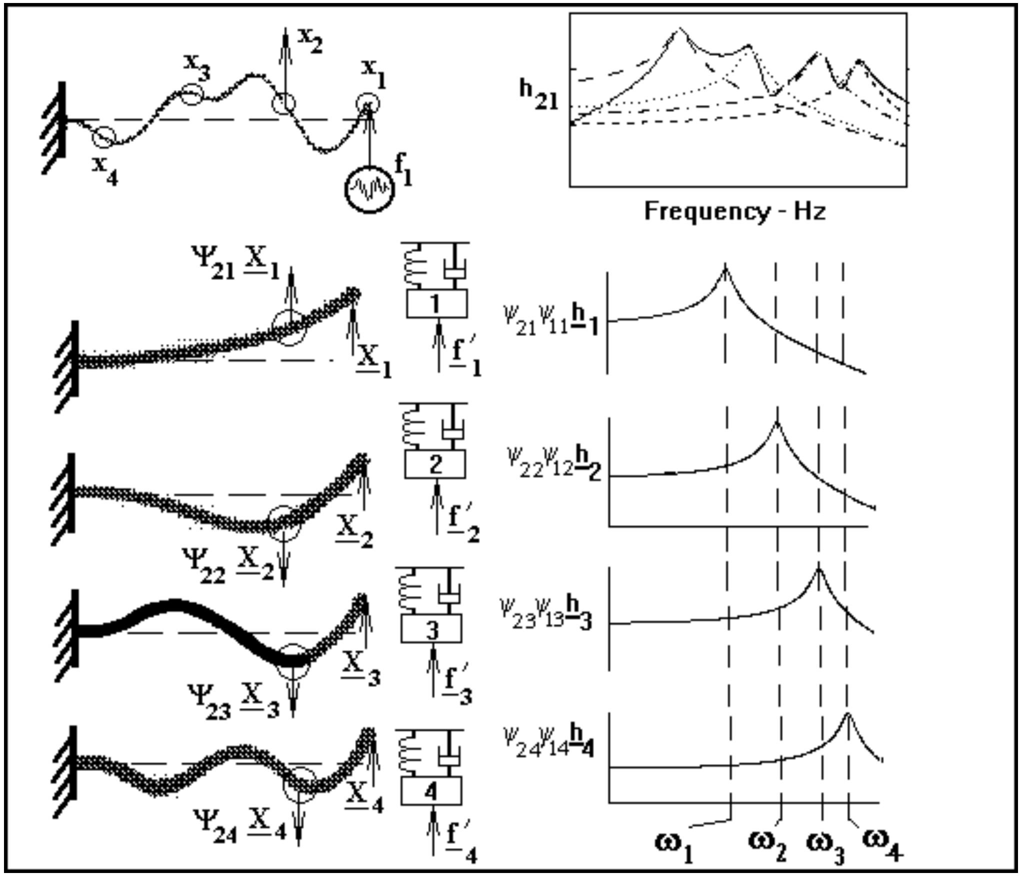
Standing waves and higher harmonics (II)

Any motion = sum of motion of all the harmonics



Source: http://signalysis.com

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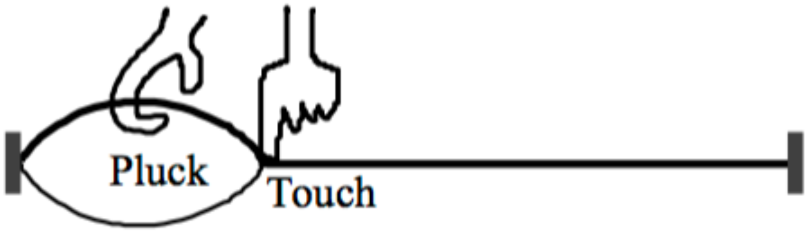
Music instruments: string (e.g. violin)

$\lambda=2L/n$

$f_n=n\cdot v/2L$

$v=(\text{tension}/\text{mass-length})^{1/2}$

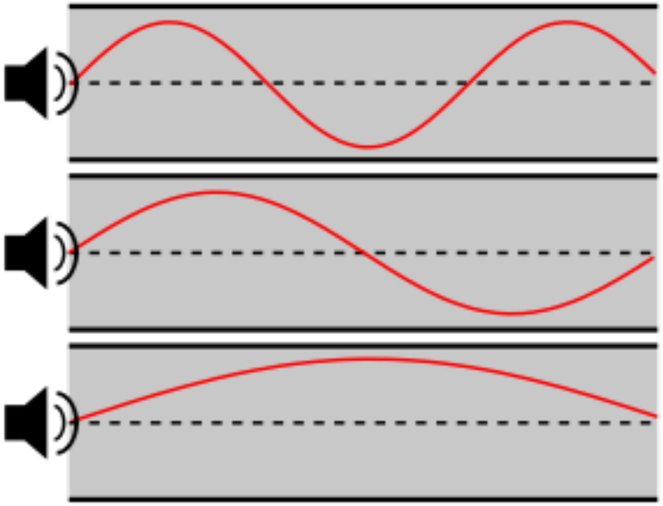


When ones plays → Changes L

Knobs (tune) → Vary tension

To discuss: Piano?

Music instruments: wood-wind



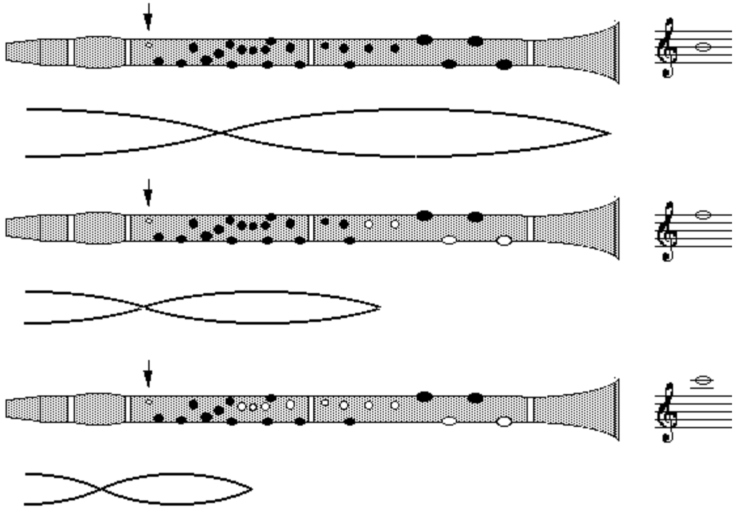
Open-open / Closed-closed:

$\lambda=2L/n$

$f_n=n\cdot v/2L$

NOTE: Open-closed vary

$v=(\text{temperature}/\text{molecular weight})^{1/2}$



Change of v (molecular weight)

Cover holes → Vary L

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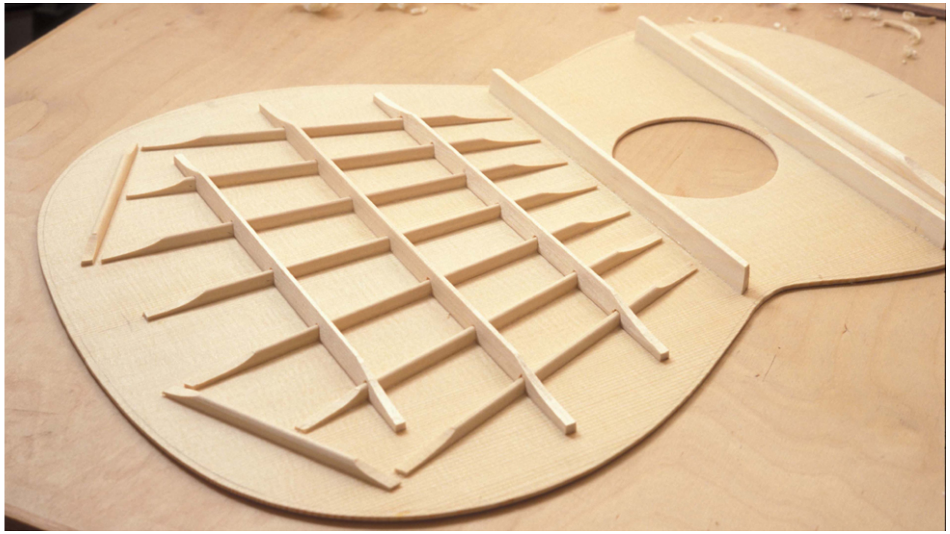
Music instruments: soundboards

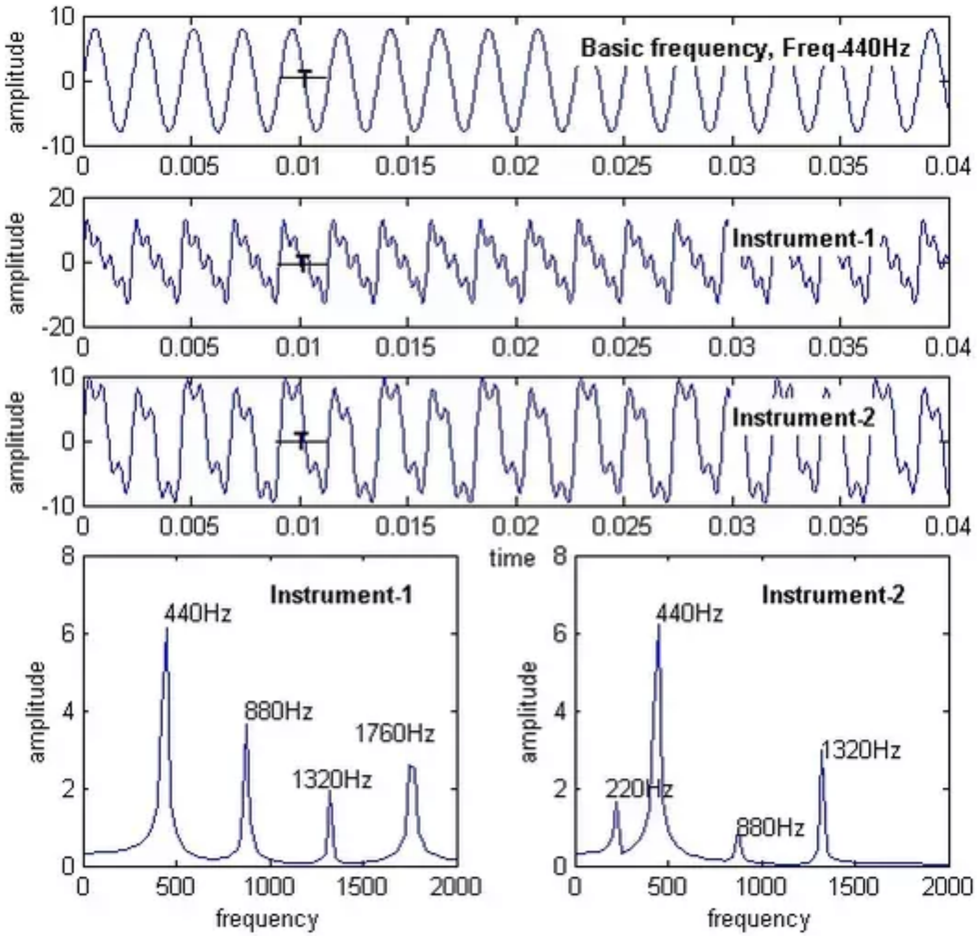


Standing waves and timbre (I)

- Characteristics of sound:
  - Loudness (amplitude)
  - Pitch (frequency)
  - Quality or Timbre
    - » "Cocktail" characteristic of every instrument/source
    - » Different combination of higher harmonics
    - » What makes us distinguish one instrument from another

- To discuss: helium & voice
  - Change of molecular weight (i.e. v) → natural frequency goes up

Standing waves and timbre (II)



Source: Quora

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Summary

- Wave propagation in solid media
- Wave equation solution
- Wave propagation in fluid media
- Wave phenomena
  - Interference (constructive/destructive)
  - Standing waves
    - » Resonances
    - » Eigenmodes
- How musical instruments work
  - → Read: T.E.Vigran, Building Acoustics - Ch. 3

REFERENCES: Animations retrieved from Dan Russell's website

Thank you for your attention!

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