An introduction to Statistical Energy Analysis

LUND UNIVERSITY

BREKKE STRAND

Guest lecture

11/12/19

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Myself

- 2002-2005 BSc in Telecom Eng, Politecnico Milano
  - voice activity detector GSM networks, Siemens
- 2005-2008 MSc in Sound Eng and Design, Politecnico
- 2006-2008 MSc in Sound and Vibration, Chalmers
  - parametric array loudspeakers
- 2008-2013 PhD in Technical Acoustics, KTH
  - SEA, porous materials, variational principles

Myself

- 2013-2016, Consultant, Lloyd’s Register Consulting
  - N&V; marine sector
- 2016-2017, Consultant, Creo Dynamics
  - N&V; R&D, ultrasonics, nonlinear acoustics, Volvo cars.
- 2017-2019, Consultant, WSP
  - N&V; urban and industrial sectors.
- 2019 – today, Consultant, Brekke & Strand (Malmö)

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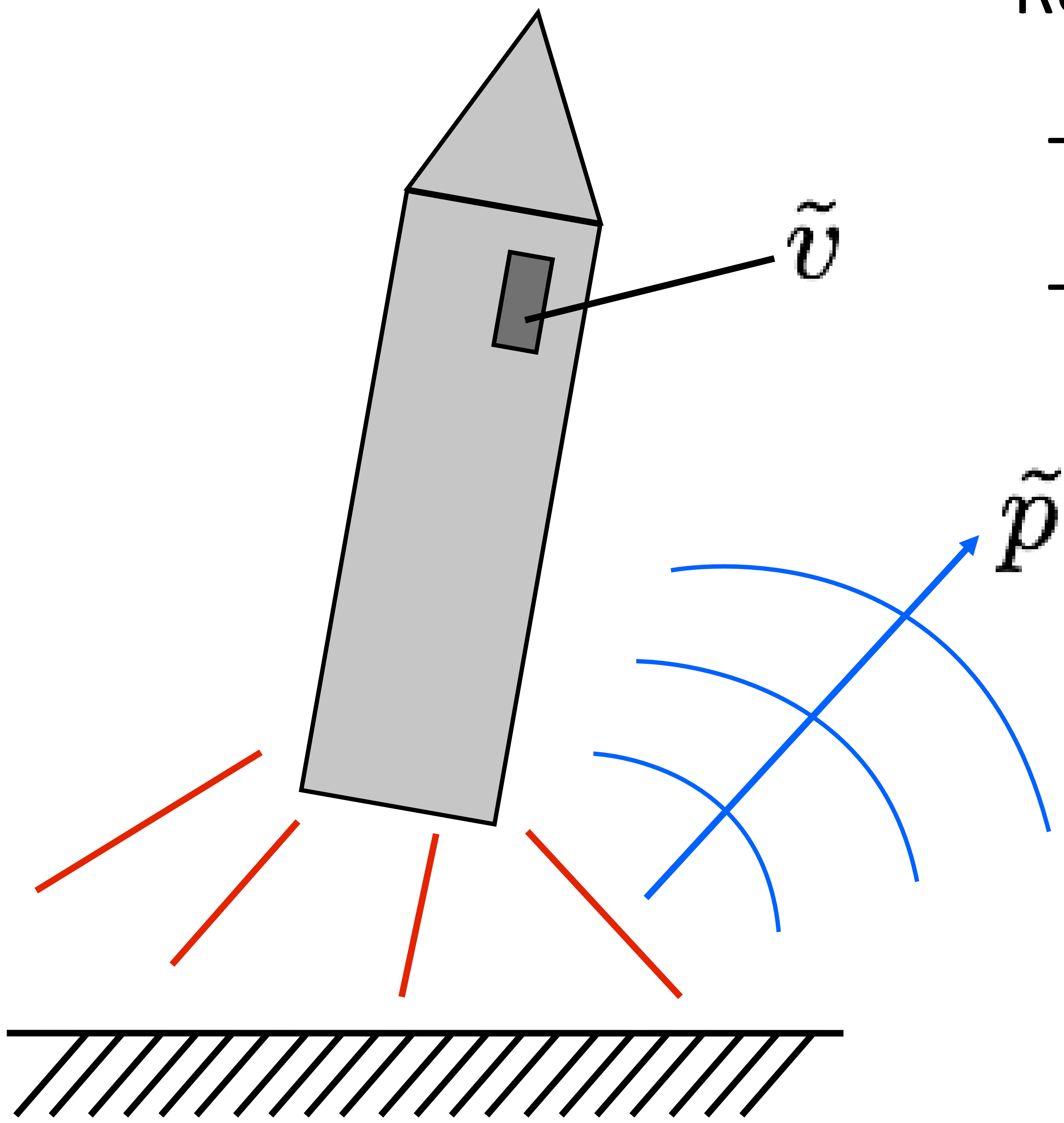
This lecture

- Material produced at KTH
  - internal courses
  - seminars/lectures within Mid-Mod EU project
  - together with my supervisor Svante Finnveden (former professor at KTH, now at ACAD)

Motivation



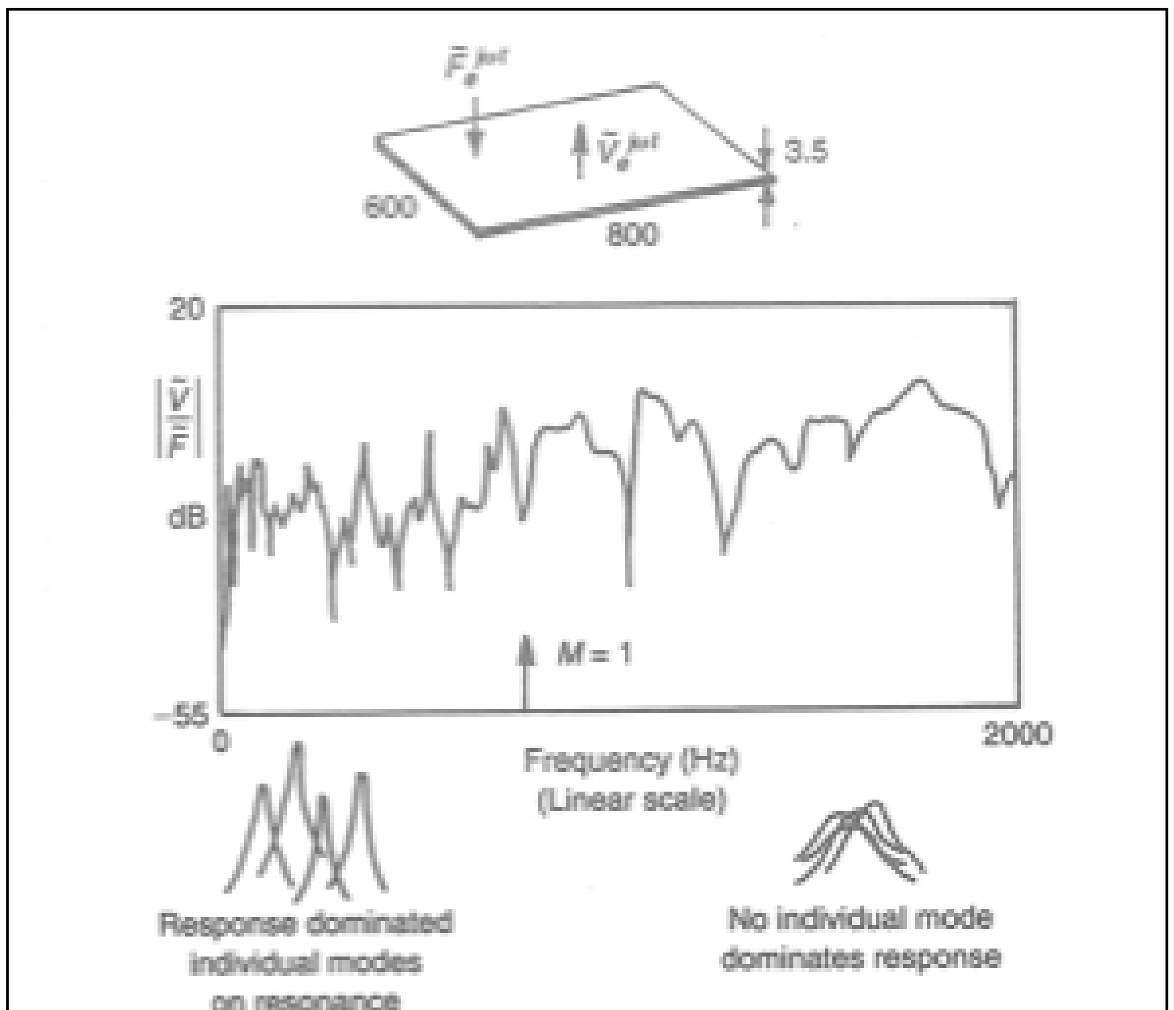
- Rocket to the moon (Smith, 1962)
  - some 200k modes
  - modelling effort

Motivation

- Vibroacoustic predictions
  - low frequency: measurements, FEM
  - high frequency: today's topic
- Definition of high frequency

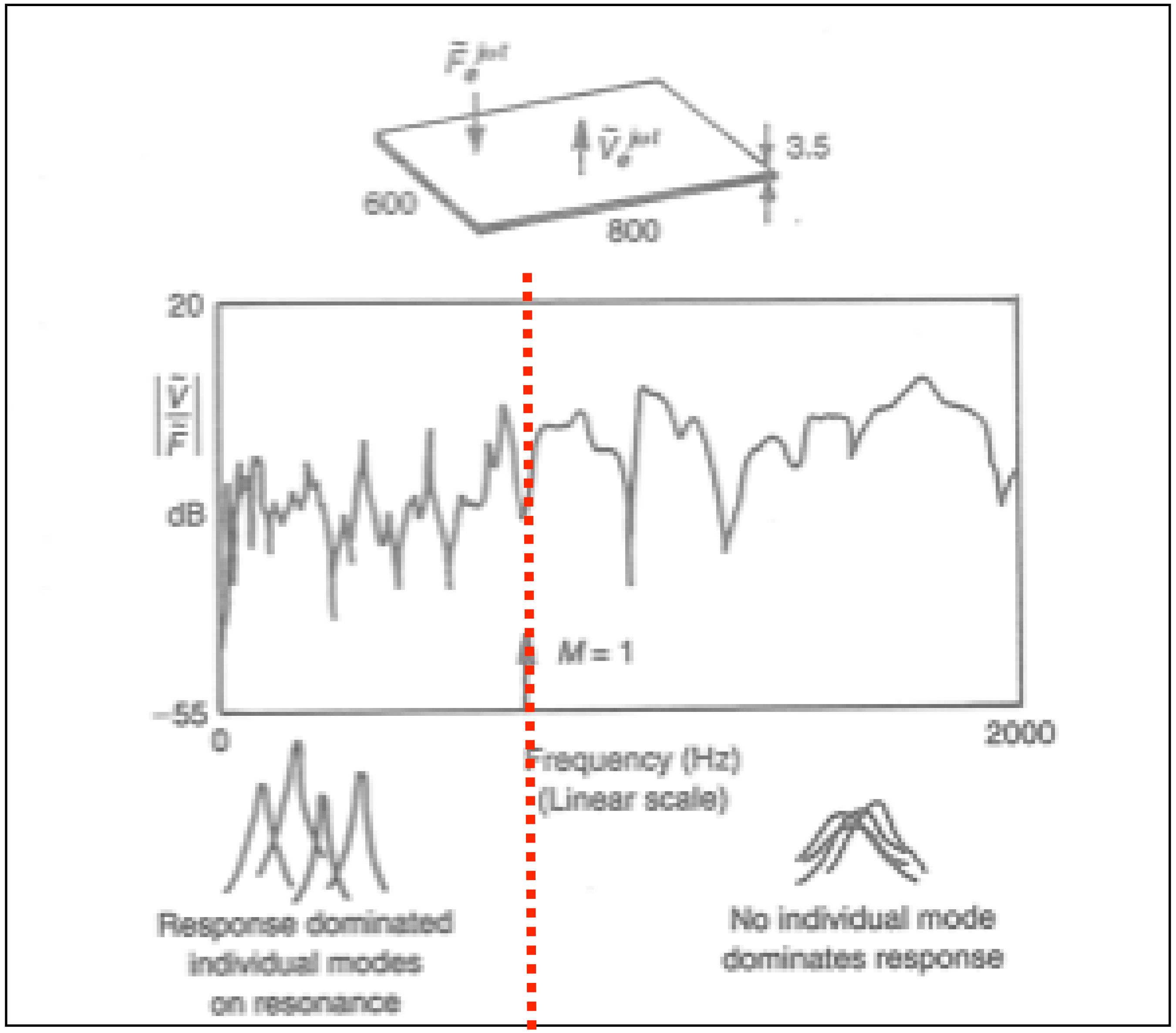
Motivation

- Broadband excitation of a structure
- Not too high damping ⇒ reverberant field
- Peaks, natural frequencies ⇔ modes



Fahy, Gardonio

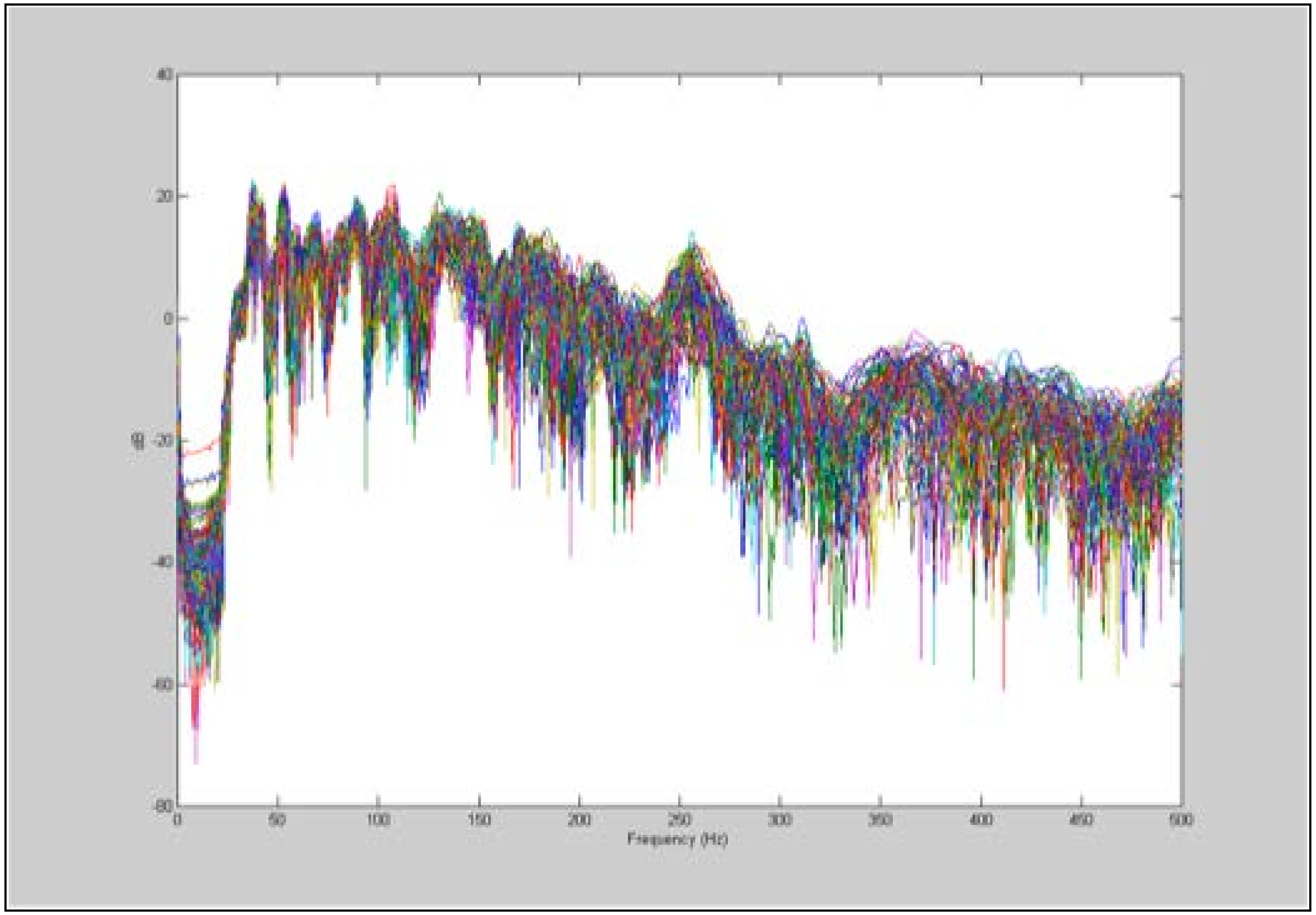
Motivation



- Clear peaks
- Peaks characteristic of the structure

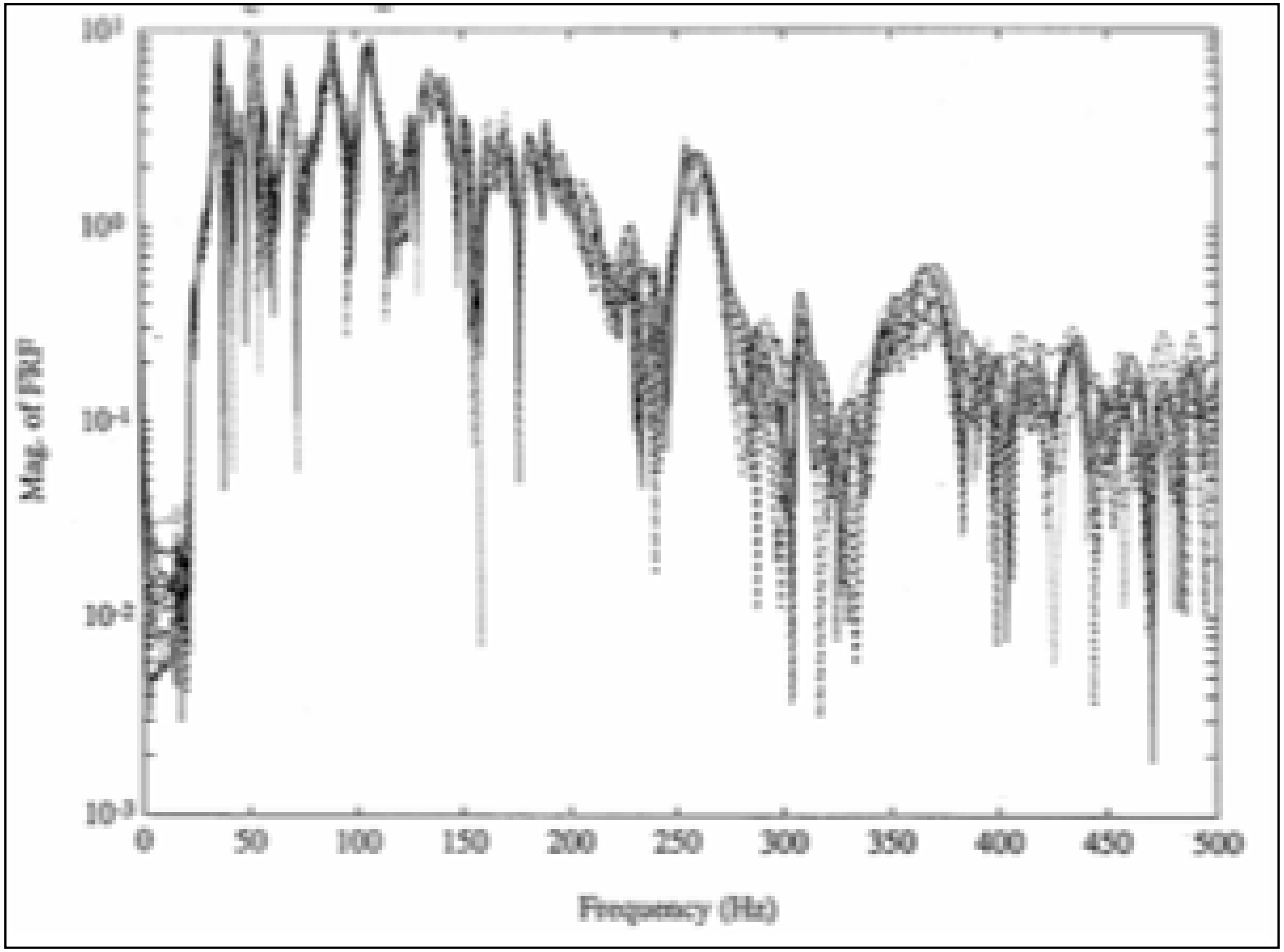
- Not clear peaks
- Peaks dependent on small geometric variations, discontinuities

Motivation



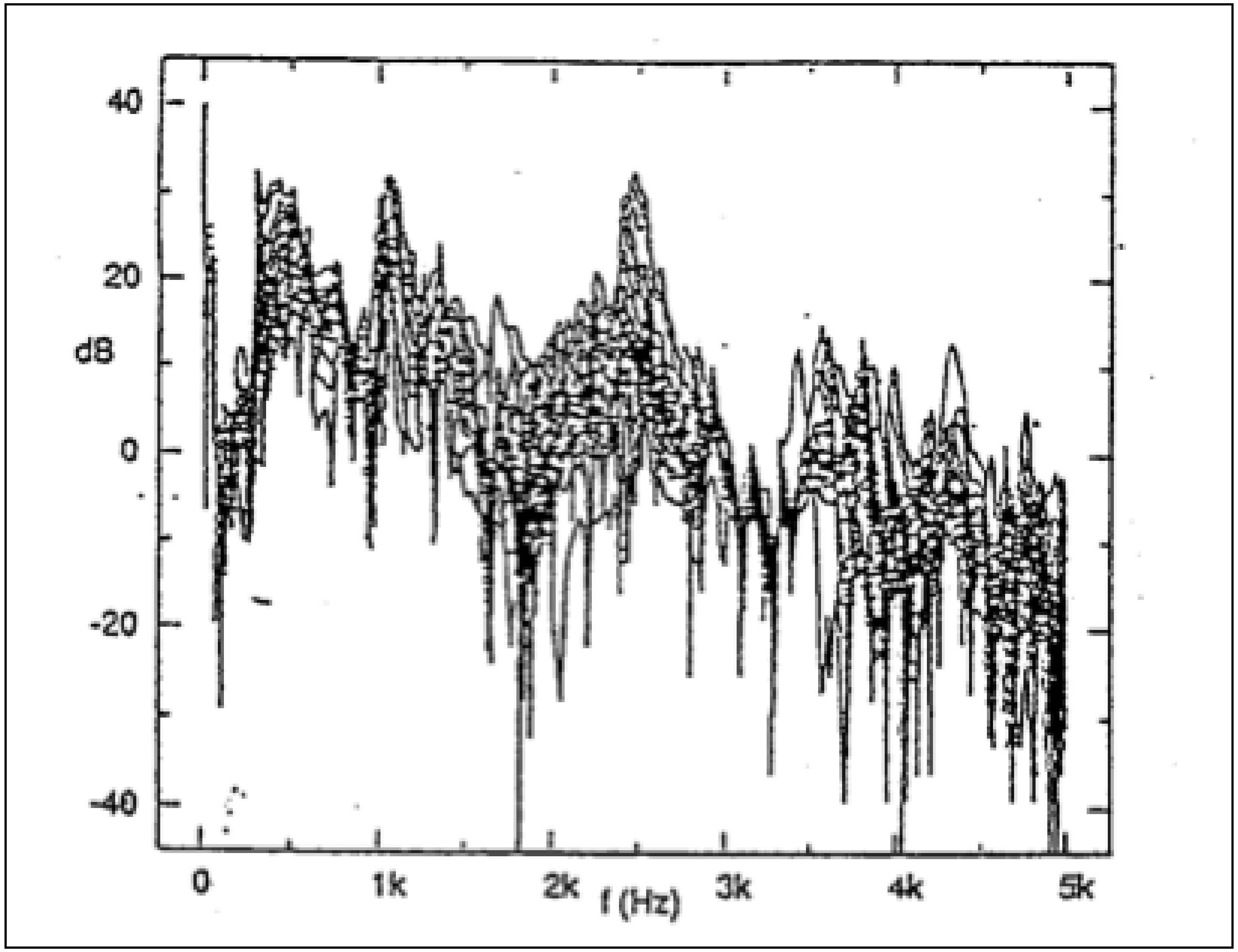
- Magnitudes of the structure-borne frequency response functions of 98 nominally identically cars (Isuzu Rodeo); force applied on the wheel hub, microphone at the driver's seat. (Kompella, Bernhard)

Motivation



- 12 measurements on the same car (Kompella, Bernhard)

Motivation



- Vibroacoustic responses of 141 identical beer cans (Fahy)

Motivation

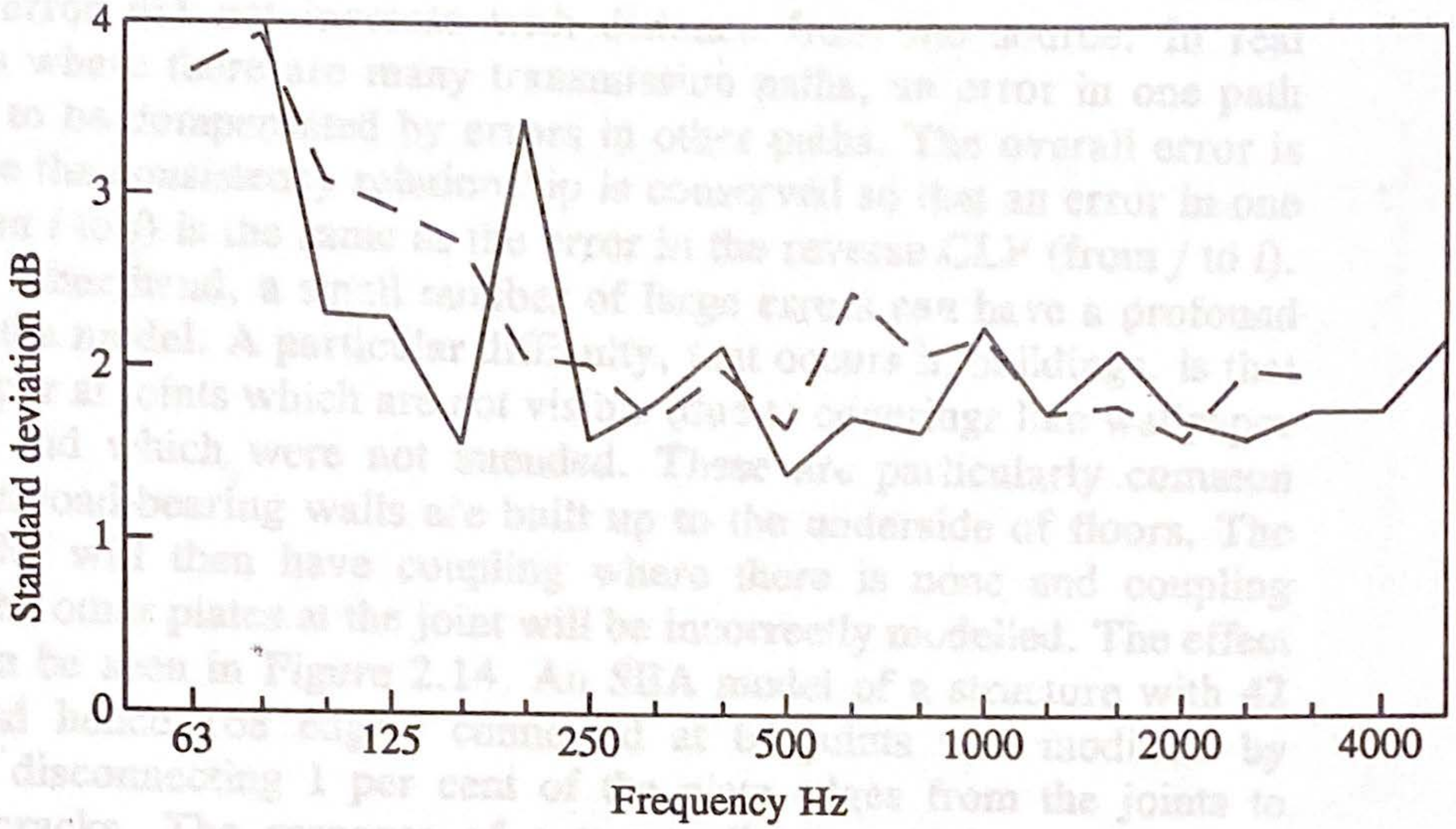


Figure 2.12 Standard deviation of ten measurements of airborne transmission through a concrete floor and ten measurements of structural transmission between a wall and a floor. ———, airborne transmission; — — — —, structure-borne transmission (Craik and Steel, 1989; Craik and Evans, 1989)

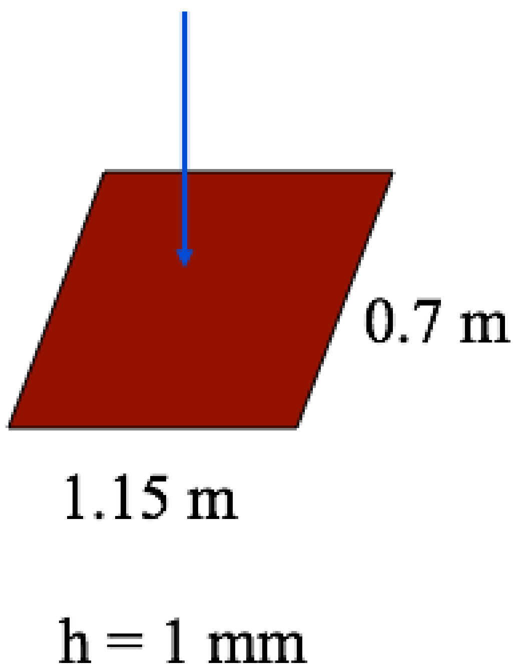
- Vibroacoustic responses of *identical* building elements in same building

Motivation - What did we see?

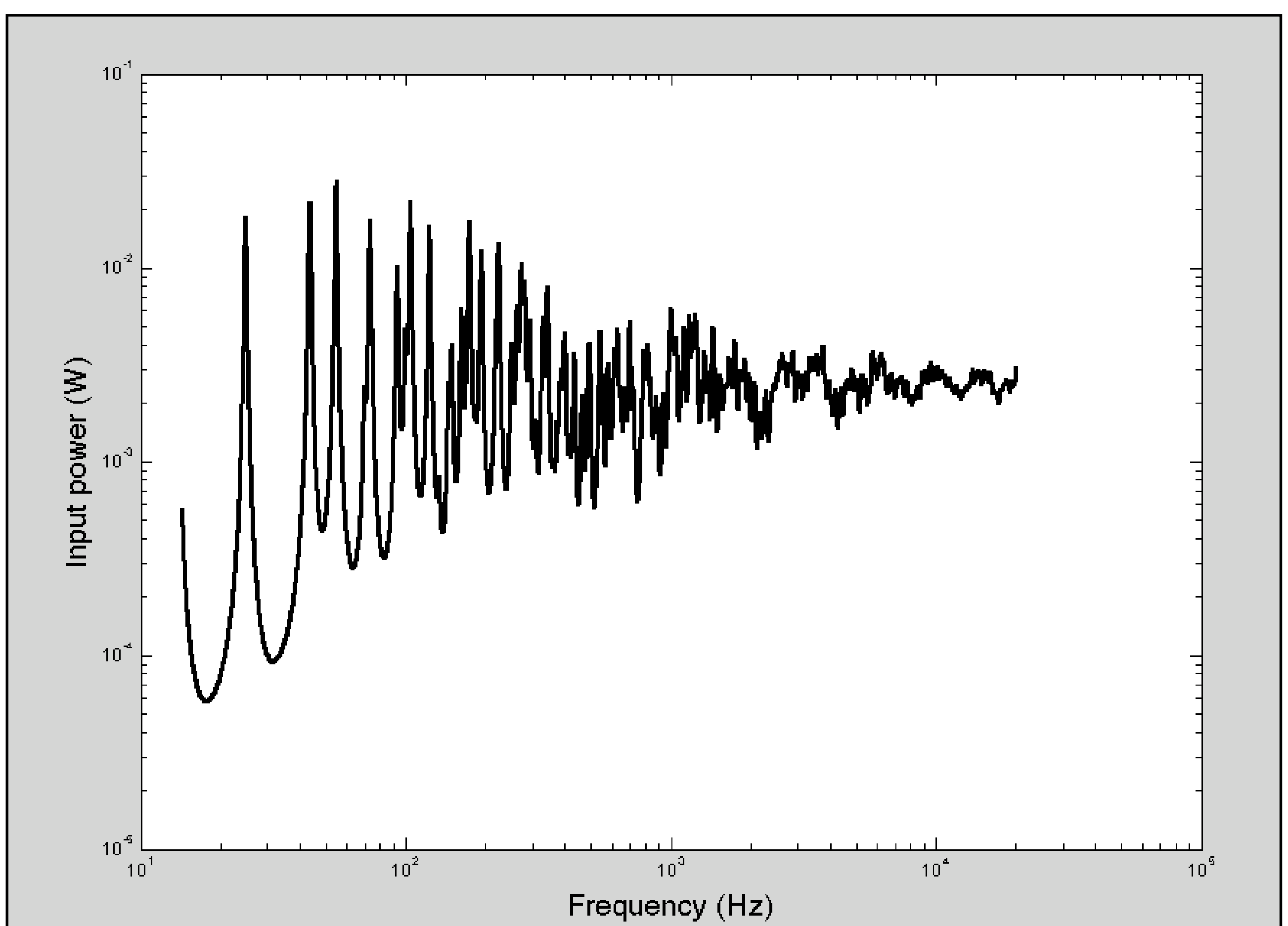
- What is changing between various cars from the same production line?
- What is changing between measurements on the same car?

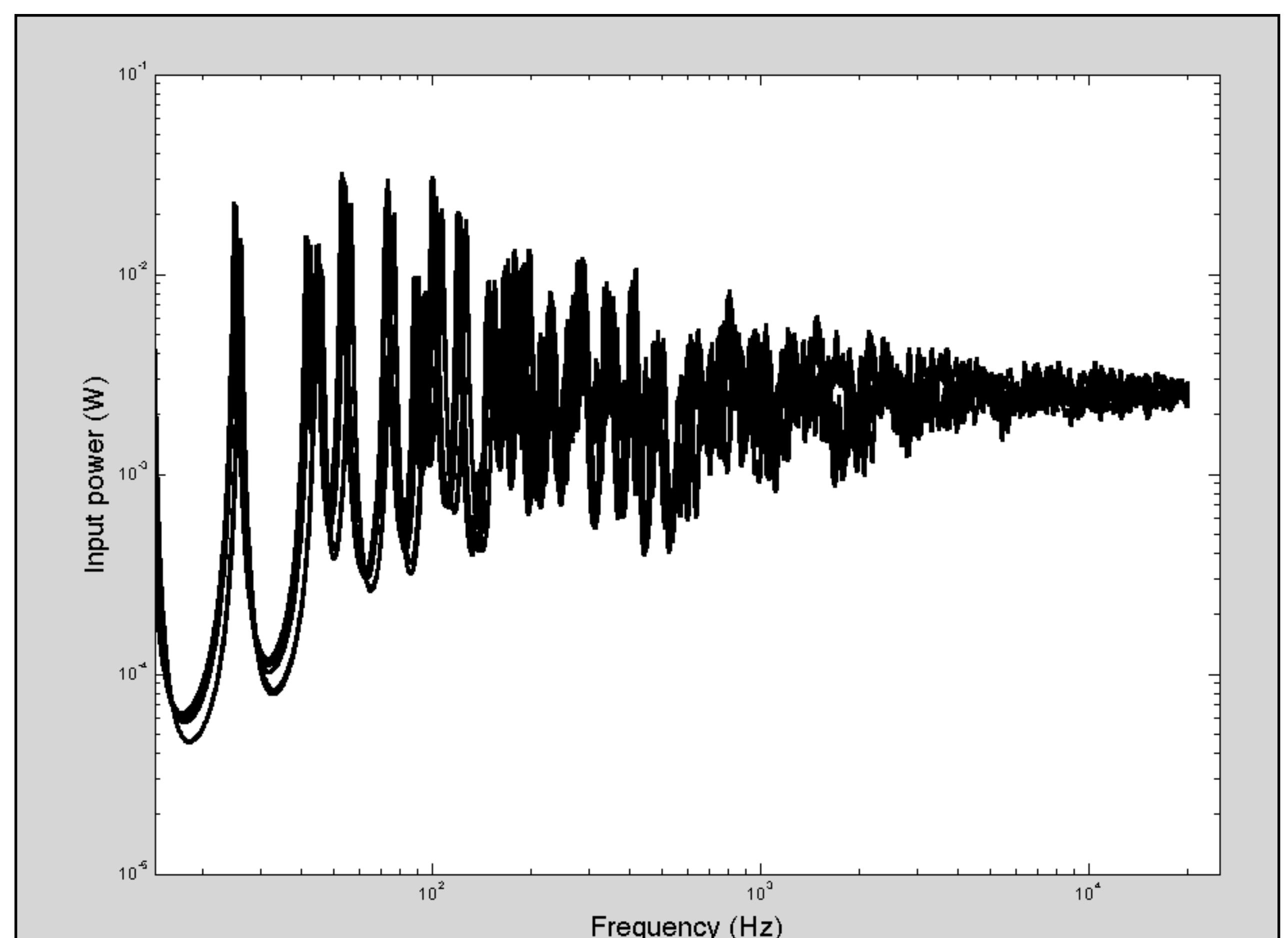
- We may have a feeling about what high frequency means
- We may have a measure for this

Motivation



- Input power to a plate with nominal dimensions (left), to 8 plates with nominal dimensional + std(0.02) on h (right)





Motivation - What can we say?

- Two problems with high frequency models:
  - Technical: small mesh compared to wavelength (FE)
  - profound: chaotic response
    - deterministic;
    - apparently random;
    - unpredictable.

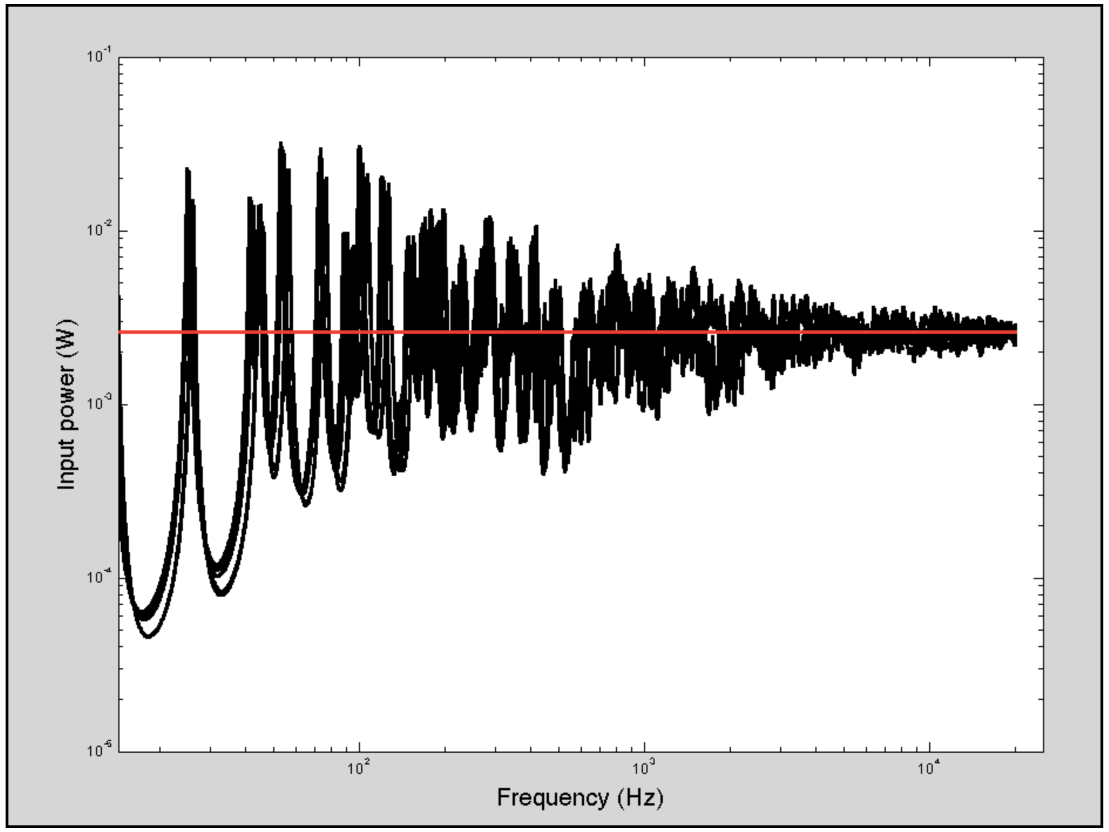
Motivation - What can we say?

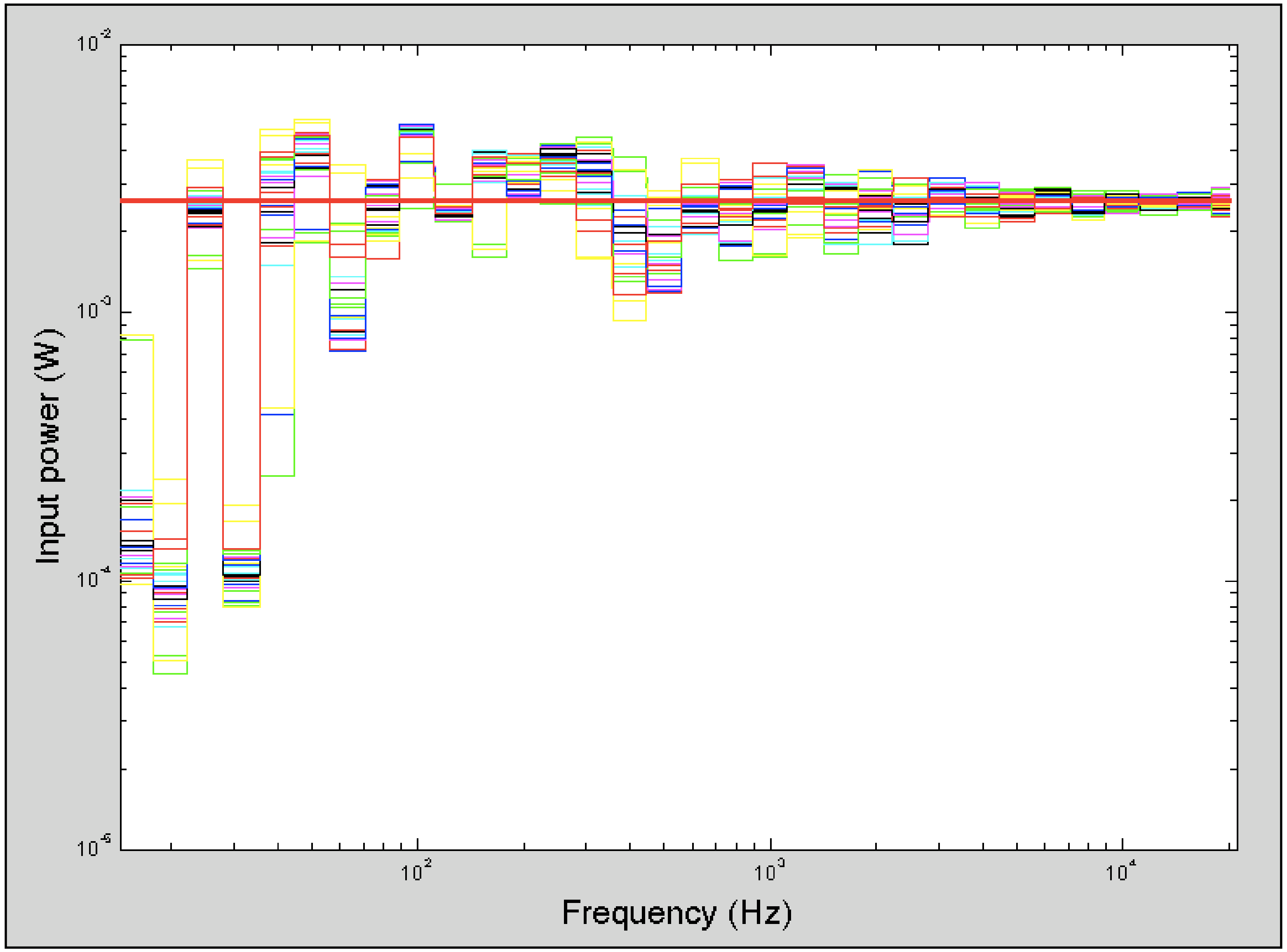
- Useful approach - i.e. not the only but a useful one for specific needs:
  - quick and efficient
  - accounting for statistical variation
  - employing a measure resistant to variations

Answer

- Statistical Energy Analysis
  - statistical
  - energy
  - analysis
  - **S**eems **E**asy, **A**in't
- Frequency, space, ensemble average response

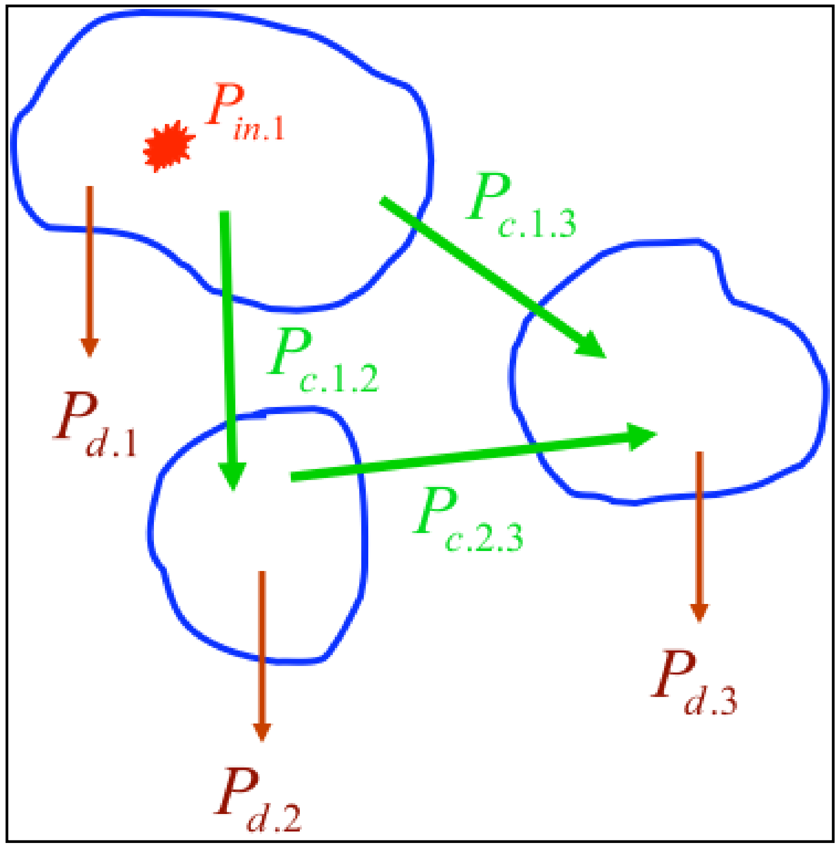
Answer

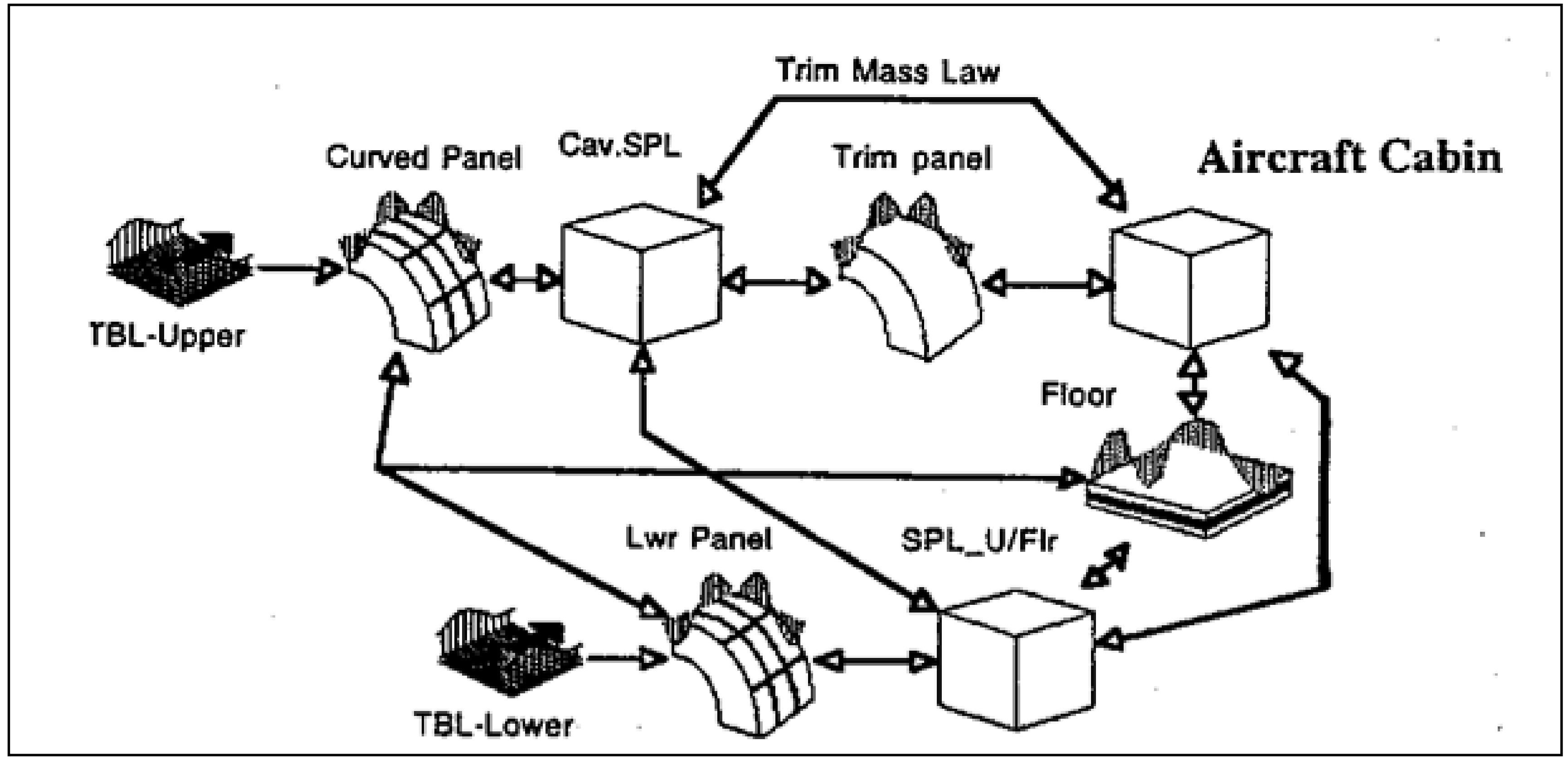




- Input power to 8 plates with nominal dimension + std(0.02) (black); SEA prediction (red)

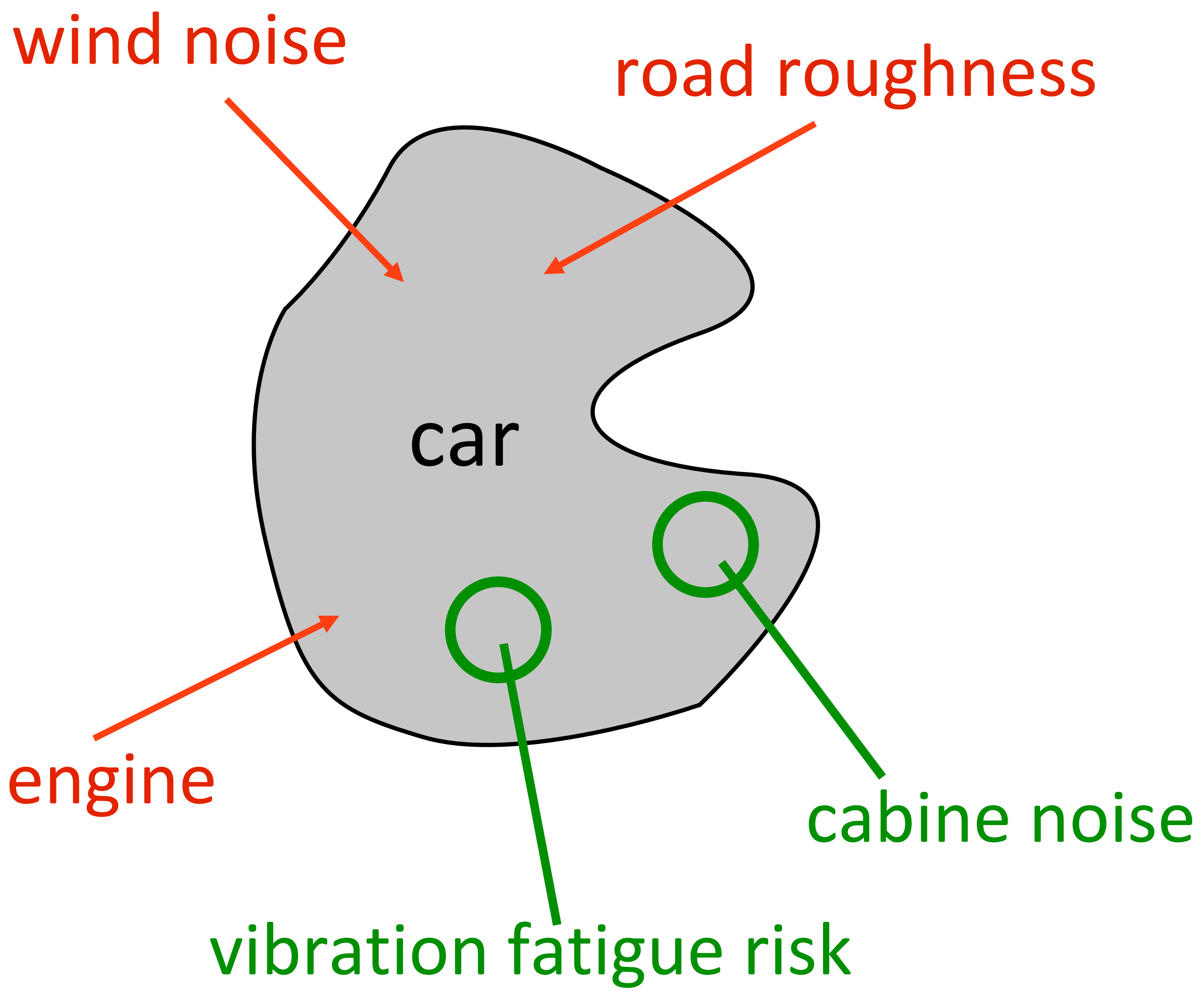
Formulation





nowadays models may look like CAD drawings...

Formulation: 0th step



- To start
  - a structure
  - identify sources
  - linear motion

Formulation: 1st Step

- Subdivide structure/vibroacoustic field into SEA Elements

  - elements may be substructures
  - elements may be elements of the response
  - elements are quite large
  - very difficult task!

Roof bending	Engine
Roof longitudinal	Cabin air volume
⋮	⋮

Formulation: 2nd step

- Power balance for element i

$$P_{in}^{i} = P_{diss}^{i} + \sum_{j \neq i} P_{coup}^{ij}$$

- SEA built upon conservation of energy!
- Now we need to define the terms in the summation

Formulation: 3rd step

- Dissipated power

$$P^i_{diss} = \eta\omega E$$

- η damping loss factor
- ω angular frequency
- E total vibroacoustic energy
- nothing really new (Lord Rayleigh)

Formulation: 4th step

- Coupling power

$$P_{coup}^{ij} \propto \left(\frac{E_i}{\Delta N_i} - \frac{E_j}{\Delta N_j} \right)$$

  - ΔN_i number of modes in a band
  - introduce:
    - $n_i = \dfrac{\Delta N_i}{\Delta \omega}$ modal density [1/rad/s]
    - $\hat{e}_i = \dfrac{E_i}{n_i}$ modal energy (power!) [W]
    - $C^{ij} = \omega n_i \eta_{ij} = \omega n_j \eta_{ji} = C^{ji}$ conductivity [-]

$$\boxed{P_{coup}^{ij} = C^{ij} \left(\hat{e}_i - \hat{e}_i \right)}$$

Formulation: 5th Step

Modal power potentials $\hat{e}_i = E_i / n_i$ [W]

Dissipated Power $P^i_{diss} = M_i \, \hat{e}_i$ [W]

Modal overlap factor $M_i = \eta_i \, \omega \, n_i$ [-]

Coupling power $P^{1,2}_{coup} = C(\hat{e}_1 - \hat{e}_2)$ [W]

C – Conductivity [-]; n – modal density [1/(rad/s)]; η - loss factor [-].

Power balance: $P_{dis} + P_{coup} = P_{in}$:

$$\begin{bmatrix} M_1 + \sum_j C^{1j} & -C^{12} & -C^{13} & \cdots \\ -C^{12} & M_2 + \sum_j C^{2j} & -C^{23} & \cdots \\ \vdots & \vdots & \ddots & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \vdots \\ \end{bmatrix} = \begin{bmatrix} P^1_{in} \\ P^2_{in} \\ \vdots \\ \end{bmatrix}.$$

Linear system of equations

SEA quantities: Energy

- Energy as dependent variable, unknown
- Frequency, space, ensemble averaged energies (ergodic assumption)
- SEA deals with:
  - resonant motion
  - reverberant wave motion
- Strain energy (density) = Kinetic energy (density)
- Energy density <u>constant</u> across the element

SEA quantities: Energy

$$L_W = 10 \log \frac{W}{W_{ref}};$$ $\longrightarrow$ power

$$L_v = 10 \log \frac{\tilde{v}^2}{v_{ref}^2};$$ $\longrightarrow$ measure of kinetic energy

$$L_p = 10 \log \frac{\tilde{p}^2}{p_{ref}^2}$$ $\longrightarrow$ measure of strain energy

- Retrieve vibroacoustic quantities!

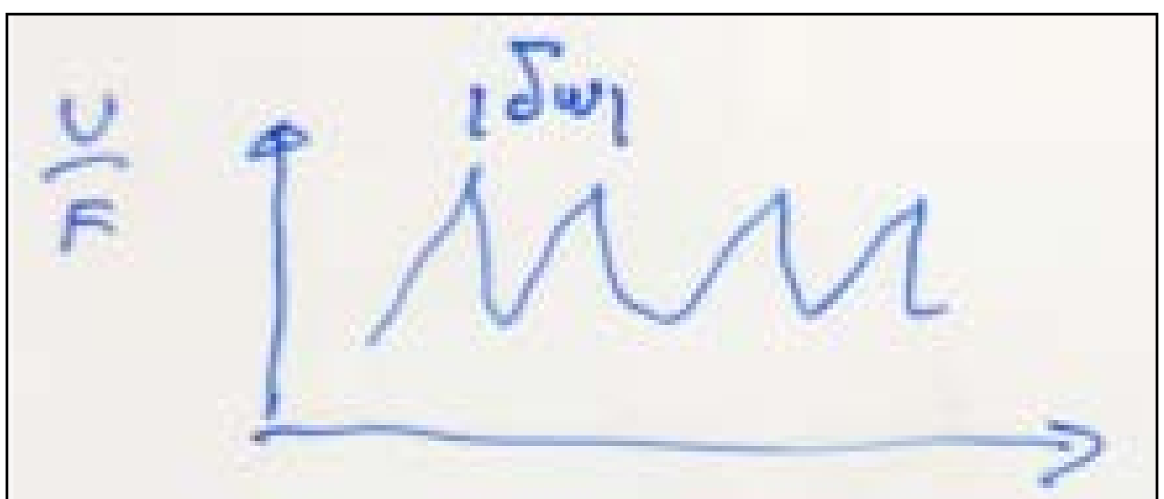
$$E = e \cdot V; \quad E = e \cdot S; \quad E = e \cdot L$$

$$e \propto \langle \tilde{p}^2 \rangle; \quad e \propto \langle \tilde{v}^2 \rangle$$ $\longrightarrow$ <> space, frequency, ensemble average!

SEA quantities: modal density

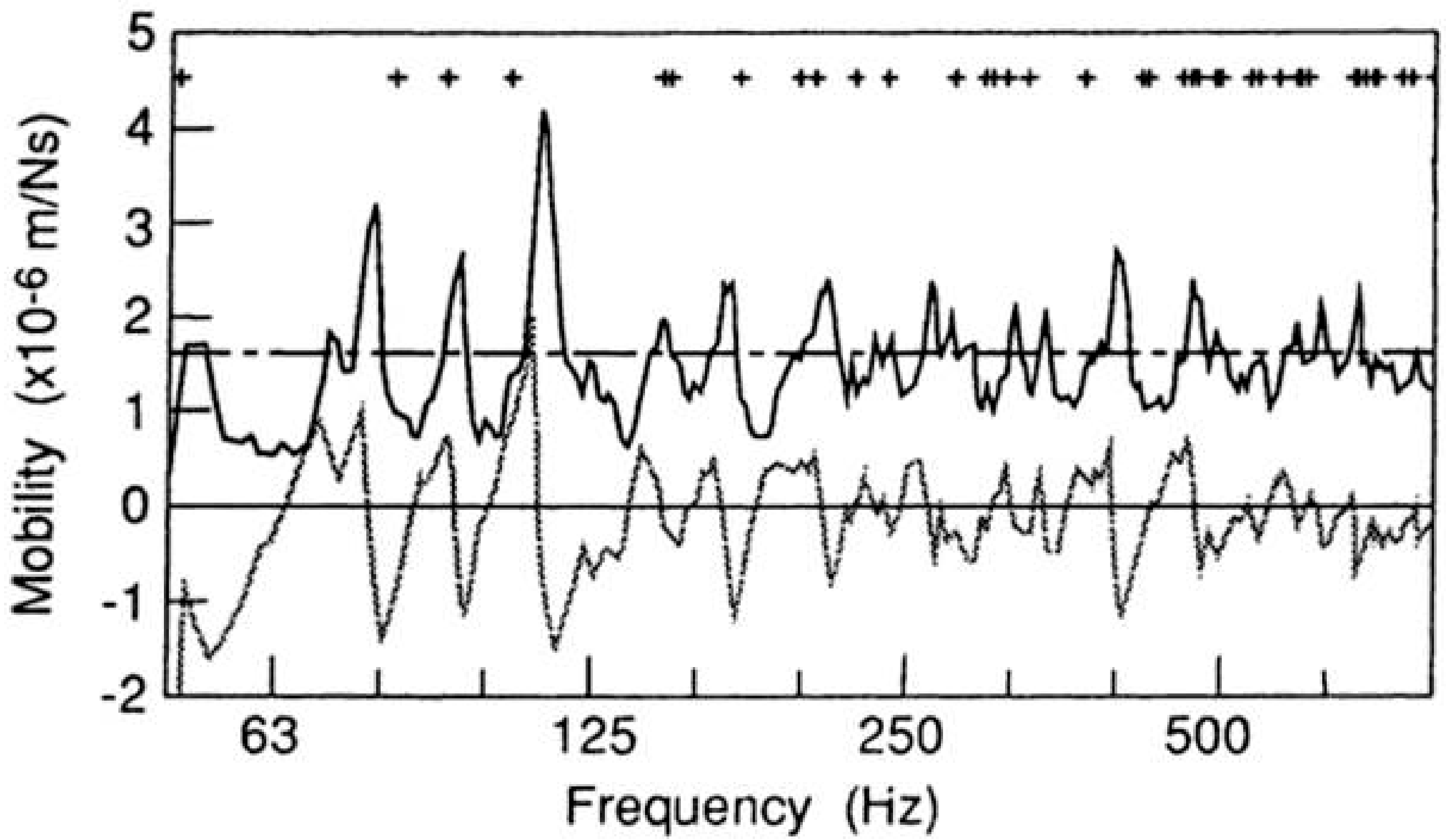
- Modal density: *expected* number of resonance per unit frequency

$$n_i = \frac{\Delta N_i}{\Delta \omega}; \qquad n = \frac{1}{\delta \omega}$$



- *expected* difference between natural frequency [rad/s]

SEA quantities: modal density



Real part of mobility for statistic ie diffuse conditions:

$$\mathrm{Re}\{Y\} = \frac{\pi}{2M}\frac{\Delta N}{\Delta\omega}$$

Figure 5.63 Measured mobility of a 6 × 4 m^2 × 200 mm thick concrete floor: —— real part; ······ imaginary part; – - – theoretical infinite plate mobility (real); + predicted modal resonance frequencies.

Craik, Sound transmission through buildings using statistical energy analysis
Fahy, Noise and vibration

SEA quantities: modal density

- Statistics implies little/no difference between various boundary conditions and/or geometries!
- Infinite structures just as good for some applications.

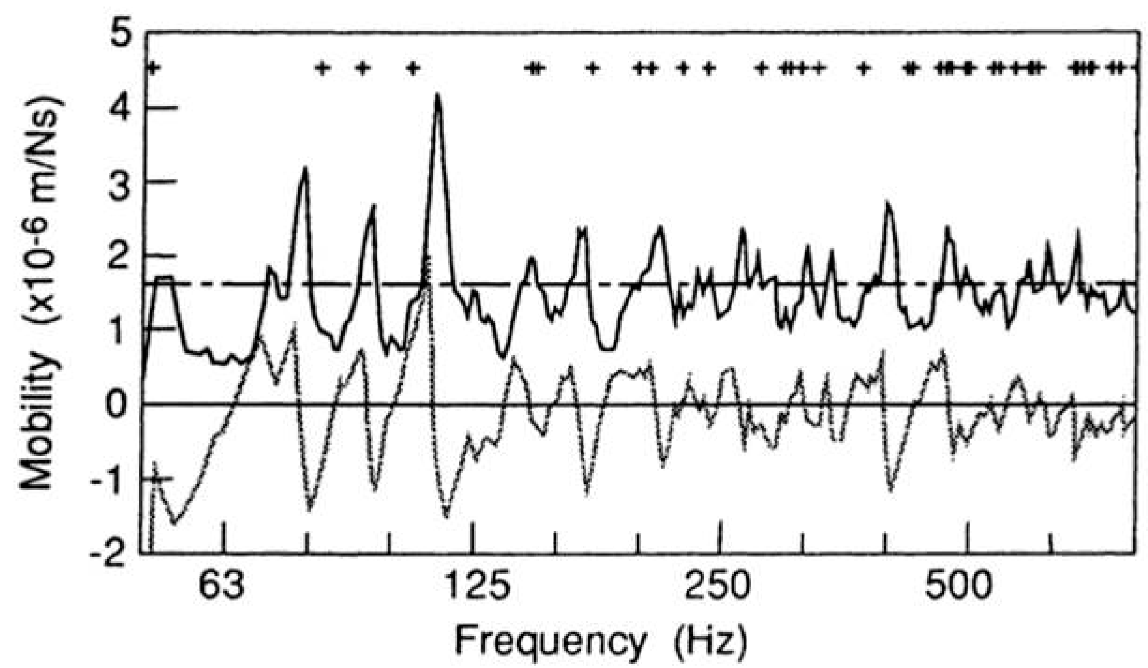


Figure 5.63 Measured mobility of a 6 × 4 m^2 × 200 mm thick concrete floor: —— real part; ······ imaginary part; – – – theoretical infinite plate mobility (real); + predicted modal resonance frequencies.

SEA quantities: modal density

Table 5.3. Number of modes and modal densities

Rod (longitudinal)	$N = k_L l/\pi = \omega l/(c_L \pi)$	$\Delta N/\Delta\omega = l/(c_L\pi)$
Beam (bending)	$N = k_B l/\pi = \sqrt{\omega} l/\left(1.7\sqrt{c_L h}\right)$	$\Delta N/\Delta\omega = k_B\, l/(2\pi\omega)$ $= l/\left(3.4\sqrt{\omega c_L h}\right)$
Plate (bending)	$N = k_B^2 S/4\pi = \omega S/(3.6 c_L h)$	$\Delta N/\Delta\omega = k_B^2\, S/(4\pi\omega)$ $= S/(3.6 c_L h)$
Acoustic volume	$N = k_0^3 V/6\pi^2 = \omega^3 V/\left(6\pi^2 c_0^3\right)$	$\Delta N/\Delta\omega = k_0^2\, V/\left(2\pi^2 c_0\right)$ $= \omega^2 V/\left(2\pi^2 c_0^3\right)$
Ring (radial)	$N = 2k_B a = 3.7\sqrt{\omega}\, a/\sqrt{c_L h}$	$\Delta N/\Delta\omega = k_B\, a/\omega$ $= 1.9a/\sqrt{\omega c_L h}$
Thin-walled cylindrical shell, $\nu < 1$	$N \approx 3\sqrt{3} l\nu^{3/2}/(2\pi h)$	$\Delta N/\Delta\omega \approx 2\sqrt{\omega}\, l a^{3/2}/\left(1.6 h c_L^{3/2}\right)$
Thin-walled cylindrical shell, $\nu > 1$	$N \approx \sqrt{3} l a\omega/(c_L h)$	$\Delta N/\Delta\omega \approx \sqrt{3} l a/(c_L h)$

k_L, c_L: wavenumber and sound speed for longitudinal wave,
k_0, c_0: wavenumber and sound speed for airborne sound,
k_B, c_B: wavenumber and sound speed for bending wave,
l length, h thickness, S area, V volume, a radius and $\nu = \omega a/c_L$.

Cremer, Heckl, Pettersson, Structure-borne sound

SEA quantities: modal overlap factor

- Modal overlap factor

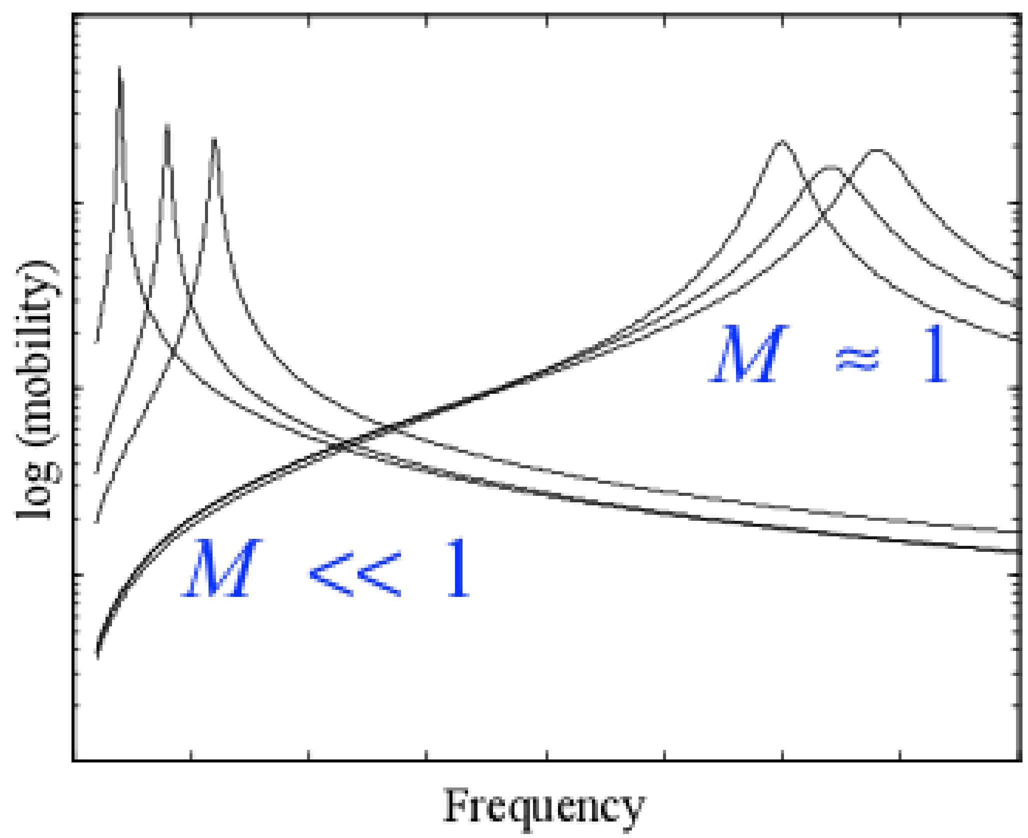
$$P^i_{diss} = \eta\omega E = M\hat{e}$$

$$M = \eta\omega n; \qquad M = \frac{\eta\omega}{\delta\omega}$$

- quantifies dissipation
- quantifies probability of a resonance

SEA quantities: modal overlap factor

Harmonic forced response of MDOF and continuous systems



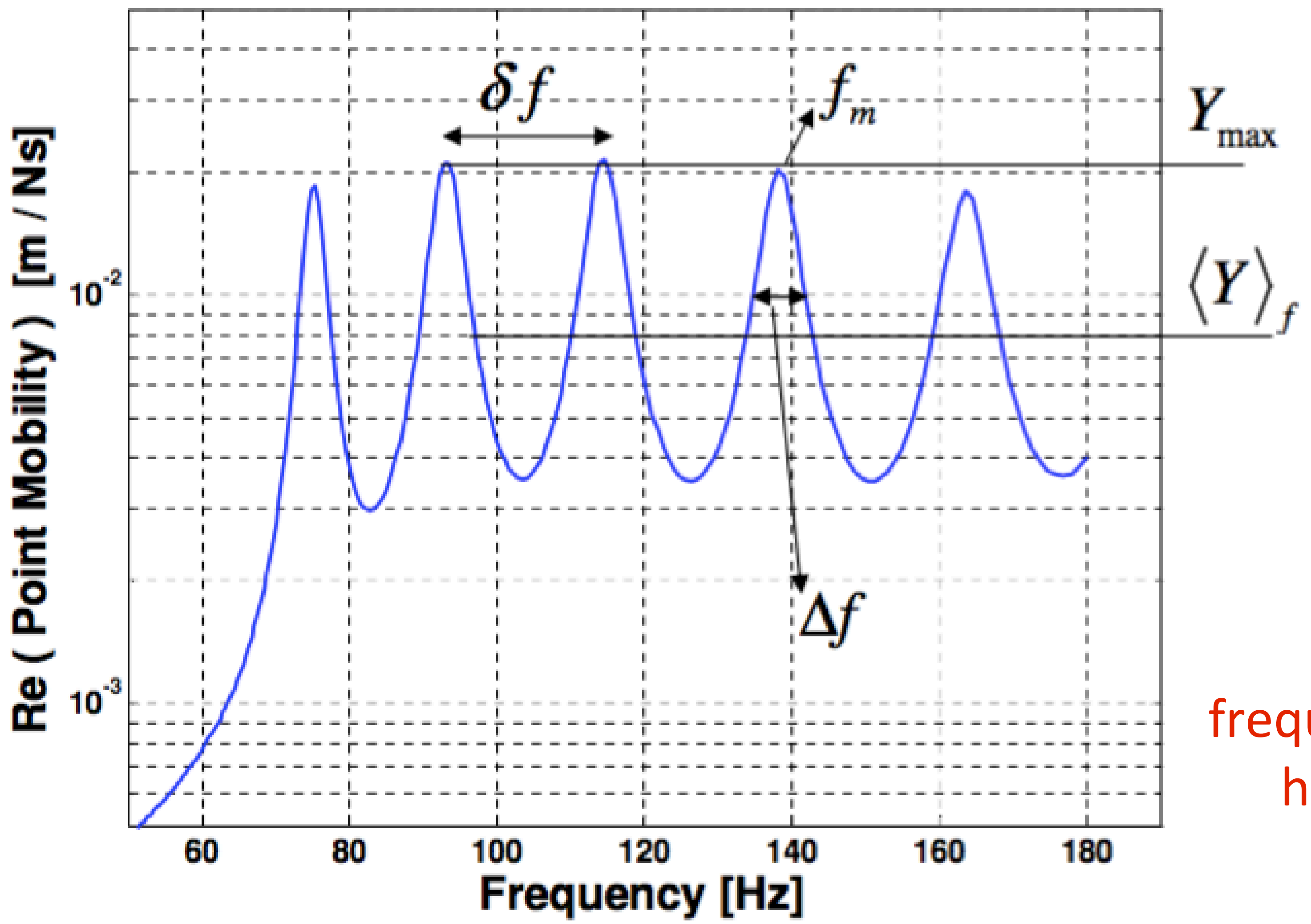
Many modes, many natural frequencies, many resonances.
Response dominated by resonant contributions.
Half power bandwidths generally increase with increasing frequency: resonance peaks *overlap*.

$$M = \eta\omega n; \qquad M = \frac{\eta\omega}{\delta\omega}$$

(Brian Mace)

SEA quantities: modal overlap factor

- “Half-power bandwidth” method



Damping loss factor (structures):

$$\eta = \Delta f / f_m$$

frequency separation where response has decreased of factor 2 (3 dB)

Fig 2-4. Real part of point mobility for a car tyre. (The real part of the mobility multiplied by the squared force magnitude gives the power injected into the structure by a point force.)

Finnveden, Energy methods (lecture notes KTH course)

approx. dampig loss factor = 7 Hz / 140 Hz = 0,05

SEA quantities: modal overlap factor

Table 9.1 Comparison of Commonly Used Damping Measures

Dissipation Descriptor	Symbol	SI Units	Relation to η
Loss Factor	η	-	η
Quality Factor	Q	-	$1/\eta$
Critical Damping Ratio	ζ	-	$1/\eta$
Reverberation Time	T_r	sec.	$2.2/f\eta$
Decay Rate	DR	dB/sec.	$27.3 f\eta$
Logarithmic Decrement	δ	nepers/s	$\pi f\eta$
Wave Attenuation*	γ	nepers/m	$\pi f\eta / c_g$
Mechanical Resistance**	R	N-s/m	$2\pi f\eta M$
Damping Bandwidth (Half-Power)	BW	Hz	$f\eta$
Imaginary Part of Modulus $\mathbf{E}_r + j\mathbf{E}_i$	$\mathbf{E}_i$	$\mathrm{N/m^2}$	$\mathbf{E}_r \eta$
Acoustical Absorption Coefficient***	α	-	$(8\pi fV / cA)\eta$

* c_g is group velocity for system in m/s

** M_g is the system mass

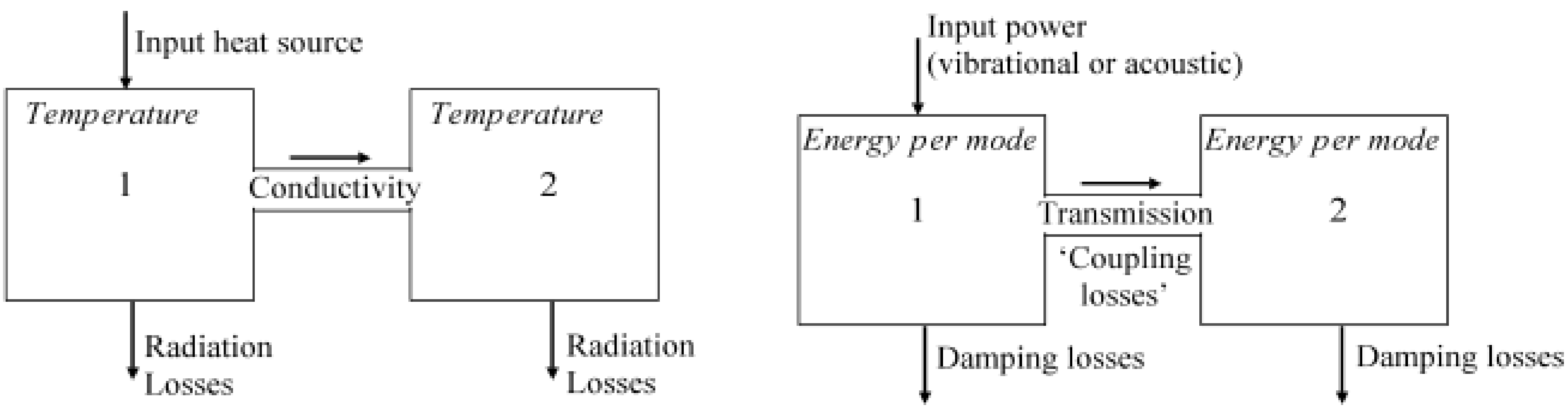
*** A is the area of walls, V is room volume, c is speed of sound

Lyon, Statistical energy analysis

SEA quantities: Conductivity

$$P^{ij}_{coup} = C^{ij}\left(\hat{e}_i - \hat{e}_i\right)$$

Potential Flow -Thermal analogy



Thermal system: heat and temperature. SEA system: power and vibrational energy.

Temperature ~ energy per mode. Total thermal energy ~ total vibrational energy.
Heat flows from "hot" to "cold".
The bigger the temperature difference, the bigger the heat flow.

(Brian Mace)

SEA quantities: Conductivity

$$P^{ij}_{coup} = C^{ij}\left(\hat{e}_i - \hat{e}_i\right)$$

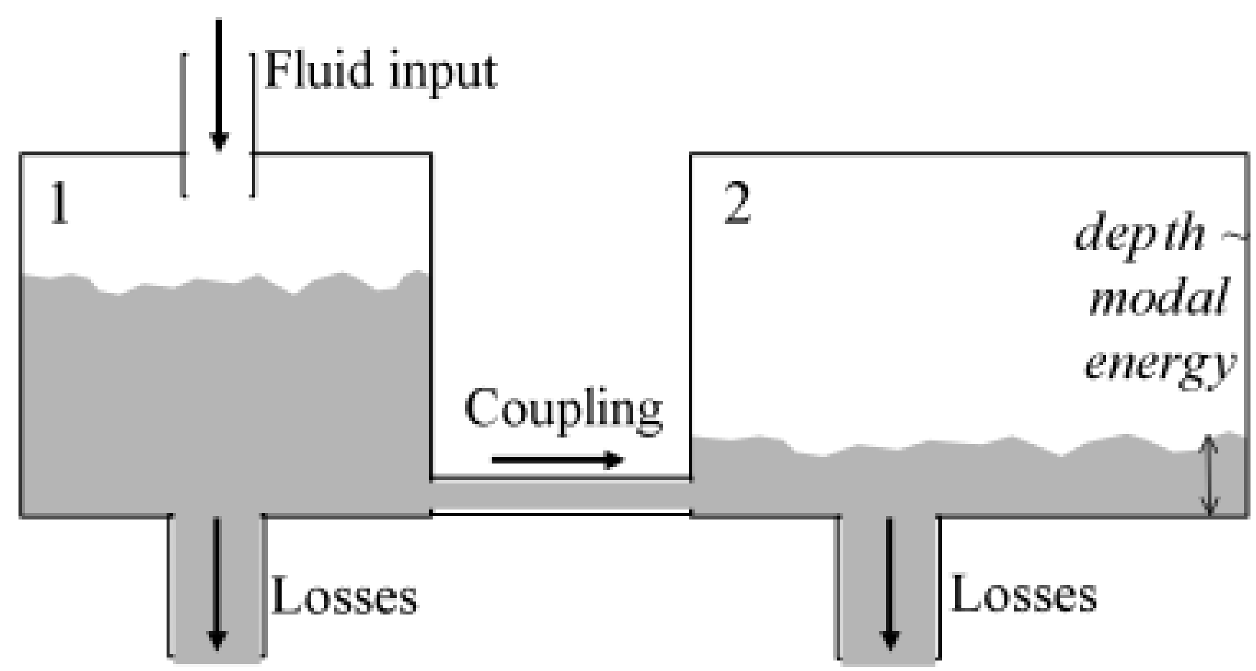
- A statistical approach based on a low population!
  - Thermodynamics: Avogadro's number, 10^23
  - SEA: vibroacoustic natural modes, 0 - 10^4?

(Brian Mace)

SEA quantities: Conductivity

$$P_{coup}^{ij} = C^{ij}\left(\hat{e}_i - \hat{e}_i\right)$$

Potential Flow - Fluid flow analogy



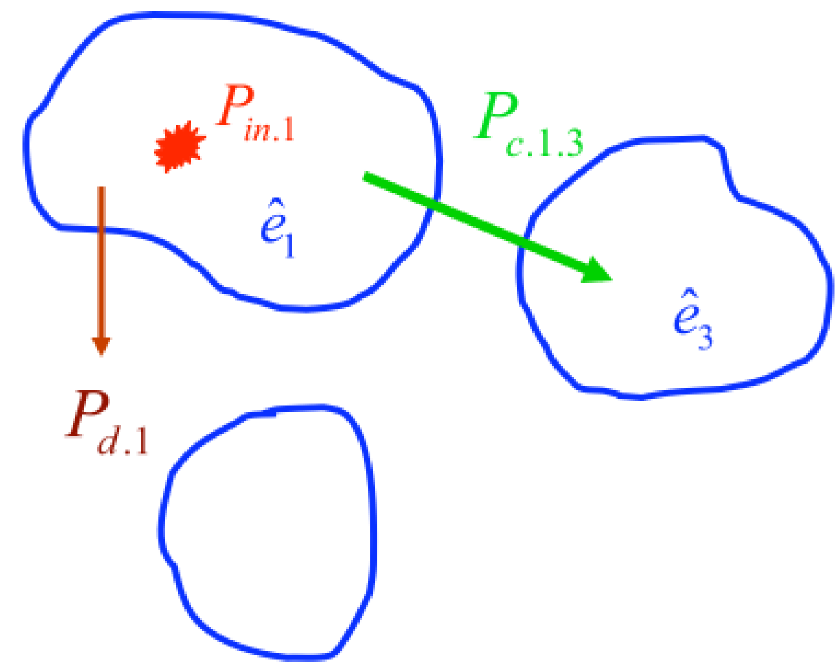
Depth of fluid ~ energy per mode. Total volume of fluid ~ total vibrational energy.
Fluid flows from "high head" to "low head".
The bigger the head difference, the bigger the volume flow.

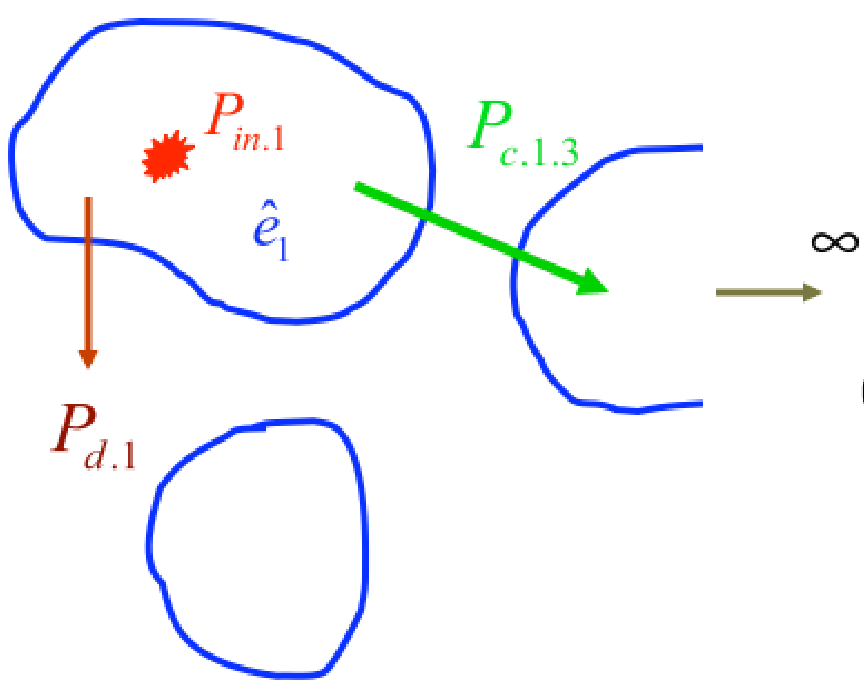
(Brian Mace)

SEA quantities: Conductivity

when calculating C

- Compact support
  - only direct couplings
- Travelling wave estimate:
  - no transfer of energy back
  - regardless of rest of the structure





Conductivity given by:

$$C^{1.3} = \lim_{C^{1.3}/M_3 \to 0} \left(\frac{P_{c.1.3}}{\hat{e}_1} \right)$$

SEA quantities: Conductivity

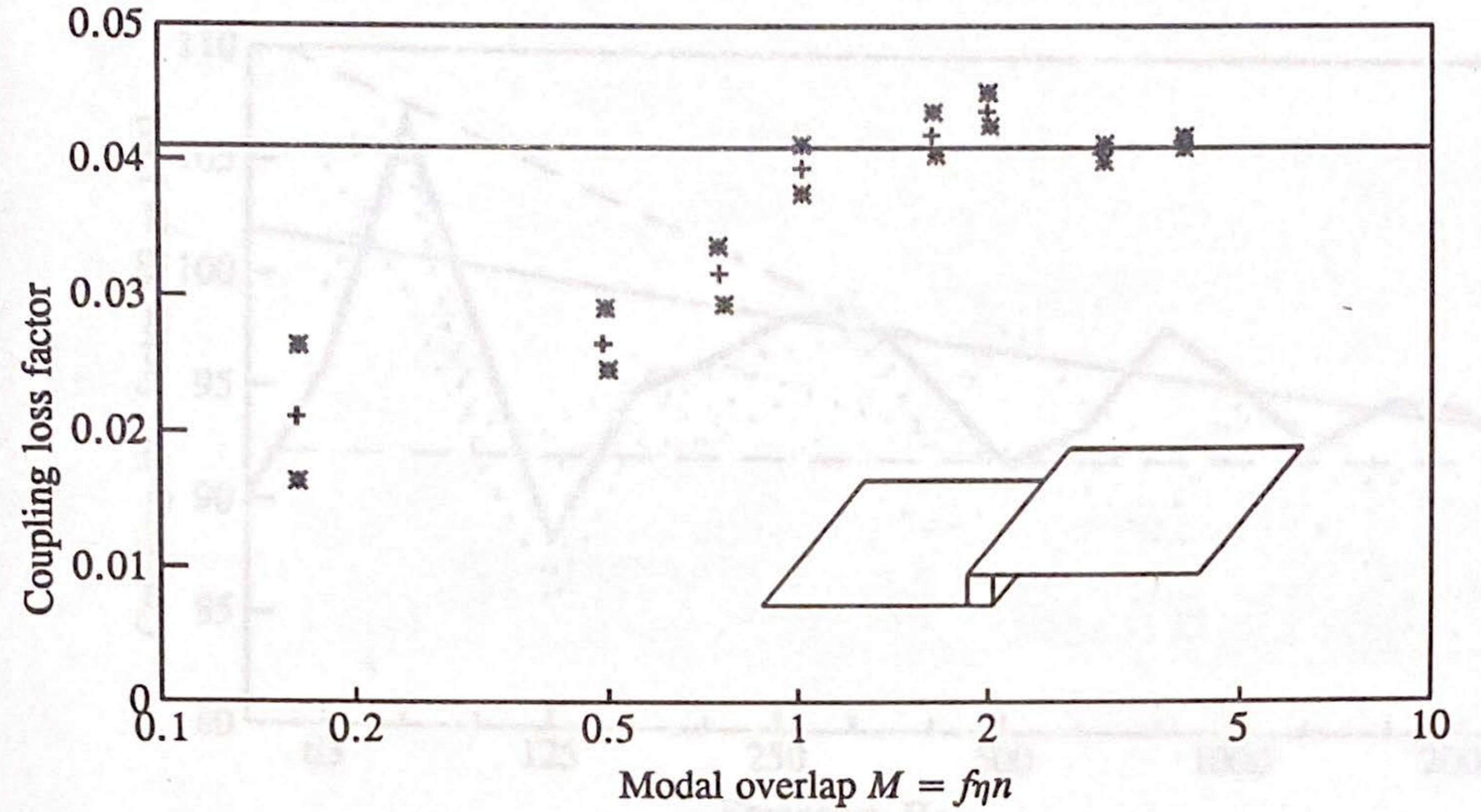
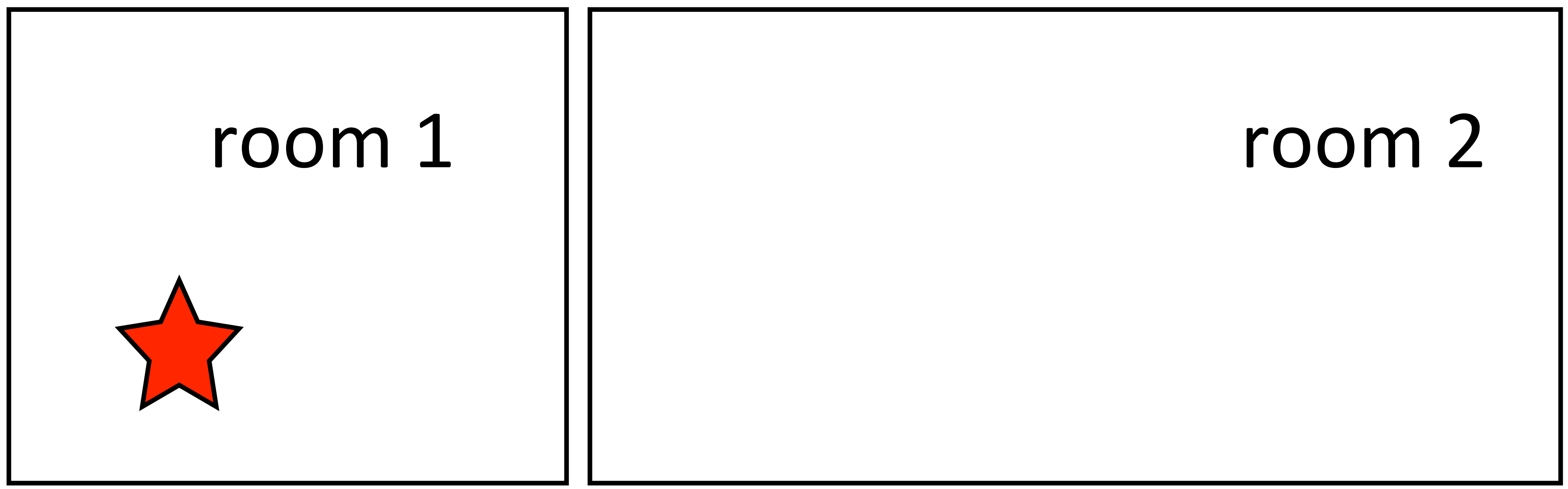
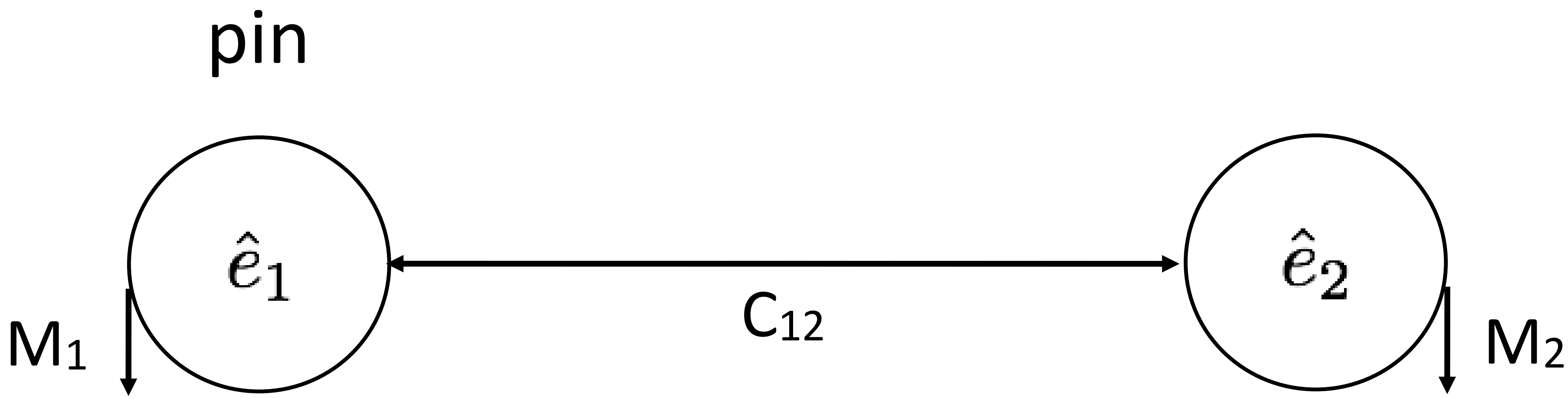


Figure 2.15 Coupling loss factor between two plates as a function of modal overlap. ———, SEA prediction; +, mean CLF; *, upper and lower confidence limits of CLF. Insert shows the layout of the two plates (Fahy and Mohammed, 1992)

- Where SEA should work due to chaotic response (M>=1), then it predicts well the conductivity (coupling loss factor).
- i.e. extending the receiving structure to infinity is sensible!

Room acoustics exercise





Room acoustics exercise

1. $E = 2\langle e_p \rangle V = 2\frac{1}{2}\frac{\tilde{p}^2}{\rho_0 c^2}V \qquad [J]\,;$

room volume V
speed of sound c
air density rho_0

2. $\hat{e} = \frac{E}{n} \qquad \left[\frac{J}{s}\right] = [W]\,;$

3. $n = \frac{k^2 V}{2\pi^2 c} \qquad [s]\,;$

wavenumber k = ω/c

4. $M = \omega\eta n = \frac{k^2 A}{8\pi^2} \qquad [-]; \qquad A = \alpha S;$

partition areas S
absorption areas A

5. $C = \left(\frac{W_t}{\hat{e}_1}\right)_{\frac{C^{12}}{M_2}\to 0} = \frac{\tau W_i}{\hat{e}_1} \qquad [-];$

incident power W_i
transmitted power W_t
transmission factor tau

6. $\tau = \frac{W_t}{W_i}; \quad R = 10\log\frac{1}{\tau}; \quad W_i = \frac{k^2 S}{8\pi^2}\hat{e}_1;$

7. $\Rightarrow C = \tau\frac{k^2 S}{8\pi^2}$

Room acoustics exercise

$$\begin{bmatrix} M_1 + C & -C \\ -C & M_2 + C \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \end{bmatrix} = \begin{bmatrix} P_{in} \\ 0 \end{bmatrix};$$

$$\hat{e} = \frac{eV}{n} = \frac{2\pi^2 \langle \tilde{p}^2 \rangle}{\omega^2 \rho_0 c};$$

- Standard texts

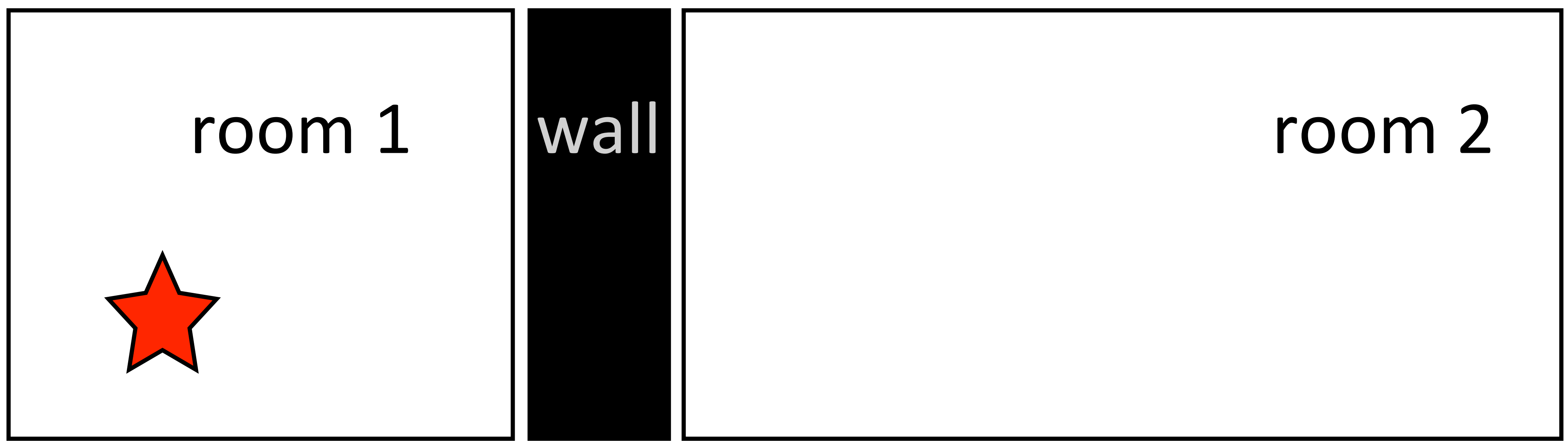
room constant

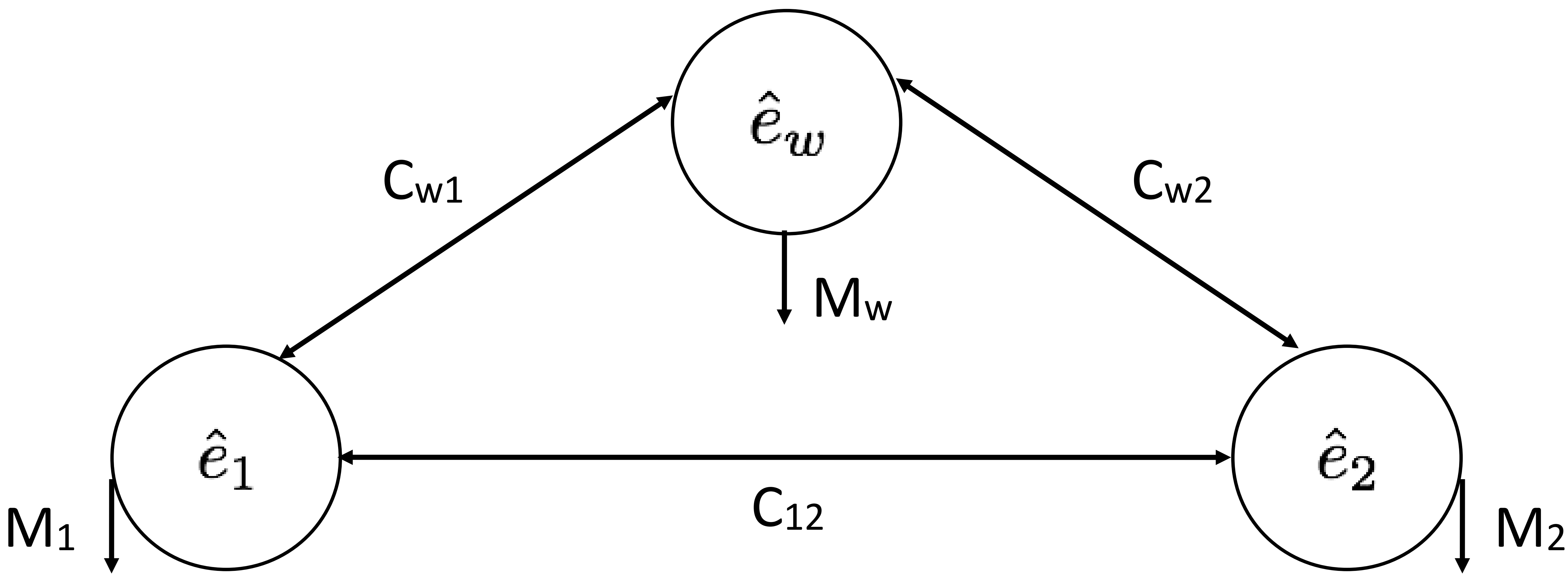
1. $L_{p1} = L_w + 10 \log \left(\cancel{\frac{\Gamma}{4\pi r^2}} + \frac{4}{A'} \right);$ direct field is not SEA
2. $L_{p2} = L_{p1} - 10 \log \left(\frac{A}{\tau S} \cancel{+ 1} \right)$ typical approximation

Room acoustics exercise

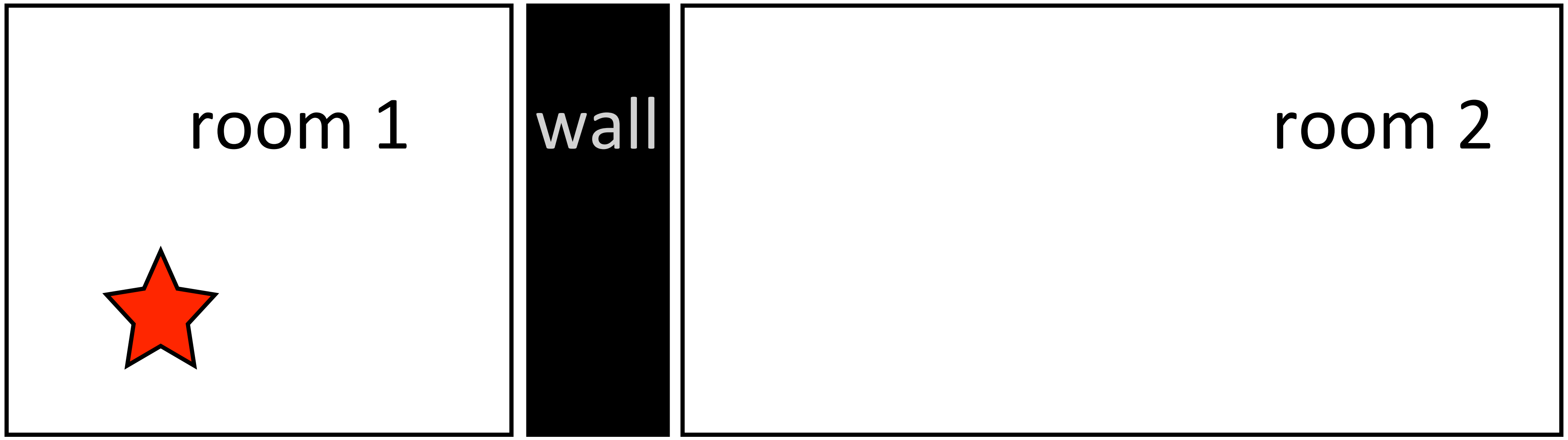
- Symmetric formulation
- Sub-structuring
- Distinction between coupling and damping losses
- (nothing really new but a good framework!)
  - (good news: you already know a bit of SEA!)

Room acoustics exercise





Room acoustics exercise



- Coincidence frequency of a plate

$$f_c = \frac{c^2}{2\pi}\sqrt{\frac{m_w}{B_w}}$$

mass per unit area

bending stiffness

Room acoustics exercise

$$M_i = \eta_i n_i \hat{e}_i$$

Radiated power by a simply-supported plate placed in an infinite baffle radiating into a semi-infinite room (Leppington)

$$C^{1w} = \left(\frac{P_c^{1w}}{\hat{e}_w} \right)_{\frac{C^{1w}}{M_1} \to 0} = \frac{\rho_0 c S \sigma^{1w} \langle \tilde{v}^2 \rangle}{\hat{e}_w};$$

$$\hat{e}_w = \frac{E_w}{n_w} = \frac{S t \rho \langle \tilde{v}^2 \rangle}{n_w};$$

plate thickness t
plate density rho

$$n_w = \frac{S}{3.6 c_L t};$$

$$\Rightarrow C^{1w} = \rho_0 c \frac{S}{3.6 c_L t^2 \rho} \sigma^{1w};$$

Leppington, The acoustic radiation efficiency of rectangular panels, Proceedings of the Royal Society of London A, 382 (1982)

Room acoustics exercise

- Radiation efficiency (k_f, fluid wavenumber; k_s structure wavenumber):

Assuming fc >> 1st eigenfrequency of plate

$$\sigma(\nu) = \begin{cases} \dfrac{c(a+b)}{2\pi^2 \nu k_f ab\left(\nu^2-1\right)^{1/2}}\left[\ln\left(\dfrac{\nu+1}{\nu-1}\right)+\dfrac{2\nu}{\nu^2-1}\right], & \nu > 1 \\ \left(0.5-0.15\dfrac{a}{b}\right)\sqrt{k_f a}, & \nu = 1 \\ \left(1-\nu^2\right)^{-1/2}, & \nu < 1 \end{cases}$$

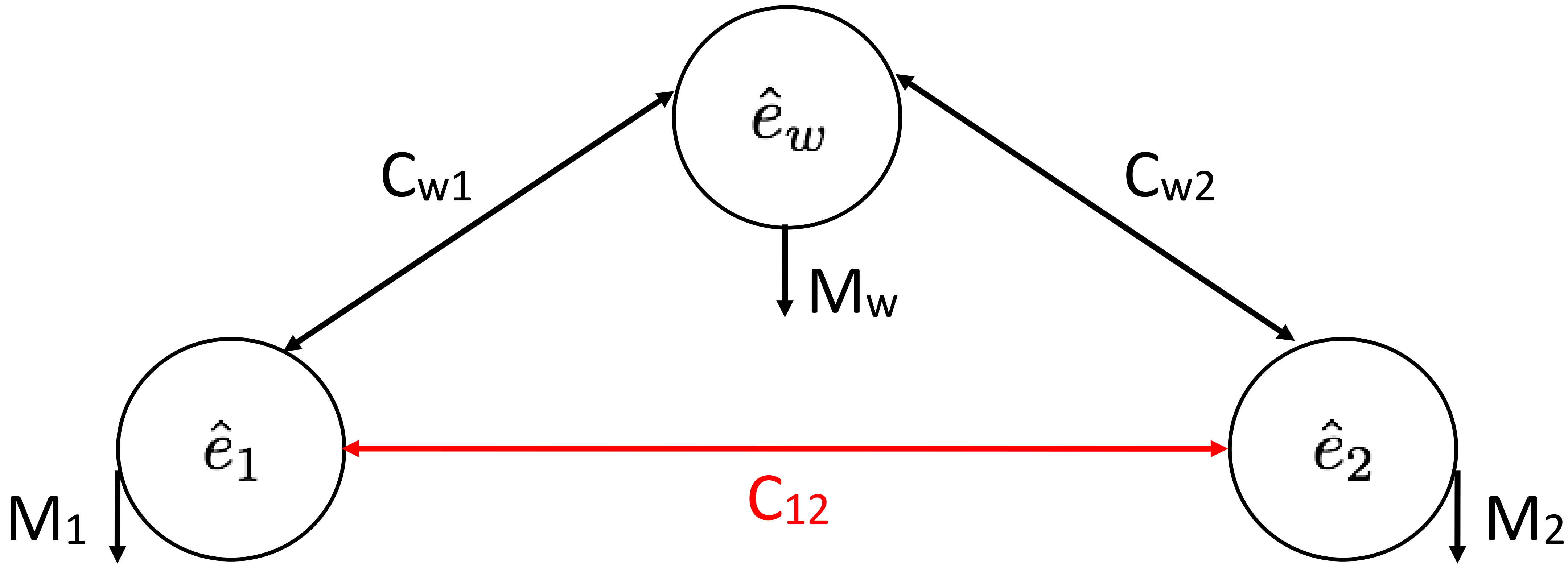
where a and b are the side lengths, so that $S = ab$, $a \le b$, and

$$\nu = k_s / k_f .$$

Leppington, The acoustic radiation efficiency of rectangular panels, Proceedings of the Royal Society of London A, 382 (1982)

Room acoustics exercise

- Mass-law, forced (non-resonant) motion below coincidence frequency (this is NOT SEA!):
  - insert this *indirect coupling* between two rooms as direct coupling!
  - (what would be an example of actual direct coupling between two rooms?)



Room acoustics exercise

- Mass-law, forced (non-resonant) motion below coincidence frequency (this is NOT SEA!):
  - insert indirect coupling between two rooms as direct coupling
- Transmitted and incident power in a room, diffuse field

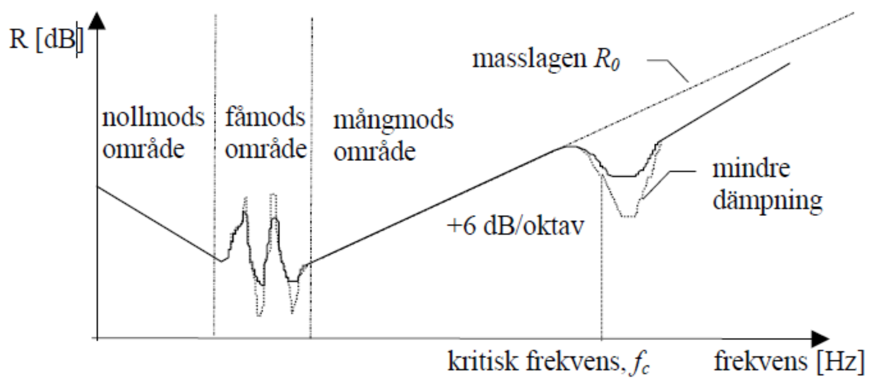
$$W_{\text{trans}} = W_{\text{inc}}\tau = \frac{Ec_0S}{4V}\tau.$$

- Coupling loss factor (as per definition):

$$\eta_{12} = \frac{c_0S\tau_{12}}{8\pi f V_1}.$$

Craik, Sound transmission through buildings using statistical energy analysis

Room acoustics exercise



Figur 43 Principiellt utseende för reduktionstal hos en enkelvägg

non-resonant transmission coefficient by Leppington

There are many approximations to this equation which are widely used (and misused) in building acoustics. The normal incidence sound reduction index, R_0, can be given by (Beranek and Ver, 1992)

$$R_0 = 10\log\left[1 + \left[\frac{2\pi f\rho_s}{2\rho_0 c_0}\right]^2\right] \simeq 20\log\frac{2\pi f\rho_s}{2\rho_0 c_0} = 20\log f\rho_s - 42 \tag{4.23}$$

and can be obtained from consideration of forced waves at normal incidence on an infinite plate.

Variations of this are the field incidence mass law,

$$R_f = R_0 - 5 = 20\log f\rho_s - 47 \tag{4.24}$$

Craik, Sound transmission through buildings using statistical energy analysis

Room acoustics exercise

- Use eq. 4.24 to obtain R_f (non-resonant sound reduction index) and use it to compute tau_{12} and then eta_{12} i.e. the coupling loss factor describing the non-resonant transmission between two acoustic volumes (two slides ago).

Room acoustics exercise

- Finalise: get sound pressure levels in the two rooms.
  - Functions of modal energy, retrieve pressure.
- Check formula in e.g. ISO standard 16283-1:2014 to retrieve R from sound pressure levels in the two rooms, wall area and absorption area of the receiving room.
- Done!

Craik, Sound transmission through buildings using statistical energy analysis

Summary

- Helmholtz number, He = kl = $2\pi/\lambda$ l (wavenumber k, typical dimension l).
- Standing wave at least, l = $\lambda/2$. Hence, He = π.

Classification	Frequency region	Character
No-modes region	He << π	No wave propagation. Discrete mechanical system.
Low-modes region	He $\approx \pi$	A few eigenmodes dominate. A complete mathematical description is possible (analytical / FEM).
Many-modes region	He >> π	A large number of eigenmodes control the vibroacoustic response. Analyses is only practicable / sensible using energy methods

- The energy method presented today is SEA, which has its range of validity – i.e. strengths and weaknesses.

Validity of SEA

Careful! If losses are too high, then energy decay will occur! Perhaps $M \approx 1$ works better.

- N>>1; many modes in a given frequency band
- M>>1; we need disorder, i.e. overlapping modes
- low energy decay within an element == constant energy density within an element
- weak coupling: Cij/Mi << 1, i.e. flow of exchanged vibroacoustic energy small compared to internal losses. Think how Cij is estimated.

Le Bot, Cotoni, JSV 329 (2010)

Why SEA?

- Only method for high frequency vibroacoustics of complex systems supported by commercial software
- Efficient (numerically cheap)
  - less demand on input data
  - immediate answers
  - impossible problems

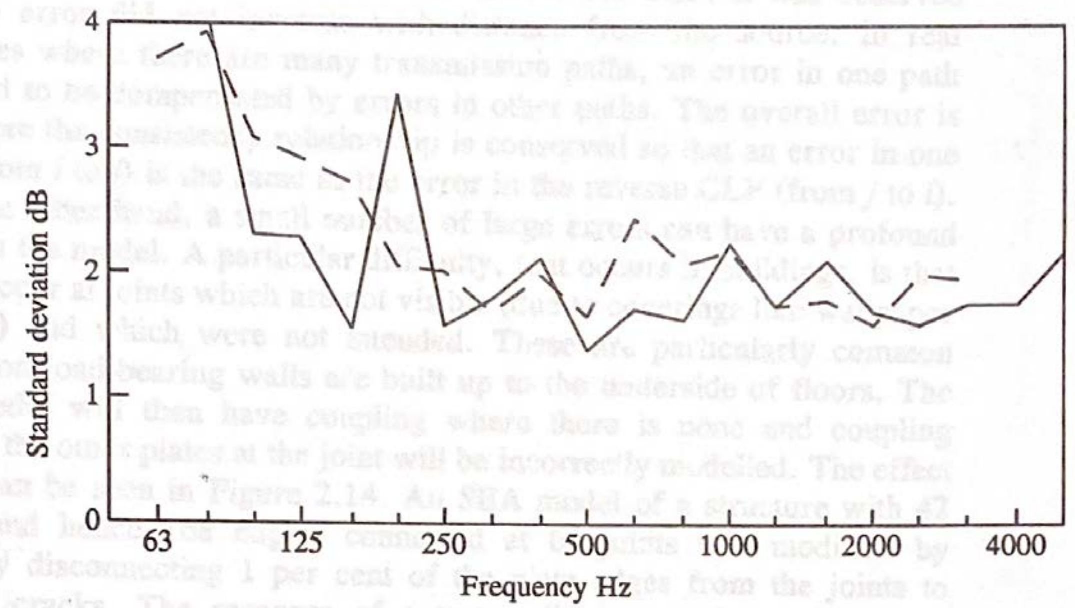


Figure 2.12 Standard deviation of ten measurements of airborne transmission through a concrete floor and ten measurements of structural transmission between a wall and a floor. ———, airborne transmission; — — — —, structure-borne transmission (Craik and Steel, 1989; Craik and Evans, 1989)

- We are ignorant of many things
  - why bother with unrealistic precision?
  - mean response of “typical” product
- Alternate view point

Why SEA?

- Successful with:
  - satellites, launch vehicles
  - airplane
  - ships
  - buildings
  - vehicles

Why not SEA?

- No details in results
  - poor frequency resolution
  - no spatial resolution
- Software
  - lack of routines for parameter estimation (not so true actually)
  - lack of bench mark examples
- Connection to deterministic methods
- Not universally applicable (narrowband excitation
  - how accurate?
  - valid for this problem?

Why not SEA?

- In none of my jobs as consultant I have used SEA!
- Instead:
- Semi-empirical method based on measured transfer functions
- Bastian – stripped down first-level SEA based on reduction index and impact sound isolation
- Just guess high frequency performance based upon experience.

Why not SEA?

- In none of my jobs as consultant I have used SEA!

Untrue!

- As seen today, SEA is often a point of view, a framework and in that sense I have carried it with me all the time.
- Plus, room acoustics builds on SEAs thinking, common software (Bastian, EN 12354) too.

References

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- Le Bot, Cotoni, Validity diagrams for statistical energy analysis, JSV 329 (2010)