adverse effects (Y-axis); e.g., percentage of the people expressing annoyance or satisfaction.

It has also been tried to establish dose-response relationships between airborne sound insulation or impact sound pressure level and the percentage of people being annoyed by noise from neighbours. For example, as the acoustic requirement increases, the percentage of satisfied people increases, and dissatisfied people decrease, and so on. Several of the most reliable results show an average slope (i.e., between acoustic requirement and satisfaction level of exposed people) around 4 % per dB (Rindel, 1998). For instance, an increase of 5 dB in the acoustic requirements indicates that the percentage satisfied people will go up by 20, and percentage dissatisfied people will go down by 20. These results can be further used to assess the acoustic quality level of a certain set of sound insulation requirements, or they may be used as a basis for specifying the desired acoustic quality of new buildings. Approximately, the same slope is found when linear regression has a high correlation for airborne neighbour noise, impact noise from neighbour and noise from heating or plumbing (or traffic). It is suggested that the level of acoustic requirements for dwellings may be deemed "acceptable" when for $R_w' \approx 56$ dB, $L_{n,w}' \approx 57$ dB, L_A (heating) ≈ 32 dB, since 50% of exposed people evaluate as good and 20 % as poor. It can be further deemed "satisfactory" when for $R_w' \approx 60$ dB, $L_{n,w}' \approx 52$ dB, L_A (heating) ≈ 27 dB since 70% of exposed people evaluate as good (Rindel, 1998).

For transportation noise, the dose-response relationships can indicate the relationship between the percentage "highly annoyed" and the noise exposure level have been produced. From a statistical point of view, the noise exposure level can account for 70% of the variance in annoyance, at the community level. However, at the individual level, the variance in annoyance due to noise exposure is typically only about 20% (Berglund et al, 2000). The world health organisation (WHO, 1980) suggested that the bedroom noise level should not exceed 35 dB(A) to "preserve the restorative process of sleep". This figure have subsequently been revised down to 30 dB(A), to ensure that negative effects on sleep are avoided (Berglund et al, 2000). There is also recognition that short lived noisy events could have an effect on the sleeping process, and recommends a maximum level of 45 dB(A) not to be exceeded in bedrooms; see also European Commission (2002).

Obviously, the abovementioned results can not be made universal since the satisfaction level of exposed people can vary according the time and location under concern. Therefore, it is recommended that the acoustic requirements be updated on a continuous basis, taking into account the dose-response relationships for the region, under concern.

Chapter 4

Airborne Sound Insulation

In sound insulation, one must consider the many paths that sound can take from one room to another: (a) direct path: sound passes through common partition; (b) flanking: sound passing around common partition, e.g. along roof rafters ; and (c) structure-borne: direct vibration of structure by source (e.g. boiler pumps), sound passes through structure. One procedure for noise control is to provide an acoustic barrier or partition to reduce the transmission of sound. For design purposes, one must be able to predict the transmission loss (sound reduction index) for the partition over a wide range of frequencies. In this chapter, it is attempted to examine the general case of airborne sound transmission through a panel or partition; particularly, the direct path of airborne sound transmission. The other paths will be further examined is the subsequent chapters.

4.1 Background

The sound insulation of a partition is determined, by its properties, the attached constructions, and the nature of partition's boundary conditions. In addition, the parameters of the incident sound energy play also an important role in the calculation of the sound insulation. It should, however, be observed that the influence of the connected or attached constructions can be big. A measurement of the construction's sound insulation can, therefore, be misleading for the insulation, which occurs *in- situ* (at the field). To some extent, a consideration can be taken to the connected constructions in the calculations; this is why the calculated values can be more secure than the lab measurement. This is especially for the case of heavy constructions. Ignoring flanking and leakage, the basic mechanism of sound transmission through a wall is that sound in the source room forces the exposed surface to vibrate; this vibration is transmitted through the structure of the wall to the other surface, which in turn vibrates producing sound in the receiving room. If the two surfaces of the wall are rigidly connected so that they vibrate as a unit (e.g., a single-leaf partition), the transmission loss or sound reduction depends only on the frequency and the mass per unit area, stiffness, and intrinsic damping of the wall. If the partition consists of two unconnected walls separated by a cavity (a double-leaf partition), the transmission loss depends mainly on the properties of the two walls and on the size of the cavity and its absorption. By considering some common panel constructions, techniques for estimation of the sound reduction index, R (= transmission loss, TL) curve, are presented.

The material presented in this chapter applies for transmission of sound through panels (e.g. a plate of glass, brick, wood, concrete, etc.) of homogenous or inhomogeneous materials, of infinite or finite size, of isotropic or orthotropic properties. Note that double constructions can include also e.g., double glazing windows, double doors constructions and not only the "conventional" walls or floors.

In many cases in building acoustics practice, the panels are considered thin, of constant thickness and infinite extent in which the panel responds to a bending forced vibration (covering the framework); further, the panel material is homogeneous and isotropic. For the panels to be considered thin, the bending wavelength of the forced bending vibration of the plate is at least six times the panel thickness; see Chapter 2. For the panels to be considered to be of infinite extent, the lateral dimensions must be much greater than the bending wavelength of the forced vibration. The case of plate of inhomogeneous material and orthotropic geometry is treated in Sec. 4.7 whilst the case of finite thick plate as well as thin plate of finite size is treated in Sec. 4.8, and finally the case of infinite thick plate is treated in Sec. 4.9. In addition, for the cases treated in this chapter, flanking transmission is not considered in developing the prediction schemes. The case of flanking transmission is treated separately in Chapter 6.

4.2 Sound Reduction Index (Transmission Loss) of Single-Leaf Partitions

For the general case, when an acoustic wave travelling in one medium encounters the boundary of a second medium, reflected and transmitted waves are generated. For example, when sound strikes upon a solid partition, part is reflected, part absorbed within the material, and part transmitted to the other side or to elsewhere in the building, as discussed in Chapter 2. When a partition is stroke by an airborne (plane) sound wave, i.e., longitudinal wave, the ratios of the pressure amplitudes and intensities of the reflected and transmitted waves to those of the incident waves depend technically on the following factors: angle of incidence, φ; densities of the two media; and speeds of sound in the two media.

Physically, sound attenuation or transmission loss in ordinary building materials is the result of an interplay between mass, stiffness and damping. In many cases in practice, a diffuse sound field in the room is assumed, and that the sound insulation is analysed between two rooms of normal size.

The general variation of the sound reduction index with frequency for a homogeneous infinite partition is shown in Fig. 4.1. As can be seen, there are three main regions of behavior for the wall or panel:

(a) Region I: Stiffness-controlled region
(b) Region II: Mass-controlled region
(c) Region III: Damping-controlled region (upper stiffness region)

Note, however, that the boundaries between these regions are approximate, especially with respect to the prediction methods. For instance, the mass controlled region can start effectively at frequencies well above the lowest resonance frequency, f_{11}. Further, the mass-controlled region can extend up to the coincidence region as the mass law is affected by resonance at lower frequencies and coincidence at higher frequencies.

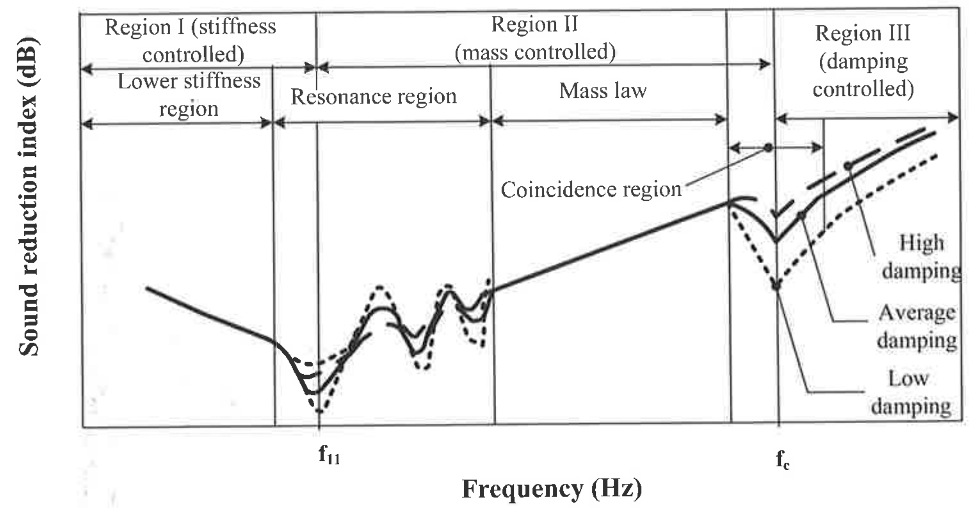


Fig. 4.1 Approximate behaviour of sound reduction index (or transmission loss) of a thin homogenous single wall, of infinite size. Distinct regions showing the way a single leaf solid partition will react to different frequency sounds, f_c is the wall's critical frequency, $f_{1,1}$ is the wall's lowest *eigen* frequency. The boundaries between the main regions (I, II, III) are approximate.

Instructively, the behaviour of transmission loss of an infinite plate in the whole frequency domain is complicated and the prediction methods are simplified and approximated; see also Sec. 4.9.

4.2.1 Region I: Stiffness-controlled region

At low frequencies, the wall or plate can be considered as very thin and so it vibrates as a whole. The sound transmission through the plate (or panel) depends mainly on the stiffness of the wall, while damping and mass having little effect, especially in the lower stiffness region.

Consider the plate shown in Fig. 4.2 in which the medium is the same on both sides (air) of the plate, and the panel is very thin. The expressions for the acoustic pressure and particle velocity on each side of the plate may be written as follows (see Sec. 2.9, Chapter 2):

$$p_1(x,t)=(p_{1+}e^{-jkx}+p_{1-}e^{jkx})e^{j\omega t} \qquad (4.1)$$

$$p_2(x,t)=p_{2+}e^{-jkx}e^{j\omega t} \qquad (4.2)$$

$$v_1(x,t)=(1/\rho_0 c)(p_{1+}e^{-jkx}-p_{1-}e^{jkx)})e^{j\omega t} \qquad (4.3)$$

$$v_2(x,t)=(1/\rho_0 c)\,p_{2+}e^{-jkx}e^{j\omega t} \qquad (4.4)$$

where ρ_0 is the air density, and c the sound speed in air.

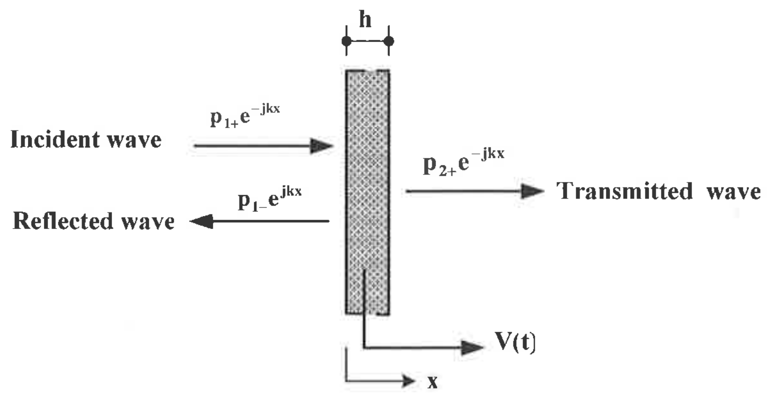


Fig. 4.2 Vibration of a plate in the stiffness-controlled region due to normal incidence of sound wave. The quantity $V(t)$ is the vibration velocity of the whole plate.

At the surface of the panel, $x = 0$, the particle velocities are both equal to the instantaneous vibration velocity of the plate, $V(t)$: $V(t) = v_1(0,t) = v_2(0,t)$. Thus, it follows from Eq. (4.1)-Eq. (4.2) that:

$$p_{1+}-p_{1-}=p_{2+} \quad \text{and} \quad V(t)=p_{2+}e^{j\omega t}/\rho_0 c \qquad (4.5\text{a, b})$$

In general, the plate and the surrounding medium may be modelled at very low frequencies, as a simple mass-spring system. The governing equation of motion for this system is expressed by Eq. (11.3) (Chapter 11). Eq. (11.3) may be rewritten in terms of plate velocity as

$$F(t)=m\frac{\mathrm{d}V}{\mathrm{d}t}+r_m V(t)+k_s\int V(t)dt \qquad (4.6)$$

The quantity r_m is the coefficient of viscous damping or the mechanical resistance, m is the mass, F is the applied force on the plate, and k_s is the stiffness constant. Assume sinusoidal excitation by force $F_0e^{j\omega t}$, so the velocity is also sinusoidal, and Eq. (4.6) may be simplified as:

$$F=j\omega mV(t)+r_m V(t)+\frac{k_s}{j\omega}V(t) \qquad (4.7)$$

Since the damping and mass has little effect, especially in the region below the first or fundamental resonant frequency, f_{11}, (the lower stiffness region), in comparison to the stiffness, it follows that Eq. (4.7) will only contains the term with k_s:

$$F\approx\frac{k_s}{j\omega}V(t) \qquad (4.8)$$

Thus, the reaction of plate to the net (acoustic) forces acting on it is only represented by the spring force of the plate, F, assuming that the plate has a finite stiffness, k_s; see also Fig. 11.1, Chapter 11. Consequently, substituting Eq. (4.5b) into Eq. (4.8), and applying a force balance at the surface of the plate, the result becomes:

$$p_1(0,t)-p_2(0,t)=-F=-\frac{k_s p_{2+}e^{j\omega t}}{j\omega\rho_0 c} \qquad (4.9)$$

The stiffness, k_s, is here in unit area of the plate (SI unit: N/m^3). The latter expression may be simplified by replacing the acoustic pressure forces with Eqs. (4.1) and (4.2):

$$p_{1+}+p_{1-}-p_{2+}=+\frac{jk_s p_{2+}}{\omega\rho_0 c} \qquad (4.10)$$

Now, combining Eq. (4.5a, b) with Eq. (4.10) will result in:

$$\frac{p_{2+}}{p_{1+}} = \frac{1-(jk_s/2\omega\rho_0 c)}{1+(k_s/2\omega\rho_0 c)^2} \tag{4.11}$$

The sound power transmission coefficient for normal incidence is defined as the ratio of the transmitted acoustic power I_{tr} to the incident acoustic intensity I_{in}, at the same interface (see Sec. 2.9.6). Accordingly,

$$\tau_n \equiv \frac{I_{tr}}{I_{in}} = \left|\frac{p_{tr}}{p_{in}}\right|^2 = \left|\frac{p_{2+}}{p_{1+}}\right|^2 = \frac{1}{1+(k_s/2\omega\rho_0 c)^2} \tag{4.12}$$

The sound reduction index for normal incidence, R_n, may now be written as follows:

$$R_n \equiv 10\log(1/\tau_n) = 10\log(1+K_s^{-2}),\ K_s = \frac{2\omega\rho_0 c}{k_s} \tag{4.13a, b}$$

The previous case is derived on the assumption that there is a normal incident wave striking a wall. If one follows the analysis for the oblique incidence of the sound wave, as developed in Sec. 2.9.7, it an be shown that the sound power transmission coefficient for an angle of incidence, φ, is obtained:

$$\tau(\varphi) = \frac{1}{1+\left(\cos\varphi \dfrac{k_s}{2\omega\rho_0 c}\right)^2} \tag{4.14}$$

At $\varphi = 0$ (normal incidence), Eq. (4.14) reduces to Eq. (4.12).

In typical applications of building acoustics, it is often assumed a random sound incidence (diffuse field) in the room in which the sound waves strike the wall surface at all angles of incidence. The average (mean) sound power transmission coefficient for random incidence of the sound waves is given by the Paris formula as:

$$\bar{\tau} = 2\int_0^{\pi/2} \tau(\varphi)\cos\varphi\sin\varphi\, d\varphi \tag{4.15}$$

Substituting Eq. (4.14) into Eq. (4.15) leads, after manipulation, to the following expression for the mean sound power transmission coefficient in the stiffness-controlled region, Region I:

$$\bar{\tau} = K_s^2 \ln(1+K_s^{-2}) = -K_s^2 \ln(\tau_n) \tag{4.16}$$

The sound reduction index (= transmission loss) for the stiffness-controlled region is given by the following:

$$R \equiv 10\log(1/\bar{\tau}) = -20\log K_S - 10\log\{\ln(1+(K_S)^{-2})\} \tag{4.17}$$

The latter expression may be simplified by knowing that $\ln x = 2.303 \log x$. This implies that: $R_n = 10\log(1+K_s^{-2}) = 4.343\ln(1+K_s^{-2}) \Rightarrow \ln(1+K_s^{-2}) = 0.23R_n$, Eq. (4.13a). Consequently, the random-incidence transmission loss for Region I, is related to the normal incidence case as

$$R = -20\log K_s - 10\log R_n + 6.4 \tag{4.18}$$

The quantity K_s is expressed by Eq. (4.13b), and R_n by Eq. (4.13a).

For a rectangular plate, the stiffness constant, k_s, is given by the following expression:

$$k_s = \frac{\pi^8 E h^3 (1/a^2 + 1/b^2)^2}{768(1-\mu^2)} \tag{4.19}$$

The quantities a and b are the width and height of the panel; h is the thickness of the panel; and E and μ are the Young's modulus and Poisson's ratio for the panel material, respectively.

For a circular panel with a diameter D and thickness h, the specific mechanical compliance is given by:

$$k_s = \frac{256 E h^3}{3D^4(1-\mu^2)} \tag{4.20}$$

Note that the specific mechanical compliance or mechanical compliance is the reciprocal of stiffness.

Some properties of various common materials used in building constructions are given in Table A (Appendix). Note that the given values are not final and can vary according to manufacturing method and/or geological conditions. Therefore, the manufacturer data should always be consulted first.

To study closely the behaviour of plate in the lower stiffness region, Eq. (4.7) may be rewritten as

$$\frac{V}{F} = \frac{1}{j\omega m + r_m + (k_s/j\omega)} \tag{4.21}$$

At the lower stiffness region (Fig. 4.1), the $k_s/j\omega$ term dominates so $V/F \approx j\omega/k_s$. Thus, if a force constant in frequency is applied, then the vibration of the panel increases at 6 dB/octave. Consequently, the effectiveness of stiffness in the attenuation of sound transmission or R decreases by 6 dB up to the lowest panel resonant frequency, f_{11}, for every doubling of frequency (one octave). Since the impedance or stiffness is inversely proportional to frequency, the sound reduction index increases with decreasing frequency, Fig. 4.1. At the first few resonant frequencies, the magnitude of the sound reduction index is strongly dependent on the damping at the edges of the panel (boundary losses).

Resonance region

As the frequency of the incident wave increases, the plate or panel will resonate mechanically at a series of frequencies, called the resonant or *eigen* (natural) frequencies. This is because of the fact that the panel has a finite boundary and edge fixings. The resonant frequencies consist of a fundamental frequency (having the greatest effect), and integer multiples of this fundamental called harmonics (having less and less effect). The resonant frequencies are function of the plate dimensions, as discussed in Chapter 2. For a rectangular plate, simply supported, having dimensions a (width) × b (height)× h (thickness), the resonant frequencies are given by Eq. (2.360), Chapter 2, which may be simplified as

$$f_{nm} = 0.4534\, c_L h\{(n/a)^2 + (m/b)^2\} = (c^2/4f_c)\{(n/a)^2 + (m/b)^2\} \quad (4.22)$$

where factors m and n are integers, 1, 2, 3,..., c_L is the speed of longitudinal sound waves in the solid panel material, Eq. (2.129) and f_c is the critical frequency, Eq. (2.332). Eq. (4.22) concerns a panel, which is simply supported but not permanently fixed along its four edges. For a permanent fixation, the values of the resonant frequencies are approximately twice the values of Eq. (4.22). For small values of n and m, the resonance frequencies lie fairly long from each other.

Typically, the lowest resonant frequency (the fundamental frequency) is the most predominant frequency. This frequency is obtained by setting $n = m = 1$ in Eq. (4.22):

$$f_{11} = (\pi\, c_L h/4\sqrt{3})\{(1/a)^2 + (1/b)^2\} \quad (4.23)$$

The fundamental resonant frequency for a circular plate of diameter D and thickness h with a simple supported edge, the fundamental resonant frequency is given by (Roark and Young, 1975):

$$f_{11} = \frac{5.25 c_L h}{\pi\sqrt{3}D^2} \quad (4.24)$$

Similarly, for a circular plate clamped at the edge, the fundamental resonant frequency is:

$$f_{11} = \frac{10.2 c_L h}{\pi\sqrt{3}D^2} \quad (4.25)$$

The lowest resonant frequency, f_{11}, characterizes, approximately, the transition between Region I and Region II behavior.

Discussion

At the fundamental resonance, f_{11}, the sound transmission through the plate is enhanced by the resonant response and the transmission loss, R, drops

significantly. The damping (natural or added) at plate edges reduces the resonance amplitude. Increasing the amount of damping applied to the panel will not alter the frequencies of resonance and coincidence but will act to reduce their effect. Typically, for most building elements, the fundamental frequency is much below 100 Hz (e.g., 10-50 Hz) and it is too low to be of practical importance for typical building acoustics applications, especially, the frequency region: $f < f_{11}$ (lower stiffness region). Therefore, the lower stiffness area mainly is interesting for studies of infrasound (pertaining to frequencies below the audible range, i.e., sub-20 Hz.) and insulation against supersonic booms.

At frequencies, well above that of the lowest resonant frequency, the wall tends to behave as an assembly of much smaller masses and is then said to be mass controlled. It is within this range that the mass law directly effectively applies, and one enters thereby in the Region II, as discussed below.

4.2.2 Region II: Mass-controlled region

In the mid frequency range of a plate, higher than the first resonant frequency, f_{11}, the sound transmission is controlled by the mass inertia of the plate and is independent of the stiffness of the panel. In this region, part of the acoustic energy is transmitted through the panel and the remainder is reflected at the panel surfaces. This physical situation is analysed in Sec. 2.9.8, Chapter 2. Obviously, the greater the mass of the wall, the greater the sound energy required to set it in motion.

As derived in Sec. 2.9.8 (Chapter 2), the sound power transmission coefficient for normal incidence is obtained as

$$\frac{1}{\tau_n} = 1 + \left(\frac{\pi f M_s}{\rho_0 c}\right)^2 \quad (4.26)$$

The quantity M_s is called the surface mass, or the panel mass per unit surface area: $M_s = \rho h$, where the quantity ρ is the density of the wall or panel, and ρ_0 and c are the density and speed of sound in the air around the panel, respectively. The sound reduction index for normal incidence is related to the sound power transmission coefficient for normal incidence:

$$R_n = 10\log(1/\tau_n) = 10\log\left(1 + \left(\frac{\pi f M_s}{\rho_0 c}\right)^2\right) \quad (4.27)$$

In practice, a random field-incidence (diffuse sound field in the source room assuming waves incident from 0 to 78°) occurs in the room. In this case, it has been found experimentally that R, in the mass-controlled region, is typically 5 dB less than that theoretically calculated at normal incidence, R_n, (Beranek, 1971). Thus, Eq. (4.27) becomes:

$$R \approx R_n - 5 = 10\log\left(1+\left(\frac{\pi f M_s}{\rho_0 c}\right)^2\right) - 5 \qquad (4.28)$$

The latter expression is also called field-incidence mass law; see also Sec. 4.9.
In many cases in practice, the second term is Eq. (4.27) is much larger than 1. In these cases, the reciprocal of the sound power transmission coefficient for normal incident is proportional to f^2 and M_s^2, which means that R increases by 6 dB per doubling of the surface density, and by 6 dB per octave and 5 dB increase in frequency; e.g., 20 log (2) = 6 dB/octave. However, a 5 dB/octave and 5 dB per doubling of mass density is more common in practice. This behaviour is termed the "mass law" of homogeneous, isotropic materials. The mass law applies strictly to limp (low bending stiffness), non-rigid partitions and is obeyed up until half the first critical frequency, as discussed below. However, most materials used in buildings possess some rigidity or stiffness. This means that other factors must really be considered, and that the mass law should only be taken as an approximate guide to the amount of attenuation obtainable.

Note that although the field-incidence mass law, Eq. (4.28) is theoretically for $f < f_c$, it yields only accurate results for $f \le 0.5f_c$ and, therefore, for the region between $0.5f_c$ and f_c, the mass law will not yield the required accuracy and one may resort in this case to approximate methods (see Sec. 4.3); see also Example 4.4 and Sec. 4.9.

Approximation

Eq. (4.28) may be simplified using an approximate expression that is often used to determine R in the mass controlled region. For frequencies above about 60 Hz, the term ($\pi M_s f/\rho_0 c$), Eq. (4.28), is usually much larger than 1; consequently, Eq. (4.28) may be approximated by the following expression:

$$R = 10\log\left(\frac{\pi f M_s}{\rho_1 c_1}\right)^2 - 5 = 20\log(M_s) + 20\log(f) - 20\log(\rho_0 c/\pi) - 5 \qquad (4.29)$$

At air pressure, 101.3 kPa (14.7 psia) and standard temperature, 22°C (72°F), the density and sonic velocity of air are: ρ_0 = 1.196 kg/m^3 and c = 344 m/s, so the characteristic impedance of air, $\rho_0 c$ = 411.4 Pa.s/m. Inserting this value into Eq. (4.29), the following expression the sound reduction index in the mass-controlled region is found:

$$R = 20\log(M_s) + 20\log(f) - 47.3 \qquad (4.30)$$

Here, the specific mass, M_s, is in kg/m^2 and the frequency f is in Hz. For normal ranges of temperature and pressure, the variation in $\rho_0 c$ would not alter the R estimate more than ± 1dB. This variation is within the range of interlaboratory variation and may be neglected for all practical purposes. At any rate, Eq. (4.30) can be used when air temperature varies between 20°C and 30°C.

Practical matters on the mass law

In the absence of test data, standard calculation methods exist, although these tend to be conservative and not necessary yield exact results. A wall that obeys the mass law, Eq. (4.28) throughout the frequency range from 125 to 4000 Hz has a sound transmission class as (Kinsler et al. 2000):

$$STC = 20\log M_s + 10 \qquad (4.31)$$

The calculation is based on a best-fit relationship between wall weight and *STC* based on a wide range of test results.

In brick structures, the density of the bricks and the finish layers has an essential effect on the airborne sound insulation of the structure. The standard method (TMS 0302, 2000) outlines procedures for determining *STC* values of concrete masonry walls. As with Eq. (4.31), the calculation is based on a best-fit relationship between wall weight and *STC* based on a wide range of test results, as follows:

$$STC = 0.18M_s + 40 \qquad (4.32)$$

where M_s is here in psf (lb/ft^2) units. This equation is applicable to uncoated fine- or medium- textured concrete masonry. Coarse-textured units, however, may allow airborne sound to enter the wall, and therefore require a surface treatment to seal at least one side of the wall. Coatings of acrylic, alkyd latex, or cement-based paint, or of plaster are specifically called out in TMS 0302 (2000), although other coatings that effectively seal the surface are also acceptable. Eq. (4.32) also assumes the following: (1) walls have a thickness of 3 in. (76 mm) or greater; (2) hollow units are laid with face shell mortar bedding, with mortar joints the full thickness of the face shell; (3) solid units are fully mortar bedded; and (4) all holes, cracks, and voids in the masonry that are intended to be filled with mortar are solidly filled with mortar. If *STC* tests are performed, the standard requires the testing to be in accordance with ASTM E 90 (ASTM, 1999), for laboratory testing or ASTM E 413 (ASTM, 1987) for field testing; refer to Chapter 3.

Eq. (4.31) and Eq. (4.32) indicate that a direct relationship exists between wall weight and the resulting sound insulation. Heavier concrete masonry walls have higher *STC* or R_w values. In practice, however, when a structure is very heavy, its sound insulation cannot be essentially improved by any small increase in mass. Thus, the addition of mass will only be economically feasible from the sound insulation point of view when the structure is originally light. As indicated earlier that when the surface mass of a structure is doubled, its airborne sound insulation improves by 5 dB. Structures that insulate sound on the basis of their mass include concrete and similar massive structures. For example, a properly designed single-leaf partition still requires a large mass per unit area to obtain a respectable *STC*. For example, a 0.15 m (6 in.) thick concrete wall with 13 mm (0.5 in.) plaster on each side has a mass per unit area of 390 kg/m^2 (19.8 psf) and is rated at *STC* 52. To obtain high acoustic isolation without excessive weight, it is necessary to employ double-leaf construction, as discussed later on.

Instructively, it is noticed that Eq. (4.31) doesn't often yield reasonable results for single-leaf partitions. For instance, for the previous example, Eq. (4.31) predicts *STC* = 62, which is 10 point higher than the measured one. Therefore, Eq. (4.31) should be used as a guideline only. The porosity of the material and factors related to the stiffness of the panel, are neglected in developing Eq. (4.31) and Eq. (4.32), although the effect of stiffness is low in the mass-controlled region.

In general, the mass law is affected by resonance at lower frequencies and coincidence at higher frequencies. Increasing panel mass also lowers resonant frequencies and raises the critical frequency.

4.2.3 Coincidence region and the transmission loss

As the frequency of the incident sound wave increases in the mass-controlled region, the wavelength of bending waves (frequency-dependent) in the material approaches the wavelength of the sound waves in the air. In this frequency range, the sound reduction index is adversely affected by the coincidence phenomenon (equality of the wavelengths), as shown in Fig. 2.45, Chapter 2. The coincidence first occurs at grazing incidence (an angle of incidence of 90°) when the trace acoustic wavelength in air matches with the bending wavelength of the plate. When this condition occurs, the incident sound waves and the bending waves in the panel strengthen each other and the wall will vibrate with an amplitude, which approximately corresponds to the particle displacement in the incident sound wave. Because of this matching, the panel offers very little resistance to the sound transmission and the wall would, thus, radiate a sound wave into the receiving room, which has about the same amplitude as that of the incident wave. Consequently, the resulting panel vibration causes a sharp decrease (dip) in the panel sound reduction index or transmission loss at this frequency, which is termed the critical or wave coincidence frequency, f_c. Approximately, this point

corresponds to the transition from Region II behavior to Region III behavior. Because coincidence can be considered analogous to resonance, the *R* of the panel as well as the depth of this coincidence dip in this frequency range depends mainly on the intrinsic damping of the panel, as shown in Fig. 4.1.

For a homogenous plane plate, the frequency of the first coincidence dip or critical frequency, f_c, is expressed by Eq. (2.332), or Eq. (2.331); *E* in these equations is normally the dynamic *E*, which is approximately equal to the static *E* for rigid materials. Note also that Eq. (2.332) concerns the critical frequency for thin plates. However, when the plate becomes thick, i.e. at high frequencies, the propagation of bending waves occurs rather slowly. The frequency above which the plate becomes thick is given in Eq. (4.155) and corrected critical frequency is given by Eq. (4.156). Further, Table A (Appendix) permits one also to calculate $M_s f_c$ from which the critical frequency (the lowest frequency at which coincidence can occur) for different materials can be calculated. Alternatively, Table 2.8 (Chapter 2) offers approximate values of the critical frequency as a function of thickness for some common building materials. Instructively, the values of f_c of materials can also vary according to manufacturing and geological conditions, which should explain why the values of f_c are not exactly the same for the same material, and one may resort to Eq. (2.331) for more accurate results, taking into account that *E* and ρ of the material are known.

In case that a structure has several superimposed layers of panels that are not glued to each other, the coincidence frequencies shall be determined for each panel layer separately. If, on the other hand, they are glued to each other and behave as one unit, then it is sufficient to calculate the whole glued panel.

The mathematical expression for the sound reduction index in the coincidence region is quite complicated and it is rarely used in acoustic projecting. Alternatively, one may use approximate methods, as reviewed in Sec. 4.3 or the numerical calculation of the transmission coefficient, as discussed in Sec. 4.9.

Summary of coincidence features

The coincidence effect has the following characteristics (see Fig. 2.45).

1. The condition for coincidence is that $\lambda_B = \lambda/\sin(\varphi)$. The lowest coincidence frequency (sometimes called limiting or critical frequency, f_c) at which coincidence can occur is when the angle of incidence of the sound is at 90° (grazing incidence), which implies that $\lambda_B = \lambda$. When φ decreases the coincidence frequency f_c increases according to the following expression:

$$f_c(\varphi) = \frac{f_c}{\sin^2(\varphi)} \tag{4.33}$$

The critical frequency f_c can be read from Eq. (2.332) and the coincidence frequency $f_c(\varphi)$, which is function of φ can then be calculated using Eq. (4.33). Accordingly, the problem is not confined to a single frequency; at each angle of incidence φ (oblique incidence), there is a coincidence frequency defined by Eq. (4.33) at which a dip occurs in the transmission loss curve and that the first frequency at which coincidence occurs is when sin $(\varphi) = 1$; see also Fig. 4.31.

2. In the coincidence phenomenon, the sound waves penetrate the panel, and its sound insulation mainly depends on the loss (damping) mechanisms of the panel and the structure.

3. Above the critical frequency, transmission is dominated by coincidence. The coincidence effect continues at higher frequencies but the loss of insulation is gradually reduced.

4. When coincidence occurs, it gives rise to a far more efficient transfer of sound energy from one side of the panel to the other, hence the big coincidence-dip at the critical frequency. In many thin materials (such as glass and sheet-metal), the coincidence frequency begins somewhere between 1000 and 4000 Hz, which includes important speech frequencies.

5. Above the critical frequency, stiffness begins to play an important role again.

6. Coincidence frequencies for different materials occur in different parts of the acoustical spectrum, sometimes outside the normal range used in building acoustics. For best design practices, f_c should not fall in the middle of building acoustic frequency region, i.e. the unfavourable frequency region, approximately: 160 Hz-2000 Hz.

Design of partition with respect to f_c

Every simple panel or plate structure has a coincidence frequency, f_c; the coincidence phenomenon occurs at frequencies higher than this, and the sound insulation of the structure decreases. Therefore, in order to obtain high sound insulation close to that predicted by the mass law one should try either to make f_c as low as possible, below 100-125 Hz or very high, above 3150-4000 Hz; the frequency range 100 Hz-4000 Hz is important for human hearing. In the former case, it is implied that the wall becomes thick with low density and high Young's modulus, e.g. 15 cm (5.9 in) concrete. In the latter case, it is required that the wall instead become thin and has high-density with low Young's modulus, e.g. 13 mm (1/2 in) plasterboard wall, according to Eq. (2.332). Walls of medium thickness that lie in between roughly 1 cm (0.4 in) and 8 cm (3.2 in) are therefore often less favourable, from the acoustical viewpoint. In this context, the coincidence phenomenon is generally not a problem with thick and heavy (massive structures). On the other hand, its effect on the sound insulation of thin, simple concrete and brick structures must be considered in design. The

coincidence frequency of thin building panels is usually in the range of 2,000 Hz to 3,000 Hz. The higher the coincidence frequency of a building board, the smaller the effect of the coincidence phenomenon on the sound insulation of a structure.

To demonstrate how the coincidence effects the sound insulation experimentally, Fig. 4.3 shows idealized transmission loss curves, including coincidence dips, for some common materials. Materials 1, 2, and 3 all have the same mass per unit area, but quite different *STC* ratings ($\approx R_w$) because of differing coincidence effects. In the thicknesses commonly used in practice, the critical frequencies of concrete and plywood lie within the frequency range that is important in building acoustics (100 to 4000 Hz), and therefore they are more susceptible to *STC* reductions due to the effects of coincidence. For gypsum wallboard, the coincidence frequency is quite high and the effect on the *STC* is usually less; wallboard is also commonly known as drywall as gypsum board, and plasterboard.

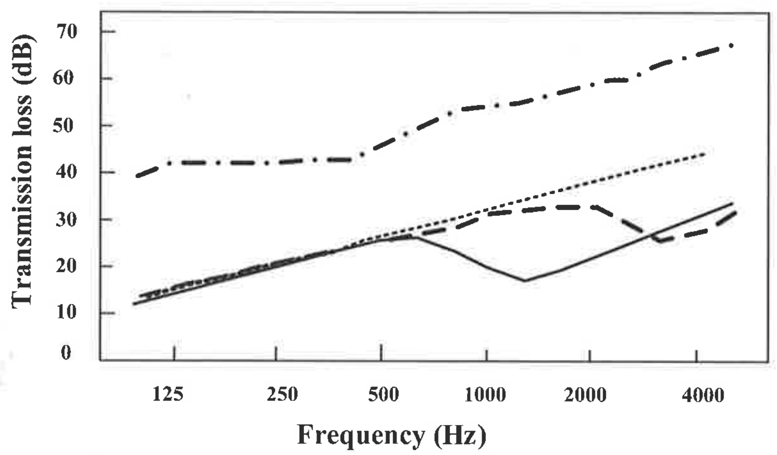


Fig. 4.3 Measured sound reduction index of typical single-leaf walls. ——— 16 mm plywood, 10 kg/m², *STC* = 21; — — — 13 mm wallboard, 10 kg/m², *STC* = 28; ••••• 1.3 mm steel, 10 kg/m², *STC* = 30; — • — • 100 mm concrete, 235 kg/m², *STC* = 52 (after Quirt, 1985; Warnock, 1985).

The depth of the coincidence dip is determined by the damping (energy losses) in the material and at its edges where it is in contact with other materials in the supporting structure. The greater the damping, the shallower the coincidence dip and the less the effect on the sound insulation. Coincidence dips are a problem for materials with low internal damping and high bending stiffness (such as metals or glass). In old building practice, lead sheet or leaded vinyl were broadly used. In the modern practices, loaded vinyl (impregnated with non-lead metal) is a superior option.

4.2.4 Region III: Damping-controlled region

For frequencies well above the critical frequency, the sound reduction index is strongly dependent on the frequency of the incident sound waves, surface density and stiffness of plate, and the damping of the plate material. Therefore, this region is also called the upper stiffness region.

For sound waves striking the plate at all angles (random incidence) at frequencies greater than the critical frequency, the following empirical field-incidence expression applies for the sound reduction index in the damping-controlled region ($f \geq f_c$) (Beranek, 1971):

$$R = R_n(f_c) + 10\log(\eta) + 33.22\log(f/f_c) - 5.7 \tag{4.34}$$

The quantity $R_n(f_c)$ is the sound reduction index for normal incidence at the critical frequency:

$$R_n(f_c) = 10\log\left(1 + \left(\frac{\pi M_s f_c}{\rho_0 c}\right)^2\right) \tag{4.35}$$

The quantity η is the total loss factor or damping coefficient for the panel. Eq. (4.34) is also called the modified mass law, and can be simplified in one expression, after inserting the necessary constant values, as

$$R = 20\log M_s - 13\log f_c + 33\log f + 10\log\eta - 48 \tag{4.36}$$

The latter expression can be used when air temperature varies between 10°C and 30°C. Note that an equivalent form to Eq. (4.36), which can be seen in some literature is: $R \approx 20\log M_s - 10\log f_c + 30\log f + 10\log\eta - 44.5$, where the constant 44.5 is sometimes approximated to 45.

Surface factor

When the panel is a part of a construction and the excited part of the plate, S, is less than the area of the whole plate S_{tot} ($S_{tot} \geq S$), then the surface factor, $\{-10\log(S/S_{tot})\}$ should be added to the right-hand side of Eq. (4.36) or Eq. (4.34). This case is emphasized in a typical dwelling in which the walls that separate the two apartments are load-bearing (e.g., concrete) while the internal walls in the apartments are made of lightweight materials (plasterboard walls) that are not load-bearing. This implies that at vertical measurement, S is the excited area of the plate in the source room and S_{tot} is plate's total area from façade to façade and from load-bearing wall to a load-bearing wall. This addition in terms of surface factor is to compensate for the loss of vibration energy in the plate S_{tot} due to the intersection of lightweight partition and heavyweight partition in which the lightweight partition contributes in this case inconsiderable energy loss. If, on the other hand, $S_{tot} = S$ as in the case of intersected heavyweight partitions of same properties, the surface factor turn to zero. It is assumed here that the vibration level (energy density for bending waves) is approximately the same over the whole surface, S_{tot} (Kihlman, 1982).

Plate size

For most thin homogeneous materials commonly used in building construction, the mass law, Eq. (4.28) and the modified mass law, Eq. (4.34) provides a good prediction of the transmission loss over most of the frequency range. However, an additional effect may be taken into account to obtain good accuracy at low frequencies; this is radiation efficiency of forced waves due to the influence of plate size. Subsequently, to improve the accuracy at low frequencies, a correction factor, which takes into account the influence of plate size (finiteness) is suggested (Sewell, 1970) as

$$\Delta R = -10\log\{\ln(kS^{1/2})\} + 20\log\{1 - (f/f_c)^2\} \tag{4.37}$$

where S is the plate area and k is the wave number of air. Using experimental testing, it was noticed that this correction is to be used for $f < 200$ Hz for a normal test constructions of area: 10-12 m^2 (Ballagh, 2004).

Discussion

- The loss factor, η, in any expression of R is the total loss factor, which depends not only on the wall's internal (material) losses or damping but also on the energy loss along the wall's edges (boundary or edge damping) or so called coupling loss factor; see Sec. 4.10. The internal losses are, for typical building materials, very small, usually in the interval (0.3%-1%). The energy losses across edges (coupling loss factor) yield often greater contribution to the loss factor than the internal losses. However, it is dependent on the how the wall's boundary conditions are made. For example, a simply supported wall at edges has losses essentially lower than if the wall is fixed at edges, which results in a lower R in the former case. A difference amounted to above 5 dB can most likely occur between extreme cases. If, on the other hand, the panel is not attached to adjoining structure, then the total loss factor is approximately due to the material losses only, assuming typical building materials.

• For the damping-controlled region and assuming that η is constant, the sound reduction index is proportional to 33.22log (f). If the frequency is doubled, the sound reduction index is increased by 33.22log (2) = 10 dB/octave. In practice, however, the loss factor drops with increasing frequency. Thus, the slop in region III (also called upper stiffness region) should conservatively be considered as 7.5-9 dB /octave.

• The sound insulation of thin plate increase (7.5-9 dB/octave) in region III (above f_c) is valid as long as the wall vibrations are considered a 2-dimensional. At higher frequencies, the oscillations become 3-dimensional in which special effects of thick plate appear and Eq. (4.36) or Eq. (4.34) will not be valid in this case. Consequently, the above simple expressions are not adequate to describe very thick and heavy panels such as concrete or brick. For these types of panels, shear waves become the dominant flexural waves at high frequencies. This affects the transmission loss at high frequencies reduces the frequency dependence to 6 dB/octave at high frequencies instead of the 7.5-9 dB/octave behaviour of thin panels above the critical frequency. Instructively, at high frequencies, one may resort to direct numerical methods suitable for thick plates, as discussed in Sec. 4.9.

Example 4.1: A door made of pine wood has dimensions of 1 m (39.4 in) wide by 1.7 m (67 in) high by 40 mm (1.6 in) thick. The air on both sides of the door has a temperature of 20°C (68°F). Determine the sound reduction index for the following frequencies: (a) 63Hz, (b) 250 Hz, and (c) 1000 Hz. Assume only material losses occur, and the pine door has high stiffness.

Solution

The following properties of the pine door are obtained from Table A (Appendix):
Density ρ= 640 kg/m^3 (40 lb$_m$/ft^3)
Damping factor η = 0.02
Young modulus E= 13.4 GPa (=1.88x10^6 psi)
Poisson's ratio μ= 0.15
From Table 2.4: c = 343.2 m/s (1126 ft/sec), ρ_0 = 1.204 kg/m^3 (0.0752 lb$_m$/ft^3), so the characteristic impedance reads, $z_0 = \rho_0 c$ = 413.3 rayls.

The propagation velocity of the quasi-longitudinal is obtained from Eq. (2.129):

$$c_L = \left(\frac{13.4(10^9)}{640(1-0.15^2)}\right)^{1/2} = 4628 \text{ m/s}$$

The critical or wave coincidence frequency is found from Eq. (2.332):

$$f_c = \frac{c^2\sqrt{3}}{\pi c_L h} = \frac{(343.2)^2(3^{1/2})}{\pi(4628)(0.040)} = 351 \text{ Hz}$$

The surface density is:
$M_s = \rho h$ = 640(0.04) = 25.6 kg/m^2
The first resonant frequency is found from Eq. (4.23):
f_{11}= (343.2)^2/4(351) [(1/(1)^2)+((1/(1.7)^2)] =113 Hz

(a) For f = 63 Hz.
The frequency, f = 63 Hz < 114.3 Hz = f_{11}.
Thus, this case lies in Region I, the stiffness-controlled region. The spring constant may be evaluated from Eq. (4.19)

$$k_s = \frac{\pi^8(13.4)(10)^9(0.04^3)\{(1/1^2)+(1/1.7^2)\}}{768(1-0.15^2)} = 1.459\times10^7 \text{ N/ m}^3$$

The value of the parameter defined by Eq. (4.13b) is as follows:

$$K_s = \frac{4\pi f z_0}{k_s} = \frac{4\pi(63)(413.2)}{1.459(10^7)} = 0.022$$

The mean sound power transmission coefficient may be calculated from Eq. (4.16):
$$\bar{\tau} = K_s^2 \ln(1+K_s^{-2}) = (0.022)^2 \ln(1+(0.022)^{-2}) = 0.0037$$
The sound reduction index for a frequency of 63 Hz, thus, becomes:
$$R = 10\log(1/0.0037) = 24.3 \text{ dB}$$

(b) For f = 250 Hz.
For this case, f_{11}= 113 Hz < 250 Hz < 351 Hz = f_c.
Accordingly, the operating region is Region II, the mass-controlled region. The sound power transmission coefficient for normal incidence is found from Eq. (4.26):

$$\frac{1}{\tau_n} = 1+\left(\frac{\pi f M_s}{z_0}\right)^2 = 1+\left(\frac{\pi(250)(25.6)}{413.3}\right)^2 = 2368$$

The sound reduction index for normal incidence is found from Eq. (4.28):
$R_n = 10\log(1/\tau_n) = 10\log(2368) = 33.7$ dB
The sound reduction index associated with random incidence is found from Eq. (4.28):
$R = R$ −5 = 33.7−5 = 28.7 dB

(Note that this result is very approximate since the validity of field-incidence mass law is up to $f \leq 0.5f_c$)

(c) For $f = 1000$ Hz.
The frequency, $f = 1000$ Hz > 351 Hz $= f_c$.
Accordingly, this case lies in Region III, the damping-controlled region. The sound reduction index for normal incidence at the critical frequency is found from Eq. (4.35):

$$R_n(f_c) = 10\log\left(1 + \left[\frac{\pi(25.6)(351)}{413.3}\right]^2\right) = 36.7 \text{ dB}$$

The transmission loss for a frequency of 1000 Hz is found from Eq. (4.34):

$$R = 36.7 + 10\log(0.02) + 33.22\log(1000/351) - 5.7 = 29.1 \text{ dB}$$

Alternatively, using Eq. (4.36):

$$R = 20\log(25.6) - 13\log(351) + 33\log(1000) + 10\log(0.02) - 48 = 29.1 \text{ dB}$$

Example 4.2: A steel plate (density 7700 kg/m^3) has dimensions of 1 m (39.4 in) wide by 1.7 m (67 in) high. The air on both sides of the plate has a temperature 20°C. After checking the sound insulation requirements, it is found that if the transmission loss (or sound reduction index) of the plate becomes 33 dB at a frequency of 500 Hz, then the sound insulation will fulfill the requirements. What is the required thickness of the plate?

Solution
From Example 4.1: $c = 343.2$ m/s (1126 ft/sec), $\rho_0 = 1.204$ kg/m^3 (0.0752 lb$_m$/ft^3), $z_0 = \rho_0 c$ = 413.3 rayls. From Table A (Appendix), the properties of steel are selected as: $\rho = 7700$ kg/m^3, $E = 200$ GPa, and $\mu = 0.3$.

The problem under concern doesn't clarify in which region the required the sound reduction index is located in, so this problem involves iteration. However, it is clear that at 500 Hz, the transmission loss can either be in Region II or Region III since f_{11}, the limit of Region I, cannot be higher than 500 Hz, for typical applications.

Let us first consider Region II, the mass-controlled region.
The required sound reduction index for normal incidence is given by Eq. (4.28):

$$R_n = R + 5 = 33 + 5 = 38 \text{ dB}$$

Thus,

$$R_n = 10\log\{1 + (\pi M_s f / z_0)^2\} = 38 \text{ dB} \Rightarrow (\pi M_s f / z_0)^2 = 10^{38/10} - 1 = 6309$$

The surface mass is:

$$M_s = \frac{(6309)^{1/2}(413.2)}{\pi(500)} = 20.90 \text{ kg/m}^2 = \rho h$$

Thus, the required thickness (if the R region is Region II) is as follows:

$$h = \frac{20.90}{7700} = 0.00271 = 2.71 \text{ mm (0.11 in)}$$

Now, let us check the assumption of Region II behavior.
The propagation velocity of the quasi-longitudinal is obtained from Eq. (2.129):

$$c_L = \left(\frac{200(10^9)}{7700(1-0.3^2)}\right)^{1/2} = 5343 \text{ m/s}$$

The critical frequency is found from Eq. (2.332):

$$f_c = \frac{c^2\sqrt{3}}{\pi c_L h} = \frac{(343.2)^2(3^{1/2})}{\pi(5343)(0.00271)} = 4485 \text{ Hz} > 500 \text{ Hz} = f$$

Let's check also the first resonant frequency:

f_{11}= (343.2)^2/4(4485) [(1/(1)^2)+((1/(1.7)^2)] = 8.8 Hz

Subsequently, the frequency $f = 500$Hz lies in Region II, because $f_{11} < f < f_c$, and the required panel thickness is:
$h = 2.71$ mm = 0.11 in.

4.2.5 Practical design guideless for a single wall

Resonance and coincidence effects cannot be eliminated. As indicated before, if the designer aims to create the maximum transmission loss, an attempt should be made to get resonant frequencies as low as possible (preferably well below the audible range) and the critical frequency as high as possible (preferably well above the audible range). In a broader sense, the ideal barrier material should have a high density and low bending stiffness (i.e. very limp). Dense, limp materials have the tendency to push the coincidence frequency upward and out of the range of interest. As it is not possible to apply a generic solution to all single panels, the following guidelines are applied:

• Reducing the stiffness of a panel lowers its resonant frequency and raises its critical frequency, basically increasing the region for which the mass law applies. One desires resonant frequencies be below range of human hearing.
• Increasing panel mass also lowers resonant frequencies and raises the critical frequency.

• Decreasing panel thickness raises the critical frequency but generally reduces panel mass. One also should consider that high density gives high R in mass-controlled region.
• Increasing the amount of damping applied to the panel will not alter the frequencies of resonance and coincidence but will act to reduce their effect. High internal damping can prevent resonant modes from "ringing".
• Materials with very low stiffness such as sheet lead effectively do not show coincidence dips.

Good insulation is therefore a combination of low stiffness, high mass and high damping, taking into consideration the cost constraints. Consequently, the ideal material for high R is sheet lead, which has both high density and low stiffness. Unfortunately, due to health concerns as well as pollution to environment, lead can no longer be used. For the same reasons, gypsum board is a good barrier material and is more effective than plywood (which is stiffer and not as dense as gypsum board); see e.g., Fig. 4.3. Loaded vinyl, or vinyl impregnated with metal filings, is a common material for high R.

The most common method of adding damping is to apply a thick layer of mastic-like material to one side of the panel. This type of treatment is only effective on materials that have low mass and an inherent lack of damping. It would be useless on thick concrete walls, for example, but very effective on metal automobile panels.

The insulation of a single-leaf panel can be improved in a number of ways, but this process can only continue up to a certain point given the exponential increase in mass required. Consider the example of a single brick wall with an R= of 22 dB. To increase this to an overall 40 dB in all regions, the mass must be increased to 8 times the original (2^3). This is clearly impractical from a building perspective. Consider, on the other hand, the fact that the wall already has an R= 22 dB. If one were to build another brick wall right next to it, in theory one could achieve a further drop of 22 dB. A situation approaching this is possible if the two walls are completely separated from each other with no common links, footings or edge supports, and an air gap greater than 1 m between them. However, this is often just as impractical as it is vastly increasing the mass of the wall and/or disagreeing with the architectural design. In practice, walls do have common supports at the edges. It is also rare to find a cavity wall with more than few centimetres of air gap. In this context, the following practical guidelines may be considered for better sound insulation for a wall with a cavity and/or attached layers; see also Sec. 4.4.
• Well sealed cavities can result in an increase in sound insulation well above mass law, assuming the cavity depth is not small, e.g., at least 100 mm deep.

• Use of layers of different thickness can greatly assist in mismatching resonant and critical frequencies across the panel.
• The use of absorbent materials within the cavities can help to further reduce sound transmission.
• Only resilient elastic materials should be used as wall ties and suspension members such as elastic elements to reduce any direct connection between layers.
• If required, only widely spaced and staggered studs should be used within partitions.
• Caulking (i.e., the process of sealing a gap between two surfaces for the purpose of making it air or watertight using e.g., acrylic latex caulk) and sealants should be used to eliminate perimeter sound leaks. This point is rather important in pratice as it alludes to flanking. One of the highest achievable R value for a partition is about 55-60 dB. Above 45-50 dB, flanking paths become more and more important. This explains why multiple-layer (three or more) partitions do not offer any significant improvement over double-leaf construction, as discussed later on.

At grazing incidence where the wavelength of the sound in air is the same as the bending wavelength of the partition, the transmission of the sound is high with consequent loss of insulation due to coincidence effect, as discussed earlier. Typical plasterboard has a superficial mass of about 18 kg/m^2 and a critical frequency of 2000 Hz. Stiff materials show a reduced coincidence effect and improved insulation at the resonances if external damping is provided by the fixings. Lead clearly has many disadvantages, it can flow under its own weight, but it can be incorporated within the sheet material. Note, however, that the environmental impact of lead should be investigated carefully before any application is decided.

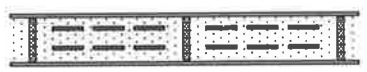
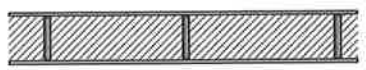
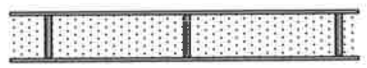
4.2.6 Examples of sound insulation of single-leaf walls

In order that a single wall should have a good sound insulation, which is interesting in relation to the dwelling houses, offices, etc it is preferred that it is made relatively thick and stiff. Ordinary materials are, therefore, concrete, brick and the like. In Table 4.1, it is presented examples of measured insulation index for some typical constructions. The measurements have taken place under laboratory conditions, i.e., with relatively small areas (in general 10 m^2) and fixed edges, i.e. high energy loss at edges. In the field, there are usually other conditions, which seem to decrease the insulation such as flanking. Typically, a difference between the two cases can amount to 5 dB. Other examples of single partition with ratings in R_w and *STC* are presented in Appendix 2 of the book.

4.3 Approximate Methods for Estimating the Sound Reduction Index for Single Leaf Partitions

In preliminary design, it is often required to estimate the sound reduction index spectrum for a panel. However, since the prediction of sound transmission loss over the whole frequency range is complicated, especially at the coincidence region, it is suggested a number of simplified schemes to calculate the sound reduction index R (= transmission loss TL) for sing-leaf partitions. Two of these methods are, herein, presented.

Table 4.1 Examples of insulation index for single-leaf walls measured in the laboratory.

Construction type		Features
150 mm 180 mm		15 cm Concrete: $R_w \approx 58$ dB 18 cm Concrete: $R_w \approx 60$ dB
230 mm		20 cm massive concrete block with plaster layer (15 mm on both sides): $R_w \approx 58$ dB
280 mm		25 cm hollow concrete block with plaster layer (15 mm on both sides): $R_w \approx 58$ dB
150 mm 140 mm		½-stone brick wall with plaster layer (10 mm) on both sides: $R_w \approx 53$ dB The same but with plaster layer on one side only: $R_w \approx 50$ dB
330 mm		30 cm lightweight concrete block with plaster layer (15 mm on both sides): $R_w \approx 50$ dB

4.3.1 First approximate method

This method is advanced to estimate the sound reduction index or transmission loss curve for Regions II and III (Watters, 1959). If the plate has dimensions a and b that are at least 20 times the panel thickness h, the first resonant frequency for the panel is usually less than 125 Hz, so the major portion of the sound reduction index curve will involve Regions II and III. This case is typical for evaluating the transmission loss in building acoustics applications. However, when using this approximate method, one should also check the importance of the Region I behavior. For frequencies below the plateau (Region II), Fig. 4.4, the sound reduction index is approximated by Eq. (4.30). As inspected from Eq. (4.30), when the frequency (a frequency change of one octave) is doubled, the change of the sound reduction index becomes: $\Delta R = 20\log(2) = 6.02 \approx 6$ dB/octave. At the coincidence region, a horizontal line or plateau is reached and the average transmission loss is practically constant. The frequency extent of this plateau depends on the damping, or the panel material. Consequently, the approximate method replaces the coincidence region (Fig. 4.1) between Region II and Region III by a plateau, as shown in Fig. 4.4. The height of the plateau (R_P) and the width of the plateau, Δf_P, depend on the panel material. Table 4.2 shows the plateau average height for some common single panel partition wall materials. Above the plateau (i.e., the damping-controlled region), the transmission loss increases again as 10 dB/octave, according to Eq. (4.34).

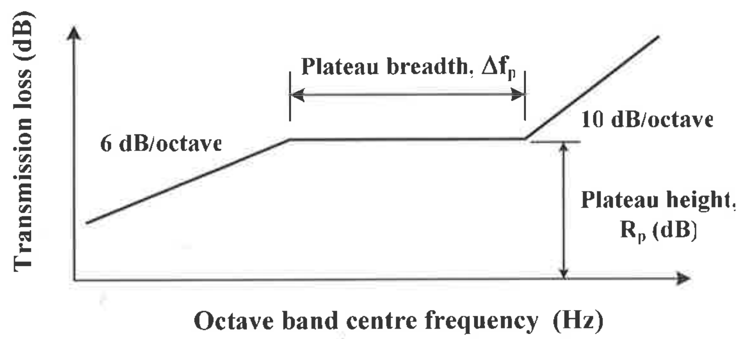


Fig. 4.4 Schematic of the approximate curve for the sound reduction index of a panel. The chart assumes a reverberant sound field on the source side and approximates the behaviour around critical frequency with a horizontal line or plateau. The plateau width is $\Delta f_P = f_2 - f_1$, where f_1 is the frequency at which the plateau begins and f_2 the frequency at the end of the plateau. The plateau width indicates region II, mass-controlled region. The slope 10 dB/octave depends on size of panel, edge damping and internal damping (total loss factor). The slope 6 dB/octave is field-incidence transmission loss.

In Region III, the slope of sound reduction curve is 10 dB/octave.

For a conservative estimate of the sound reduction index, it is recommended that the R curve in Region III be drawn with a slope of the 10 dB/octave for the first 2 octaves above the plateau. The remainder of the curve should be drawn with a slope of 6 dB/octave (Beranek, 1960). Note, however, as discussed earlier, that the practical reduction in sound transmission loss is not actually 10 dB/octave in Region III, but varies between 7.5-9.0 dB/octave.

Table 4.2 Values of the plateau height (R_P) and plateau width (Δf_P) for the approximate method of calculation of the transmission loss for panels (partially after Watters, 1959).

Material	Specific surface density (kg/m^2 per cm)	Plateau height, R_P (dB)	$\Delta f_P = f_2 - f_1$ (octave)	Plateau breadth, frequency ratio, f_2 / f_1
Aluminum	26.6	29	3.5	11*
Brick	21	37	2.2	4.5
Concrete, dense	22.8	38	2.2	4.5
Glass	24.7	27	3.3	10
Lead	112	56	2.0	4
Masonry block				
Cinder**	11.4	30	2.7	6.5
Dense		32	3.0	8
Plywood, fir	5.7	19	2.7	6.5
Plaster, sand	17.1	30	3.0	8
Steel	76	40	3.5	11*

* These materials have, in general, very low damping. The numbers are for a typical panel in place

** Hollow block. The values are determined for 6-in (150 mm) plastered block.

4.3.2 Second approximate method

This method, called also a template method resembles somewhat the first method, treated above. In this method, a consideration is given to the influence of critical frequency on the sound insulation of a structure, and the slope of Region III is set to 7.5 dB/octave. Since it is common that Region I is rarely encountered in building acoustics practices, Region I will not be considered. This method is outlined in the following steps; see Fig. 4.5.

I. The critical frequency for typical building constructions is calculate as

$$f_c \approx 1.83\times10^4 \sqrt{M_s}/\sqrt{B} \qquad (4.38a)$$

For the special case of homogenous, plane-parallel wall, Eq. (4.38a) becomes:

$$f_c \approx 6\times10^4 \sqrt{\rho}/h\sqrt{E} \qquad (4.38b)$$

where ρ and E are the density and Young's modulus of the plate, respectively.

II. Calculate the plateau height, R_p (dB) for the panel. In general, for single wall, this is expressed as

$$R_p = 20\log(M_s) + 20\log(f_c) - 58.5 \qquad (4.39a)$$

For the special case of homogenous, plane-parallel wall, Eq. (4.39a) becomes:

$$R_p = 30\log(\rho) - 10\log(E) + 37.5 \qquad (4.39b)$$

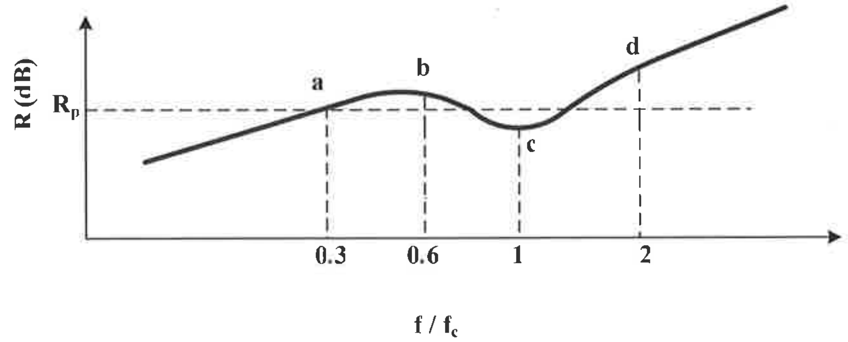


Fig. 4.5 Schematic for calculation of sound reduction index for a single-leaf wall according to the second approximation method. The frequency scale is logarithmic and the reduction scale linear. R_p is the plateau height, f_c is the critical frequency, and points a, b, c, d are used to draw the transmission loss curve.

III. Construct the low-frequency asymptote. At $f / f_c = 0.3$, draw a vertical line, which intersect with the value of R_p (calculated from Eq. (4.39)) in point a and draw from there, towards lower frequencies, a straight line with the slope 6 dB/octave, as illustrated in Fig. 4.5. The frequency scale must be logarithmic and the reduction scale linear;

IV. At $f / f_c = 0.6$, the point b must be 3 dB above R_p.

V. The point d. Put in the point d on {25+10 log (η)} dB above R_p at $f / f_c = 2$.

VI. Construct the high-frequency asymptote. Draw from d a straight line, towards high frequencies, with the slope 7.5 dB/octave.

VII. The approximation for the coincidence region. Join the points, a-d so that a dip is formed at a point c at f_c.

The value of point c (coincidence dip) is as follows: 5 dB under R_p, if $f_c \geq 200$ Hz; 4 dB, if 200 Hz $\geq f_c \geq$160 Hz; 3 dB, if 160 Hz $\geq f_c \geq$125 Hz; and 2 dB, if 125

Hz $\geq f_c \geq$100 Hz. Note, however, that a real dip in the sound reduction index's curve will not occur if f_c is below 200 Hz. In this case, the points, b and c are put directly on the plateau R_p. The application of this approximate method for estimating the sound reduction curve is illustrated in Example 4.3.

Discussion

This method is based on empirical values to estimate the sound reduction index for typical building constructions. It is widely used in Sweden and was developed by Stig Ingemansson (see e.g., Ingemansson and Elvhammar, 1977). Note that this method doesn't consider the case when the sound reduction index curve will drop, approximately, at 6 dB/octave at high frequencies, $f >> f_c$.

Example 4.3: Calculate the sound reduction index in the frequency range 100-3150 Hz for a massive concrete wall, thickness 90 mm, ρ = 2400 kg/m^3; η = 2%. and E of the wall is 26 ×10^9 MPa. Use the approximate method.

Solution

Critical frequency may approximately be calculated using Eq. (4.39b):

$f_c = 6\times10^4\,(\rho)^{1/2}/h(E)^{1/2} = 6\times10^4\,(2400)^{1/2}/0.09(26\times10^9)^{1/2} = 203$ Hz.

The plateau height is obtained from Eq. (4.39a):

$$R_p = 20\log(2400(0.09)) + 20\log(203) - 58.5 \approx 35 \text{ dB}.$$

Point a on R_p = 35 dB and at f / f_c = 0.3, it follows that f = 203 × 0.3 = 61 Hz.

Thus, is plotted a line, towards lower frequencies, with the slope 6 dB/octave (see Fig. 4.5).

Point b on R_p +3 dB = 38 dB and at f / f_c = 0.6, it follows that:

f = 203 × 0.6 = 122 Hz.

Point c is 5 dB below R_p (because f_c > 200 Hz) and at f_c.

Point d on R_p+25+10logη =35+25+10log (0.02) = 43 dB and at f / f_c = 2, it follows that f = 203 × 2 = 406 Hz.

Thus, towards higher frequencies it is plotted a line with the slope 7.5 dB/octave. These steps are illustrated in the Fig. 4.6.

4.4 Sound Reduction Index (Transmission Loss) for Composite Walls

The material presented in the previous sections applies for transmission of sound through homogeneous, single-component (leaf) panels, such as a plate of glass. In this section, more complex constructions that can be analyzed analytically will be considered.

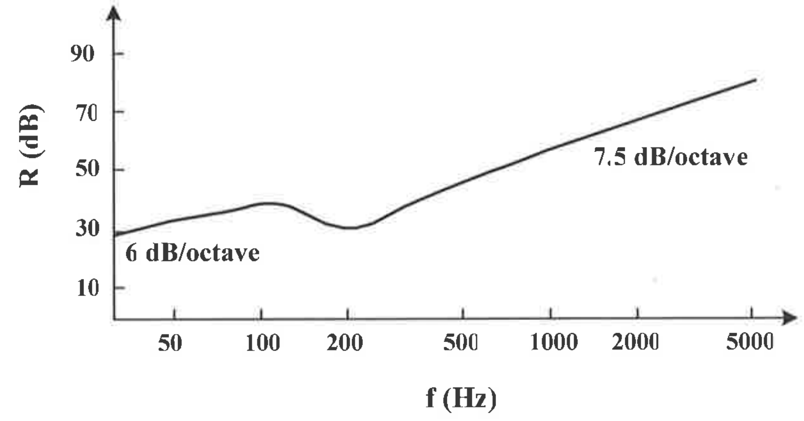


Fig. 4.6 An example application to calculate the sound reduction index for a massive concrete wall, Example 4.3.

4.4.1 Background

If one only increases the thickness of a single-leaf partition to improve the sound insulation in the mass law region, this can lead to worse performance because the coincidence frequency will reduce (Eq. (2.332)), and can fall into a more important region. Moreover, the panel becomes heavier and lightweight construction is to be preferred, not to mention the incurred costs for such designs. For such situations, an increase in sound insulation can be obtained by using double-wall construction. That is, a pair of double walls separated by an air space is very much more effective than an equivalent weight single wall. As discussed in Chapter 3, the sound insulation for a partition of double-leaf construction is considerably higher than that for a single-leaf partition of the same mass density. For instance, if one compares the transmission loss curve for two sheets of 13 mm (½ in.) gypsum board bonded together as a single leaf with that obtained for the same sheets used in a double-leaf construction, one would notice that an improvement is achieved, and when absorbing material is placed in the cavity between the boards, higher performance is obtained. A double wall construction with an air gap between (or even triple construction) would, certainly, perform better than a single one. Consequently, a method to obtain higher sound reduction index or transmission loss than what is available with normal single-leaf constructions is to use a double-leaf wall. By definition, double wall is wall that is composed of two surface layers separated by a cavity of a gap. The two surface layers (leaves or panels) can either be separated or be connected with e.g., joists or beams. The cavity can either be empty or filled with mineral wool. In practice, if the space between the walls is 30 cm or more, the overall transmission loss is approximately the sum of the transmission loss for each wall. In certain situations where a large transmission loss is required, the materials can be installed in several layers, rather than one thick one. The requirement is that there should be minimum mechanical connection between layers, other than the air space.

In general, the sound propagation though a double wall with connections can occur on three different ways, as depicted in Fig. 4.7:

A. Totally separate wall leaves: the sound passes through air cavity.

B. The wall leaves have mechanical connections with each other and the sound passes via these connections, called structure-borne sound.

C. The wall leaves are fixed on edges and so the sound passes via fixed edges. This also called structure-borne or flanking sound.

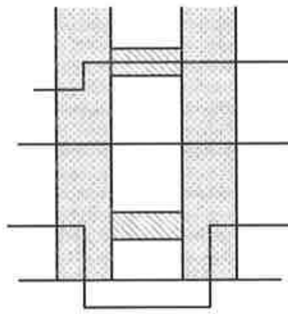


Fig. 4.7 Sound propagation through a composite wall occurs via three paths. For the path B, the fixation or connecting beam can give rise to a sound source. The structural forces excites, in turn, the plate at the fixation area at which free bending waves will spread out in the receiving leaf.

In general, the sound reduction index for a double wall can be written as

$$R = R_1 + R_2 + K \quad (f > f_0) \tag{4.40}$$

where R_1 and R_2 are sound reduction indices for respective wall's two leaves (dB), K is the coupling factor-usually negative (dB) and f_0 is the combined system *eigen* (resonant) frequency (Hz). The coupling factor K depends mainly on the properties of the air cavity: width, absorption, and mechanical connections. In the case that connections are slightly important, K only weakly depends on the leaves sound reduction index. A modification on the isolation of either leaf yields therefore a corresponding change in the double wall.

In comparison with the theories related for single walls, the mathematical derivation of the sound reduction index of double walls is substantially

complicated. Additionally, it is difficult to give the resulted expressions a form that permits some direct conclusions concerning the influence of different parameters. Note that, at present, there no comprehensive theory, which describes the sound reduction index of double walls in the whole frequency region of interest.

4.4.2 Composite wall without mechanical connections

There is two main routes for the energy flow through a double wall; radiation from the first face to the air cavity at which the second face moves and radiate sound in the receiving room, in addition to the structure-borne transmission between the leaves via mechanical connections. In this section, it is studied only the first form of transmission; i.e., path A, Fig. 4.7.

In order to fulfill the sound insulation requirement for a wall between apartments using a double-framed wall, it is not preferred to have mechanical coupling between the separate frames. Thus, there has to be an air gap between the frame halves and in addition to this, it is recommended that the studs be installed at different locations in the frame halves. Fig. 4.8 shows a double wall without mechanical connection. The impinging sound waves from a sound source make one half of the wall vibrate. The air space between the boards serves as a "spring", transmitting the oscillating motion to the other half of the wall. For this construction, the overall transmission loss is influenced by the air mass in the space, whether it is empty or filled by an absorbent, in addition to the effect of the transmission loss of each separate panel. In general, the following frequencies are important to the transmission problem:

- the coincidence frequencies of the two leaves, f_{c1} and f_{c2};
- the lowest order resonance across the air cavity, f_a;
- the lowest order structural resonance, f_0; and
- the lowest order resonance of the air cavity, f_1.

The characteristic or structural resonant frequency, f_0, defines the frequency at which the air within the cavity of a double wall acts like a spring, coupling the (limp) masses of the panels to form a resonant mechanical vibration response of the system. At f_0, the two panels move in opposite phase, bouncing on the ''spring'' of the air in the gap. This frequency may be derived as follows. Consider the representation of the double wall (Fig. 4.8) as a simple mass-spring-mass system, as shown in Fig. 4.9. Further, that the sound wave is assumed to excite the partition at normal incidence.

The equilibrium of the forces (N per unit area of panels) for this system results into two simultaneous equations as follows:

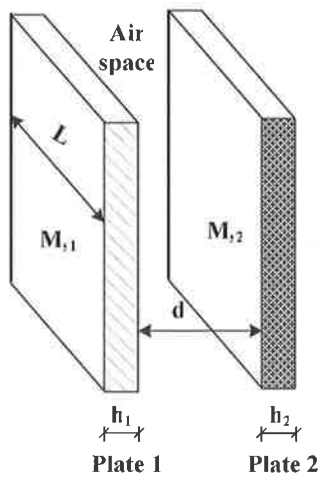


Fig. 4.8 Double partition without mechanical connection. M_{s1} and M_{s2} are the surface densities for panels 1 and 2, respectively.

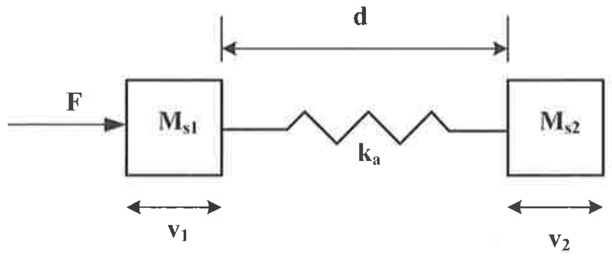


Fig. 4.9 Representation of the double wall as a mass-spring system under low frequencies. k_a is the spring stiffness, v is the horizontal velocity of respective panel, F is the applied force, and d is the spacing between the panels.

$$F - v_1 j\omega M_{s1} = (v_1 - v_2)\frac{k_a}{j\omega} \tag{4.41a}$$

$$v_2 j\omega M_{s2} = (v_1 - v_2)\frac{k_a}{j\omega} \tag{4.41b}$$

These two equations may be solved and the following equation is obtained:

$$\frac{F}{v_2} = \frac{1}{j\omega k_a}\left[(k_a - \omega^2 M_{s2})(k_a - \omega^2 M_{s1}) - k_a^2\right] \tag{4.42}$$

The term $j\omega M_s$ is the impedance of the panel (Ns/m per unit area), k_a is the dynamic air stiffness constant, M_{s1} and M_{s2} are the surface densities ($=\rho h$, where ρ is volume density) of panel 1 and 2, respectively, and v is here the panel velocity. Now, setting the impedance of panel 2: $F/v_2= 0$, and solving Eq. (4.42), the following equation is obtained:

$$\omega^2 = \frac{k_a(M_{s2} + M_{s1}) \pm \sqrt{k_a^2(M_{s2} + M_{s1})^2}}{2M_{s2}M_{s1}} \tag{4.43}$$

The (–) term with square root has no physical meaning since in this case ($\omega = 0$) and so Eq. (4.43) becomes:

$$\omega^2 = \frac{k_a(M_{s2} + M_{s1})}{M_{s2}M_{s1}} \tag{4.44}$$

Accordingly, the mass-spring-mass resonant frequency becomes:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_a(M_{s2} + M_{s1})}{M_{s2}M_{s1}}} \tag{4.45}$$

The air stiffness constant (or dynamic stiffness of the gap), k_a (N/m per unit area), is expressed at low frequencies as:

$$k_a = \gamma P_0 / d = c^2\rho_0/d \tag{4.46}$$

where γ is the ratio of the specific heat at constant pressure and constant volume ($\approx$ 1.4), P_0 is the static sound pressure (typical sea level atmospheric pressure 101 325 Pa $\approx 10^5$ Pa) and ρ_0 the air sound density, and c is the sound speed in air (Table 2.8). Substituting Eq. (4.46) into Eq. (4.45) yields f_0 (the resonant frequency of the two panels coupled by the air space):

$$f_0 = \frac{c}{2\pi}\left[\frac{\rho_0}{d}\left(\frac{1}{M_{s1}} + \frac{1}{M_{s2}}\right)\right]^{1/2} \tag{4.47a}$$

It is important to note that f_0, expressed in the latter expression agrees with the physical characteristics of two panels separated by an empty airspace such as double glass panels, as derived above. However, in practice, there was found an empirical fix that must multiplied with the right-hand side of Eq. (4.47a) to adjust with measurements. This is, particularly, for the case of a typical double wall in which the air cavity is filled with a porous sound-absorbing material (mineral wool). This factor is set to $\sqrt{1.8}$. Subsequently, f_0 in this case is expressed as

$$f_0 = \frac{c}{2\pi}\left[1.8\frac{\rho_0}{d}\left(\frac{1}{M_{s1}} + \frac{1}{M_{s2}}\right)\right]^{1/2} \tag{4.47b}$$

The sound insulation curve for a double partition consisting of two thin homogeneous panels, without mechanical connections between the two leaves and without/or with an absorbent in the cavity can be predicted according to the three regions or regimes, according to the frequency.

Regime A (or Region 1), the low-frequency regime, occurs for closely spaced panels. The walls act in this case as limp masses and air-gap acts as a spring. When the two panels are placed very near to each together, the panels act as one unit with respect to the sound transmission and so the air space between the panels has a negligible effect. In general, this behavior occurs for the frequency range:

$$\frac{\rho_0 c}{\pi(M_{s1}+M_{s2})} < f < f_0 \tag{4.48}$$

where ρ_0 and c are the density of and sound speed in the air around the panel. It is also possible reduce the frequency limit, Eq. (4.48), to only $f < f_0$ as the left term of Eq. (4.48) is very low, less than 10 Hz in most typical construction considered in building acoustics. The sound reduction index for Regime A follows mass law, with the combined mass of both partitions and is given by the following:

$$R = 20\log(M_{s1}+M_{s2}) + 20\log(f) - 47.3 \tag{4.49}$$

At f_0, there is an effective coupling between partitions across air gap, which leads to poor transmission loss, R. Therefore, one should try to use a double structure that has low resonant frequency, f_0 (below 100 Hz), so that the final R_w will not be affected, as discussed later on. In this region, the slope of transmission curve is about 6 dB/octave.

As the panels are moved farther apart, standing waves are set up in the air space between the panels, and Regime B (or Region 2) behavior is observed. This regime occurs for the frequency range, as follows:

$$f_0 < f < (f_a = c/2\pi\, d) \tag{4.50}$$

The sound reduction index (transmission loss) in Regime B is given as

$$R = R_1 + R_2 + 20\log(4\pi f d/c) \tag{4.51a}$$

The latter expression may also be simplified as

$$R = R_1 + R_2 + 20\log(fd) - 29 \tag{4.51b}$$

The quantities R_1 and R_2 are the sound reduction index values for each of the panels acting alone, which can be obtained using the material of Sec. 4.2 and/or Sec. 4.3. Eq. (4.51) indicates that a transmission loss of 18 dB/octave can be obtained in this region. However, in practice, 15 dB/octave is more normal.

At higher frequencies, the panels are moved sufficiently far apart, the two panels act independently, and Regime C (or Region 3) behavior is observed. In this case, the air space between the panels acts as a small room. There is now an effective sound transmission from panel 1 to small reverberant room to panel 2. The cavity behaves more like a usual wave field rather than a spring and the two leaves work independently to give significantly higher R than the combined mass of the entire partition. This behavior occurs for the frequency range $f > f_a$.
For the regime C, two expressions for sound reduction index are obtained:

Case 1: *Empty* (*air-filled*) *cavity* (*no absorbent in the cavity*)
The sound reduction index in Regime C (or Region 3), $f > f_a$, is given by (Beranek, 1971):

$$R = R_1 + R_2 + 10\log\left(\frac{4}{1+(2/\alpha)}\right) \tag{4.52}$$

The quantity α is the surface absorption coefficient for the panels (frequency-dependent); see the Appendix of Chapter 9.

Case 2: *An absorbent in the cavity*
It is important to include absorption in form of an acoustically absorbing blanket between the leaves to damp reverberant field (resonances) in the cavity, otherwise transmission loss is reduced. In this case, the sound reduction index reads for $f > f_a$ is obtained as (Sharp, 1973, 1978):

$$R = 20\log\left(\frac{\omega^2 M_{s1} M_{s2}}{2(\rho_0 c)^2}\right) \tag{4.53a}$$

The latter expression may also be simplified as

$$R \approx R_1 + R_2 + 6 \tag{4.53b}$$

One can obtain 12 dB/octave with absorbent in this case. Eq. (4.53b) is commonly used in building acoustics calculations.

These three main regions are shown schematically in Fig. 4.12. Moreover, at $f = f_{c1}$ or f_{c2} coincidence dips lead to more transmission, unless highly damped.
The sound reduction index expressions given in this section apply for the sound transmitted through the airspace only. If there are interconnections, then a second path that the sound may take (structure-borne flanking path), which involves sound transmission through mechanical links between the panels (Fig. 4.7). Prediction methods for this contribution to the transmission loss are given in Sec. 4.4.5.

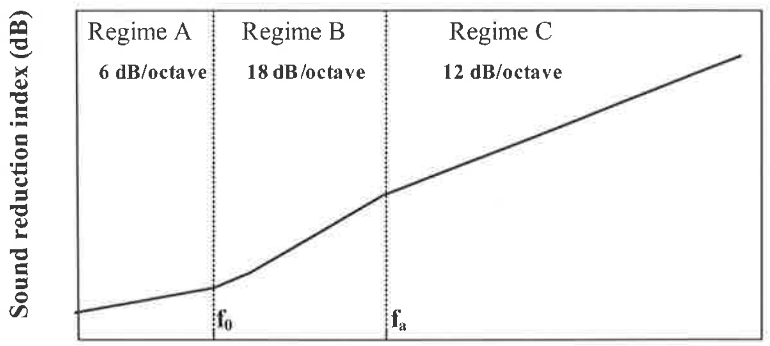


Fig. 4.10 Typical transmission loss curve for a composite wall with cavity filled with an absorbent and without mechanical connections between two leaves, with respect to main regions of the transmission.

Standing waves in the cavity

At high frequencies, resonances or standing waves appear in the air space. This is when the cavity depth between the two panels is exactly one-half the wavelength of the incident sound. The lowest order resonance of the air cavity, f_1, for normal incidence of sound wave reads:

$$f_1 = \frac{c}{2d} \tag{4.54}$$

For frequencies higher than f_1, it appears additional resonances:

$$f_n = n\frac{c}{2d} \tag{4.55}$$

where n = 1, 2, 3, etc, and d is the cavity width. These resonances cause dips in the sound transmission loss and the mass law performance will be fairly degraded. In practice, the resonances appear at lower frequencies when the cavity width increases, which imply that sound insulation rating R_w (or *STC*) will hardly increase when the cavity width exceeds about 20 cm. This concern only double walls without sound absorbent; with absorbent, the conditions changes radically, as discussed later on.

Note that one may also check the lowest order resonance along the air cavity: $f = c/2L$, where L is the longest cavity dimension (finite resonances), which may lower somewhat the transmission loss for small-size plates.

Characteristic frequencies vs. angle of incidence

The expressions for f_0 and f_1 estimate the lowest frequencies at which either panel/cavity resonance or standing waves can occur. Physically, these expressions correspond to the sound field impinging upon the construction at normal incidence, as indicated earlier. If, on the other hand, the sound field impinges upon the panel at an angle, φ, measured from the normal to the panel, the above expressions are modified by dividing the right-hand side of Eq. (4.47) and Eq. (4.54) by $\cos\varphi$. In addition, for any angle of incidence, standing waves will occur in the cavity at all integer multiples (harmonics) of the expression for f_n given by Eq. (4.55).

Weakly damped air cavity

A weak damped material in the cavity implies that higher sound pressure is built up in the cavity, which leads to higher sound transmission. This is particularly the case for window constructions in which the gap between two windows layers is lined with frame absorbent along the cavity perimeter. The sound reduction index for such constructions, at high frequencies, can approximately be determined as (Sharp, 1978)

$$R \approx R_1 + R_2 + 10\log\left(\alpha_f \frac{dU}{S}\right) \tag{4.56}$$

Where α_f is the sound absorption coefficient for the frame covering the air cavity, d the cavity depth = frame depth, U the circumference = frame length, S the panel area. The following empirical values are obtained for α_f (Brekke, 1980)

$$\alpha_f = \begin{cases} 0.5 & \text{for } d \le 20\,\text{mm} \\ 10\,\text{mm}/d & \text{for } d > 20\,\text{mm} \end{cases} \tag{4.57}$$

Practical notes on calculations

• The mathematical expression for transmission loss in the coincidence region is quite complicated and is rarely used in acoustical planning. Instead one can resort to approximate methods (e.g. see Sec. 4.3) or use empirical values. It is worth noting that the field-incidence mass law (i.e., R_n–5) is only accurate below half the coincidence frequency; $f \le 0.5f_c$. Therefore, the transmission loss of a single panel in the double wall may be estimated using the expressions for mass-controlled region. For $f \ge f_c$, the expressions for damping-controlled region are used. For the transmission loss region between $0.5f_c$ and f_c, one may approximate the situation by drawing a straight line in between. This rule is verified well using the many empirical results, especially if f_c lies relatively high in frequency; see example 4.4. Generally, the prediction of transmission loss of the coincidence region is rarely accurately managed by the approximate methods.

• At low frequencies, the finite size of the partition results in higher values than predicted for infinite partitions.

• When two layers of material such as gypsum wallboard are glued firmly together, they behave like a single thick layer with an associated lowering of the coincidence frequency, f_c. If the layers are only held together loosely (e.g., with screws) so that they can slide over each other to some extent during bending motions, then the coincidence frequency does not move to lower frequencies and the friction between the layers can also introduce some extra energy losses.

Approximate method

A method to predict the sound reduction index of a double partition in the entire frequency region, based on empirical data, has been proposed (Gösele, 1980). If the R_1 and R_2 of the single partitions that constitute the double partition are known, and there are no structure-borne sound transmission, which occurs via mechanical connections, and the gap is filled with porous sound-absorbing material, the overall sound transmission loss may be approximated as:

$$R \approx R_1 + R_2 + 20\log\left(\frac{4\pi f \rho_0 c}{k_a}\right) \tag{4.58}$$

where d is the gap thickness and k_a is the dynamic stiffness of the gap, which is expressed here according to the frequency as

$$k_a = \rho_0 c^2 / d \quad (\text{for } f < f_a = c/2\pi d) \tag{4.59a}$$

$$k_a = 2\pi f \rho_0 c \quad (\text{for } f > f_a) \tag{4.59b}$$

Eq. (4.58) can be shown to agree well with a number of performed measurements.

The method assumes the availability of measured sound transmission loss data for the leaves (single partitions of the double wall). If, on the other hand, such data are not available, the author suggested other approximate expressions, which yields good agreement only well below and well above the critical frequency but fail in the frequency region near the critical frequency.

Example 4.4: A double wall without mechanical connections between leaves has the following configuration. The leaves are composed of double layers of gypsum board (normal), 13 mm on both sides of the wall. According to the manufacturer, the gypsum board has a density 720 kg/m^3 and E-modulus is 2.6x10^9. The total spacing of air cavity is 185 mm and is filled in with 90 mm mineral wool; the depth of air space is 95 mm. There are 4 steel studs @ 45 mm× 26 mm. Draw the transmission loss curve of this construction. Measured results for this construction are shown in Fig. 4.11. Assume that energy losses are mainly due to the material, typical room condition, c = 344 m/s and ρ_0 = 1.2 kg/m^3.

Solution

According to the Appendix, the material loss factor may be selected as 1% and Poisson's ratio 0.13.

$M_s = \rho h = 720(0.013) = 9.4$ kg/m^2 for each board; each panel consists of two boards, therefore: $M_{s1} = M_{s2} = 18.8$ kg/m^2.

The mass-spring-mass resonant frequency for the double wall is found from Eq. (4.47b)

$$f_0 = \frac{344}{2\pi}\left[1.8\frac{1.2}{0.185}\left(\frac{1}{18.8}+\frac{1}{18.8}\right)\right]^{1/2} = 61 \text{ Hz}$$

c_L is obtained from Eq. (2.129)

$$c_L = \left(\frac{2.6(10^9)}{720(1-0.13^2)}\right)^{1/2} = 1917 \text{ m/s}$$

The critical or wave coincidence frequency of the gypsum board is found from Eq. (2.332)

$$f_{c1} = f_{c2} = \frac{c^2\sqrt{3}}{\pi c_L h} = \frac{(344)^2(3^{1/2})}{\pi(1917)(0.013)} = 2618 \text{ Hz}$$

The calculation of f_c is based on the assumption that the steel studs do not add to the stiffness of leaves in some great extent and so both leaves are only loosely joined.

The lowest order resonance across the air cavity is:

$$f_a = \frac{c}{2\pi d} = \frac{344}{2\pi(0.185)} = 296 \text{ Hz}$$

$$\frac{\rho_0 c}{\pi(M_{s1}+M_{s2})} = \frac{344(1.2)}{2\pi(18.8)} = 3.5 \text{ Hz} < f$$

From Eq. (4.30), transmission loss in mass-controlled Region for single panels:

$$R_1(f) = R_2(f) = 20\log M_{s1} + 20\log f - 47.3$$

$$20\log(18.8) + 20\log f - 47.3 = 20\log f - 21.8 \tag{E.1}$$

(Note that this mass law yields only accurate results for $f \leq 0.5f_c$)

From Eq. (4.36), transmission loss for $f \geq f_c$ (damping-controlled Region) may be written as

$$R_1(f) = R_2(f) = 20\log 18.8 - 13\log 2618 + 33\log f + 10\log 0.01 - 48 = 33\log f - 87 \quad \text{(E.2)}$$

Results of the double wall may now be calculated:

- $f = 50$ Hz: $f < f_0$

From Eq. (4.49), Regime A:

$R_{50} = 20\log(18.8 + 18.8) + 20\log(50) - 47.3 = 18.2$ dB

- $f = 64$ Hz: $f > f_0$, $f < f_a$, $f < 0.5f_c$

The case is regime B for double wall and mass-controlled Region for single leaf, Eq. (E.1):

$R_1 = R_2 = 20\log(64) - 21.8 = 14.3$ dB

From Eq. (4.51): $R_{64} = 14.3 + 14.3 + 20\log[64(0.185)] - 29 = 21.1$ dB

NB! The frequency $f = f_0$ can be considered in the Regime B.

- $f = 300$ Hz: $f \approx f_a$, $f > f_0$, $f < 0.5f_c$

The case is regime B, mass-controlled Region:

$R_1 = R_2 = 20\log(300) - 21.8 = 27.7$ dB

$R_{300} = 27.7 + 27.7 + 20\log[300(0.185)] - 29 = 61.3$ dB

- $f = 1310$ Hz: $f > f_a$, $f \approx 0.5f_c$

The case is regime C, mass-controlled Region:

$R_1 = R_2 = 20\log(1310) - 21.8 = 40.6$ dB

From Eq. (4.59), Regime C: $R_{1310} = 40.6 + 40.6 + 6 = 87.2$ dB

- $f = 2620$ Hz: $f > f_a$, $f \approx f_c$

The case is regime C, damping-controlled Region:

Eq. (E.2):

$R_1 = R_2 = 33\log(2620) - 87 = 25.8$ dB

$R_{2620} = 25.8 + 25.8 + 6 = 57.6$ dB

- $f = 5000$ Hz: $f > f_a$, $f > f_c$

$R_1 = R_2 = 33\log(5000) - 87 = 35.1$ dB

$R_{5000} = 35.1 + 35.1 + 6 = 76.2$ dB

The results are plotted in Fig. 4.11. As can be seen, the deviation between prediction and measurement is mostly in the coincidence region, due to the approximation used by the theory in this region. However, it can be shown that the final predicted R_w will differ only 1 to 2 dB from the measured one, which is reasonable for practical reasons.

If one change the wall so that one gypsum layer is installed on one side and three gypsum layers on the other side, then the great influences on sound insulation will be observed at higher frequencies, especially at and above the coincidence frequency.

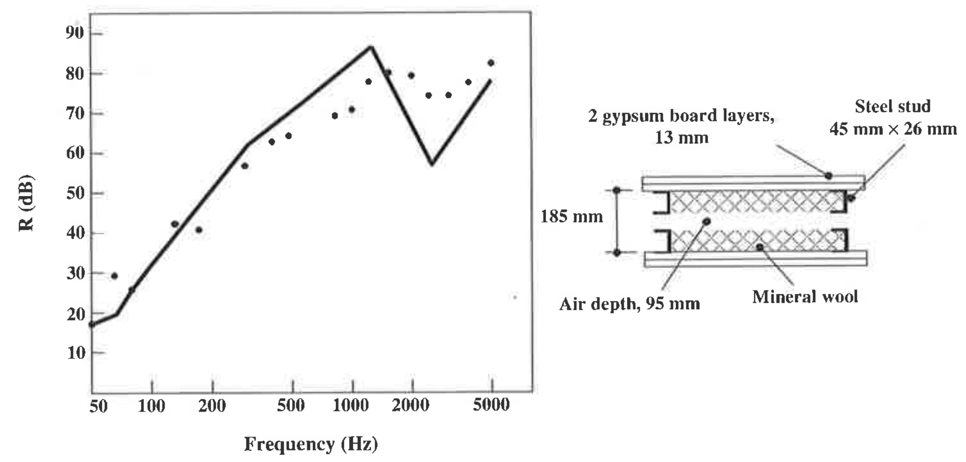


Fig. 4.11. Comparison between measured and predicted R for a wall composed of double gypsum layers on both sides and a double stud skeleton with mineral wool in between, Example 4.4. R_w is 59 dB for the measured curve. (———) calculated ; (• • •) measured.

4.4.3 Design considerations for composite walls with no mechanical connections

The insulation of the double wall at the mass-spring-mass resonant frequency, f_0, becomes low, as shown in Fig. 4.10. One should therefore strive to lower this frequency so that it should have no significance for the normal airborne insulation (R_w), i.e., under 100 Hz. At frequencies below f_0, the two leaves oscillate together in phase with each other. The insulation is determined therefore by the wall's total surface density, provided that the considered frequencies lie within leave mass region. Consequently, the sound insulation of a double wall increases rapidly above the resonant frequency range. However, within the resonance frequency range, the sound insulation of a double wall is often worse than that of a simple wall of equal mass. Thus, the mass-spring-mass resonance frequency, f_0, of a double wall should be as low as possible. These results can also be concluded from Example 4.4.

The influence of coincidence frequency

As a general rule, it can be advantageous if both leaves have different properties. It is a special case that if the frequency of first coincidence dip of the leaf lie within the considerable frequency range. The same coincidence frequency for both leaves entails that the dip in the isolation curve becomes boosted compared with a single wall. If, on the other hand, the coincidence frequencies are quite different from each other (e.g., at least one octave), the coincidence dips of the two leaves shall be eventually counterbalanced. For design purposes, the coincidence frequency for the boards in a lightweight double wall should be as high as possible so that the coincidence phenomenon weakens the sound insulation of the wall as little as possible. Thus, thin building boards have to be used in the wall and they should not be glued to each other. In practice, it is utilized very light elements as surface layers in the double wall, Example 4.4. The coincidence frequency for these elements will therefore be high at the frequency range within the wall mass region.

Since the insulation of double wall increases quickly above f_0, it is beneficial to have a big air cavity. In general, the sound insulation (R_w) value increases monotonically with the cavity size; see Table 4.3.

The influence of absorbent in the double wall's cavity

By introducing an absorbent such as mineral wall (for thermal and sound insulation) in the air cavity, the resonances (Eq. (4.55)), which reduce the sound insulation, are damped. A condition for this is, however, that the sound absorbent materials should have sufficient thickness. This thickness will be adjusted in relation to the frequencies of those resonances that shall be damped; low frequencies require big thickness and high frequencies small one. The added mineral wool can also add mass to the total construction surface density, M_s, if it completely fills the airspace of the double wall. This is the reason why in some cases, the calculated sound reduction index is smaller than the measured one.

Concerning the choice of absorbent, practical tests have shown that the density of mineral wool (Fig. 4.12) is a secondary parameter, while the most important variable is the absorbent's flow resistivity. In general, the flow resistivity is probably the most important parameter for sound absorbing materials.

Table 4.3 Airborne insulation index, R_w, for double-leaf walls with no mechanical interconnections measured in the laboratory.

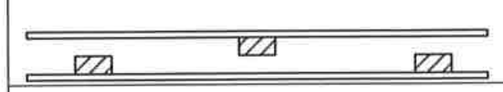
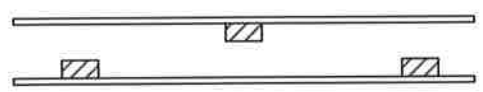
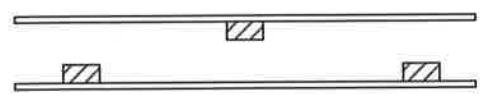
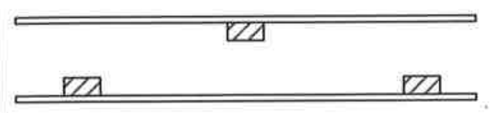
Construction sketch	Features
	Wall panels of plasterboard (13 mm) $R_w \approx 34$ dB Cavity width, 25 mm
	Same $R_w \approx 37$ dB Cavity width, 50 mm
	Same $R_w \approx 43$ dB Cavity width, 100 mm
	Same: $R_w \approx 47$ dB Cavity width, 150 mm

Fig. 4.12 Mineral wool, also known as mineral cotton, silicate cotton, stone wool, slag wool, and rock wool, is an inorganic substance used for insulation and filtering. Mineral wool fibres are used as thermal, fire, and sound insulators in buildings and HVAC- equipments. Note that in homes and offices there is a connection between landing fibres and health hazards. These large fibres can cause typically eye, skin and airway irritation. The picture on the left shows magnified fibres.

With a theoretical and experimental analysis, it can be shown that the specific flow resistivity should amount to 0.5-1×10^4 Ns/m^4 so as to dampen the lateral sound waves effectively.In turn, this implies that the density should exceed about 16 kg/m^3 for glass wool (fibre dimension ≈ 6 μm, but it be up to 10 μm) and 40 kg/m^3 for rock wool, in order to obtain a sufficient flow resistivity. Furthermore, the density should not exceed 150 kg/m^3 for completely filled air-gap in order to avoid the mechanical coupling between the two leaves. When experimentally compared, absorbents with different densities (< 150 kg/m^3) but with the same flow resistivity, the result demonstrated that the density of the acoustic absorbent has a negligible effect on the insulation rating (*STC*) of a wall (Loney, 1973); see also Sec. 4.4.4 for further details. Above f_a, one can obtain somewhat worse sound insulation than according to Eq. (4.53) if mineral wool with very little flow resistivity is used, whereas it is not preferred such high flow resistivity to effectively dampen the lateral standing waves and thereby obtaining a good sound reduction index in the frequency range f_0 to f_a. If, on the other hand, an acoustic absorbent in form of insulation mat (batt or batten) with relatively high density is used, one can also effectively dampen the standing waves that are developed in the perpendicular direction whereby the predicted sound reduction index can be obtained above f_a. As a rule of thumb, one can expect that the higher frequencies due to standing waves in the cavity can be damped with a 3 cm thick mineral wool mat. Nevertheless, with thicker mat, the damping increases and consequently, the sound reduction index increases somewhat. In addition, utilization of absorbent in the cavity limit also the coincidence phenomena, i.e., the better absorbent used the smaller pronounced becomes the coincidence dip in sound reduction index curve. The effect of filing the cavity o double wall with an acoustic absorbent is shown clearly in Fig. 4.13. Further, the influence of absorbents on the R_w value for typical cases is shown in Table 4.4.

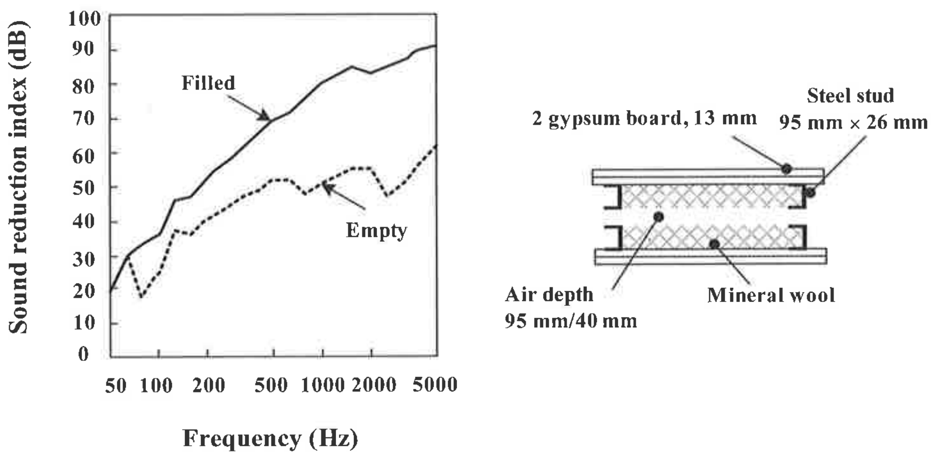
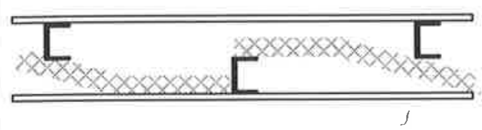
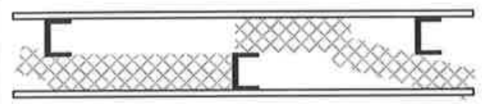
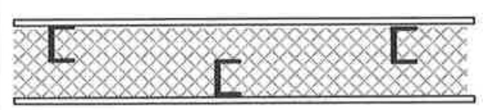


Fig. 4.13 Measured sound reduction index for a wall with or without an acoustic absorbent in air cavity. The wall is composed of double plasterboards (normal) on both sides and double 95 mm studs with or without mineral wool in between. R_w for the filled and empty cases are 62 and 50 dB, respectively.

It is important to know that the resonant frequency of the double wall, f_0, which in general appears at low frequencies, can not be damped effectively by the mineral wool even if the whole gap is filled in. Only the mass of the leaves, M_s, and depth of airspace can be changed according to Eq. (4.47) to obtain the designed resonance of the system.

Table 4.4 Influence of an absorbent, in form of a mineral wool mat, on the R_w-value for a lightweight double wall with one 13 mm plasterboard on both sides.

Construction figures	Features
	Double wall of single 13 mm plasterboard Steel beams c/c 60 cm 13.5 cm cavity without mineral wool $R_w \approx 41$ dB
	Same 13.5 cm cavity with 3 cm mineral wool, $R_w \approx 49$ dB
	Same 13.5 cm cavity with 7 cm mineral wool, $R_w \approx 50$ dB
	Same 13.5 cm cavity with 13.5 cm mineral wool $R_w \approx 54$ dB

4.4.4 Airflow resistivity and the transmission loss

Airflow resistivity indicates how absorptive a material is by evaluating how much air can pass through the material at a given volumetric flow rate. Airflow resistivity is measured according to ASTM test method C522 (1987). Alternatively, it is determined with standard equipment according to ISO/DIS 9053 (1998). As a measure, the flow resistivity depends on the porosity and friction between air particles with unit MKS rays/ m (N.s/m^4). Subsequently, it is expected that a better result can be obtained if a there is high flow resistivity, as discussed earlier. Fibrous insulations have varying degrees of resistivity that are largely dependant on the density of the material. Moreover, acoustic absorbents used within partition cavities require a thickness that fills the entire cavity for optimal sound control performance. In general, the higher the density, the higher the airflow resistivity, the greater the potential for sound absorption, particularly at lower frequencies.

Table 4.5 lists typical airflow resistivity values for the most common absorptive materials. As can be seen, an absorbent material's ability to reduce sound transmission improves with density. Because of their inherently higher density, the mineral fibre samples deliver noticeably better air flow resistivity than the glass fibre samples. Note that the manufacturer data should always be sought first since there are different commercial products with different properties than what is stated in Table 4.5. For the table, mineral fibre is inorganic fibers of glass, asbestos, or rock (mineral or rock wool). Glass fibre is a glass in fibrous form. Cellulose comes usually as spray (see Chapter 1), which is sprayed onto one surface or blown into the cavity.

Table 4.5 Density and measured airflow resistivity for common absorptive materials (partially after, Halliwell et al., 1998).

Absorptive material	Thickness of batt (blanket) (mm)	Average density (kg/m^3)	Average Airflow resistivity (N.s/m4) (= mks rayls/m)
Glass fibre	65	11.7	3600
Glass fibre	150	11.2	4300
Glass fibre	89	12.2	4800
Glass fibre	89	16.4	7900
Mineral fibre	20	25	11000
Mineral fibre	65	36.7	11400
Mineral fibre	89	32.6	12700
Mineral fibre	40	51.9	15000
Mineral fibre	75	44.2	16600
Mineral fibre	83	98.1	58800
Cellulose	wet spray	56.3	NA
Cellulose	90 (blown)	49.3	33000

The effect of the sound-absorbing material in the airspace results in refraction of the oblique-incidence sound toward the normal, thereby reducing the dynamic stiffness of the air between the plates (Beranek and Ver, 1992). The sound-absorbing material also prevents high sound energy buildup in the cavity. These

result in a substantial increase in sound transmission loss. The flow resistivity, σ, of the sound-absorbing material should be about σ = 5000 N.s/m^4 (Gösele and Gösele, 1977). Higher values of σ can yield, sometimes, only diminishing returns. The benefits available from using sound sound-absorbing materials with higher flow resistivity and density are evident at the higher frequencies but not at the low frequencies.

For a double-wall without interconnections (Sec. 4.4.2), the transmission loss expressions in the three regions do not contain any parameters to describe the variation in performance due to different acoustic absorbers in the airspace and it is a practical problem of some interest to determine this effect. To take into consideration this effect, the transmission loss may be written as (Fahy, 1985)

$$R = R_1 + R_2 + 8.6\alpha d + 20\log(\beta/k) \qquad (4.60)$$

where α and β are the attenuation constant (real part) and phase constant (imaginary part) of the propagation coefficient γ of the absorptive blanket, which is described as: $\gamma = \alpha + j\beta$ and k is the wave number. It is suggested that the propagation coefficient, γ, be taken as (Delany and Bazley, 1970):

$$\gamma = \frac{\omega}{c}0.189\left(\frac{\rho_0 f}{\sigma}\right)^{-0.595} + \frac{j\omega}{c}\left[1 + 0.0978\left(\frac{\rho_0 f}{\sigma}\right)^{-0.700}\right] \qquad (4.61)$$

where ρ_0 and c denotes the air density and speed of sound in air, respectively. Consequently, Re{γ} and Im{γ} of Eq. (4.61) represents α and β in Eq. (4.58), respectively. Recently, the model of Delany and Bazley has been discussed and empirical formulas for α and β were suggested (Komatsu, 2008). According to the author, the new model is more effective than the model of Delany and Bazley, particularly for the prediction for high-density fibrous materials. Note hat Eq. (4.60) is derived assuming that there are no connections between the two panels of the double wall. Further, Eq. (4.60) is valid for $\alpha d \geq 1$; however, it can well be used for $f > c/2\pi d$. In general, it can be shown that Eq. (4.60) yields up to 3 dB contribution to sound insulation than Eq. (4.53a).

In an experimental study in which 360 gypsum board walls with a single cavity containing sound absorbing material and having the two layers independent or resiliently connected, the following two empirical expressions are suggested for the sound insulation ratings *STC* and R'_w, based on regression analysis (Warnock and Quilt, 1995):

$$R'_w = -60.3 + 29.5\log M_g + 32.2\log d - 2.1\times10^{-4}\sigma + 0.0092b \qquad (4.62)$$

$$STC = -69.8 + 33.5\log M_g + 32.2\log d - 7\times10^{-4}\sigma + 0.017b \qquad (4.63)$$

where M_g is the total mass per unit area of the gypsum board layers (kg/m^2), d is the cavity depth (mm), σ is the airflow resistivity of the sound absorbing material (mks rayls), and b is the stud spacing (mm). The standard errors of the estimates are 2.0 and 1.6 dB, respectively. Below 500 Hz, these factors accounted for most of the variance. Above 500 Hz, stud spacing was not significant.

It is, however, difficult to generalise such formulas, but what is interesting here is the parameters that are included in these expressions, which indicate that there are some important increase in the sound insulation that can obtained at lower frequencies for lightweight double walls, by carefully selecting stud spacing and the type of sound absorbing material, in addition to wall mass and airspace.

4.4.5 Double constructions with mechanical connections

While some constructions can approach the ideal of double panels without interconnections, in practice most construction will have some type of rigid or resilient connection between the panels. It is often necessary to utilize a kind of fastenings, stiffeners, ties or fixings between leaves of double partitions so as to obtain the requisite stiffness against lateral loads. If the two panels are both connected to a single row of stud, the sound transmission loss of the construction will be less than that for the unconnected double-panel configuration. The degradation is usually attributed to the presence of "sound bridges'' formed by the studs connecting the two faces or panels. These stiffeners are normally carried out with common timber or metal studs or beams. This connection results in a more direct transmission path for the forced panel vibration (structure-borne sound) than that realized for unconnected double panels. A similar effect is obtained at the edges or boundaries of e.g., an ideal double wall if it is built up against continuous flanking construction or with a common stud. Instructively, the ideal double layer assembly is a double wall with no rigid mechanical connection between its two surfaces and can be used as reference for comparing different configurations of constructions. An asymmetric double wall has two different critical frequencies of two panels while a symmetric one has only one coincidence frequency. By incorporating resilient metal channels into one side of the construction, one attempts to decouple the direct transmission of vibration and hence reduce the reradiated sound in order to retain the higher sound transmission loss characteristics of the unconnected double-panel construction (ideal double wall).

In general, the transmission of structure-borne sound via connections to the wall's leaf of the receiving room can be approximated with point or line sound source in view of the fixation type, which can be carried out either in points or along the whole beam (stud). The line connection corresponds to the direct

attachment of the panels to the framework along the entire length of the studs using either nails or screws. The fixation can give rise to a point sound source if the contact surface between the wall leaf and the underlying stud approximates a point; in practice, this is achieved if one uses at the fixation points small (< 5 cm^2) interlayer of e.g. plywood (sheet of wood made of 3 or more thin layers of wood bonded together with glue).

4.4.5.1 Rigid studs

The structural bending force excites the plate at the fixation area and free bending waves will spread out in the receiving leaf (see Fig. 4.15). However, below the coincidence frequency of panel, the sound emission from the free bending waves is relatively ineffective, as discussed in Sec. 2.15, while the near-field vibrations around the excitation area have a substantially more effective sound emission. This results in a transmission loss or sound reduction index curve, which has a fixed number of decibels, ΔR_M, above the curve of mass law (see Eq. (4.49)) according to the following expression, which has the applicability in the frequency range, $f > f_0$, $f < f_{c1}$, $f < f_{c2}$:

$$R = 20\log(M_{s1} + M_{s2}) + 20\log(f) - 47.3 + \Delta R_M \qquad (4.64)$$

Note that in theory, Eq. (4.64) can be used in the above frequency range. However, the field-incidence mass law is practically valid up to $0.5f_c$, as discussed earlier (see also Sec. 4.9) and so the results will actually be valid for $f \le 0.5f_{c1}, f \le 0.5f_{c2}$.

The situation is illustrated in Fig. 4.14. Above the cut-off frequency, f_b, the slope of transmission loss curve is 6 dB/octave, up to the coincidence region. The transmission loss curve in the transition region between f_0 and f_b is determined mostly according to Eq. (4.64); it is obtained by a line connecting the transmission values at f_0 and f_b. In the case that the wall leaves are identical or have equal surface density (mass per unit area), the value of ΔR_M can be between 6 dB, when there is a total transmission via the connections (e.g. some rigid suds) and 30 dB when the structural transmission has been reduced to a practical minimum (e.g., using some types of resilient studs). The cut-off frequency, f_b, is the frequency at which the 6 dB/octave line starts (Fig. 4.14), and may be approximated as the frequency at which the transmission, Eq. (4.64) becomes equal to the transmission loss, Eq. (4.51); f_b typically lies between 220 Hz and 250 Hz, for typical building constructions.

Derivation of ΔR_M

Consider the case illustrated by Fig. 4.15. The following assumptions are made:

- the sound bridge is very stiff (rigid) (e.g. timbre stud), and so velocities on both sides of the stud are equal, $v_{1,b} = v_{2,b}$; the elastic connections are treated later on;
- the mass of the sound bridge is unimportant in relation to the total construction mass and is totally stiff;
- a sound absorption material is installed in the cavity; and
- the absorption of the cavity boundaries is perfect, that is, there are no reflections from the cavity boundaries that could increase the sound energy inside the cavity.

The sound insulation of a double wall is determined by summing the sound power that is transmitted via sound transmission bridges (connections), $W_{2,b}$, and the sound power that is radiated by the ideal double wall without mechanical connections (i.e. in the air cavity), $W_{2,a}$. Consequently, the total sound transmission loss of the double wall can be written as

$$R = R_I - 10\log\left(1 + \frac{W_{2,b}}{W_{2,a}}\right) \tag{4.65}$$

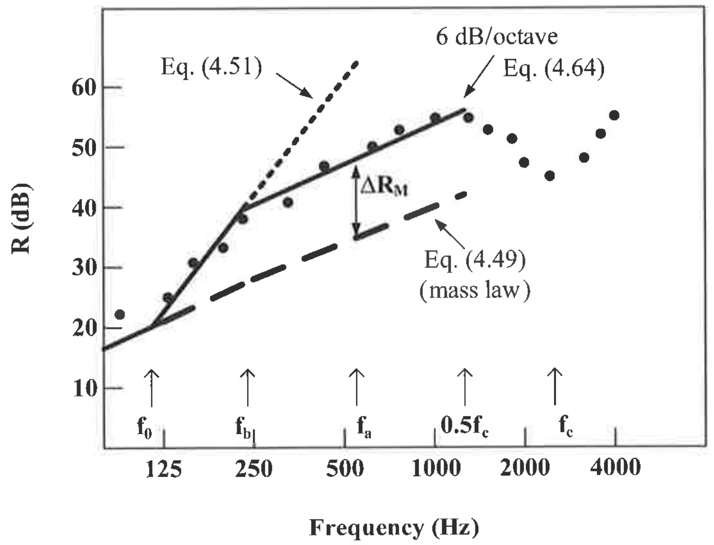


Fig. 4.14 Predicted and measured transmission loss curves for a double wall with common studs. The measurement (•) is for a double wall consists of 16 mm gypsum board (on both sides) + timber studs (50 mm × 100 mm, c/c-spacing 40 cm) + 60 mm mineral wool blanket, and a resilient interconnection between the gypsum board and stud on one side. The transmission loss adjustment, $\Delta R_M \approx 13$ dB.

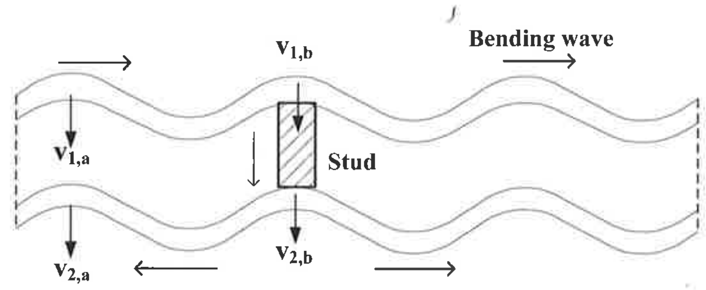


Fig. 4.15 Transmission of bending waves from panel 1 to panel 2 through a sound bridge in a double partition with connections. The two plates are assumed to be infinite laterally in which free bending waves are propagated. v_b is the local (particle) velocity of the panel at the stud and v_a is the particle velocity of the panel, assuming there is no connection between the panels (i.e., ideal double wall).

where R_I is the sound reduction index of ideal double wall (i.e., unconnected double wall). As discussed in Chapter 2 (Sec. 2.15), for frequencies less than f_c, the radiation from a panel is due to the near field, and with the help of Table 4.6, the $W_{2,b}/W_{2,a}$ is given by:

$$\frac{W_{2,b}}{W_{2,a}} = \frac{n\psi}{S}\left(\frac{\tilde{v}_{2,b}}{<\tilde{v}_{2,a}>}\right)^2 \tag{4.66}$$

$$\psi = \frac{8c^2}{\pi^3 f_{c2}^2} \quad \text{(for } f << f_{c2}\text{, point sound source)} \tag{4.67}$$

$$\psi = \frac{2cL}{\pi f_{c2}} \quad \text{(for } f << f_{c2}\text{, line sound source)} \tag{4.68}$$

where n is the number of line or point sound bridges within the total transmission surface S (m^2), $v_{2,b}$ is the vibration velocity of the panel in which the transmission forces of the sound bridges act, $v_{2,a}$ is the vibration velocity for the wall's leaf of the receiving room for an ideal double partition, f_{c2} its coincidence frequency, L is the length (m) of line source (sound bridge), and the bracket < > denotes the mean velocity in time and space; see Fig. 4.15. The assumption of line-source assumes that L is large in relation to the wavelength in air. Further, it appropriate to rewrite Eq. (4.66) as

$$\frac{W_{2,b}}{W_{2,a}} = \frac{n\psi}{S}\left(\frac{\tilde{v}_{2,b}}{<\tilde{v}_{1,a}>}\right)^2\left(\frac{<\tilde{v}_{1,a}>}{<\tilde{v}_{2,a}>}\right)^2 \tag{4.69}$$

where $v_{1,a}$ is the vibration velocity of the wall's panel of the sender-room for the ideal double wall.

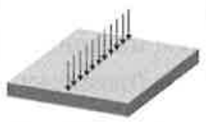
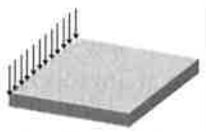
The relation between the average velocity in panel 1 and the local velocity at the sound bridge is determined by the two plate input impedances (point-or line impedances) Z_1 and Z_2 (Sharp, 1987):

$$\frac{<\tilde{v}_{1,a}^2>}{\tilde{v}_{2,b}^2} = \left|\frac{Z_1 + Z_2}{Z_1}\right|^2 \tag{4.70}$$

This expresses indicates that there is a reduced movement at the sound bridge due to common effect of plates. From Table 4.6, the impedances are expressed as

$$Z = \frac{4c^2 M_s}{\pi f_c} \quad \text{(Point impedance)} \tag{4.71}$$

Table 4.6 Important quantities for the excitation of plate through point, line or edge interconnection (after Cremer et al, 1988; Kristensen and Rindel, 1989). M_s is the surface density, f_c the critical frequency, c the sound of speed in air, ρ_0 the air density, L is the length of line source, and v_0 is the effective (rms) value of the vibration velocity at excitation.

Type of excitation	Impedance	Mobility	Sound radiation from near field	Total radiation factor, κ
 Point (number of points n)	$\frac{4c^2 M_s}{\pi f_c}$	$\frac{\pi f_c}{4c^2 M_s}$	$\tilde{v}_0^2 \rho_0 c \frac{8c^2}{\pi^3 f_c^2} n$	$1 + \frac{\pi\sigma f_c}{4\eta f}$
 Line (length L)	$2(1+j)M_s cL\sqrt{f/f_c}$	$\frac{1-j}{4M_s cL}\sqrt{f_c/f}$	$\tilde{v}_0^2 \rho_0 c \frac{2c}{\pi f_c} L$	$1 + \frac{\sigma}{2\eta}\sqrt{\frac{f_c}{f}}$
 Edge (length L_e)	$\frac{1}{2}(1+j)M_s cL_e\sqrt{f/f_c}$	$\frac{1-j}{M_s cL_e}\sqrt{f_c/f}$	$\tilde{v}_0^2 \rho_0 c \frac{c}{\pi f_c} L_e$	$1 + \frac{\sigma}{4\eta}\sqrt{\frac{f_c}{f}}$

$$Z = 2(1+j)M_s c(f/f_c)^{1/2} \quad \text{(Per unit length, line impedance)} \tag{4.72}$$

If one further assumes that $v_{1,a}$ will not be affected if the partition's leaf number 2 exists or not, then the ratio $v_{1,a} / v_{2,a}$ is determined as the difference between the transmission loss of the ideal double wall and the transmission loss of leaf number 1 as

$$10\log\left(\frac{<\tilde{v}_{1,a}>}{<\tilde{v}_{2,a}>}\right)^2 = 20\log M_{s2} + 20\log f + 20\log fd - 76.3 \quad (f_0 < f < f_a) \tag{4.73}$$

$$10\log\left(\frac{<\tilde{v}_{1,a}>}{<\tilde{v}_{2,a}>}\right)^2 = 20\log M_{s2} + 20\log f - 41.3 \quad (f > f_a) \tag{4.74}$$

For $f > f_0$, R_I of the ideal double wall is expressed by Eq. (4.51b). Above f_b, $W_{2,b} / W_{2,a} >> 1$ so that $[1+(W_{2,b} / W_{2,a})] \approx W_{2,b} / W_{2,a}$. Consequently, by combining Eq. (4.73), Eq. (4.70), Eq. (4.69), and Eq. (4.66) with Eq. (4.65), the following expression of transmission loss is obtained for $f > f_b, f > f_0$:

$$R = 20\log M_{s1} + 20\log f - 47.3 - 10\log\left[\frac{n\psi}{S}\left|\frac{Z_1}{Z_1 + Z_2}\right|^2\right] \tag{4.75}$$

The latter expression is comparable with Eq. (4.64) from which ΔR_M is given by:

$$\Delta R_M = 20\log\left|\frac{M_{s1}(Z_1 + Z_2)}{(M_{s1} + M_{s2})Z_1}\right| - 10\log\left(\frac{n\psi}{S}\right) \tag{4.76}$$

For line connections in form of parallel studs with c/c spacing, b (m), the area $S = nbL$ (m^2) and by substituting Eq. (4.68) and Eq. (4.72) into Eq. (4.76), the following expression is obtained:

$$\Delta R_M = 10\log f_{cl} + 10\log b - 23.4\,, \quad f_{cl} = \left[\frac{M_{s1}\sqrt{f_{c2}} + M_{s2}\sqrt{f_{c1}}}{M_{s1} + M_{s2}}\right]^2 \tag{4.77a, b}$$

where the c/c spacing between studs, $b=S/nL$, n is the number of line sound bridges within the total transmission (panel) surface, S, and f_{cp} is a combination of the two plates critical frequency for the case of line excitation. The constant 23.4 is the evaluation of the parameter, $10\log(2c/\pi)$. A special case is obtained if the double wall is constructed of two identical exterior surfaces with line connections. In this case, Eq. (4.77a) is reduced to:

$$\Delta R_M = 10\log f_c + 10\log b - 23.4 \tag{4.78}$$

In the same way, for evenly distributed point connections with a c/c distance, e, (m), one obtains:

$$\Delta R_M = 20\log f_{cp} + 20\log e - 44.8\,,\; f_{cp} = \frac{M_{s1}f_{c2} + M_{s2}f_{c1}}{M_{s1} + M_{s2}} \qquad (4.79a, b)$$

The quantity f_{cp} is a combination of the two plates critical frequency for the case of point excitation, and $e = (S/n)^{1/2}$. The constant 44.8 in Eq. (4.79a) is the evaluation of the parameter, $10\log(8c^2/\pi^3)$. For the special case with two identical surfaces, Eq. (4.79a) becomes:

$$\Delta R_M = 20\log f_c + 20\log e - 44.8 \qquad (4.80)$$

Edge connection

For the interconnections along the edge of a double construction, the radiated sound power is about half of the power for line excitation in the middle of the plate. The input impedance of the wall with edge line is expressed as

$$Z = \tfrac{1}{2}(1+j)M_s c(f/f_c)^{1/2} \text{ (Per unit length)} \qquad (4.81)$$

The factor, ψ, in Eq. (4.76) for edge line sound source is expressed as

$$\psi = \frac{cL_e}{\pi f_{c1}} \quad (\text{for } f << f_{c2}) \qquad (4.82)$$

where L_e is the length of edge interconnection. Consequently, substituting Eq. (4.81) and Eq. (4.82) into Eq. (4.76), ΔR_M for a double wall with edge connection becomes:

$$\Delta R_M = 10\log f_{cl} + 10\log(b_e) - 20.4 \qquad (4.83)$$

where $b_e = S / nL_e$, L_e is the length of edge interconnection, f_{cl} is expressed by Eq. (4.77b), and the constant 20.4 is the evaluation of the parameter, $10\log(c/\pi)$. Comparing Eq. (4.83) with Eq. (4.77a) shows that a difference amounts to about 3 dB between the two cases.

Discussion

The transmission loss adjustment, ΔR_M, derived above for the three cases point, line and edge sound bridges concerns the forced sound transmission and not the resonant one. They are, thus, valid for $f < f_c$ and yield good results up to $0.5f_c$. Deterioration in prediction can be seen for $f > f_c$. In this case the resonant sound transmission should be applied, as discussed below.

Total radiated power from the plate

If a large plate is excited by a harmonic force F, free bending waves will be generated and propagated away from the excitation point. Above the critical frequency, the sound is radiated from the whole plate and the radiation efficiency or factor, $\sigma \approx 1$. Below the critical frequency f_c, the pressure variations at the plate surface are eliminated and no radiation occurs except from the edges and from a region around the excitation point (near field), as discussed in Sec. 2.15, Chapter 2. When a force excites a point on the plate, the radiated sound power from this plate at frequencies lower than f_c (forced transmission or near field near field sound radiation) becomes:

$$W_n \approx \rho_0 c \tilde{v}_0^2 \frac{8c^2}{\pi^3 f_c^2} \qquad (4.84)$$

where $\tilde{v}_0$ is the effective value of the velocity at the excitation point. In this case, it is not appropriate to describe the sound radiation with a radiation factor because the radiated sound power depends on the plate area, which is principally can be infinite. Correspondingly, when a plate is excited by a harmonic force along a line with a length L, the radiated sound power becomes

$$W_n = \rho_0 c \tilde{v}_0^2 \frac{2c}{\pi f_{c2}} L \qquad (4.85)$$

The quantity W_n is the radiated sound power due to a near field radiation (near the excitation point). The total radiated power from the plate is obtained by adding the radiation power, W_r, due to plates *eigen* vibrations (or resonant/free radiation), which can be expressed with radiation factor, σ, to the near field radiation, Eq. (2.317). Accordingly,

$$W = W_n + W_r\,,\; W_r = \rho_0 c < \tilde{v}^2 > S\sigma \qquad (4.86a, b)$$

Where $< \tilde{v}^2 >$ is the square velocity average over the plate area, S, except a very small area around the excitation point or line. Consequently, the power ratio expressed by (4.66) should be reformulated in accordance with Eq. (4.86) so that both forced and resonant transmissions are included to cover the excitation frequencies above and below the coincidence frequency. How strong the *eigen* vibrations that are excited due to force effect is the subject of impact sound insulation treated in Chapter 5.

Derivation of ΔR_M with respect to total radiated power

For the frequency range above the critical frequency of both panels, expressions may be derived using statistical energy analysis; see Chapter 2. In this case, not only near field radiation occurs but also resonant one. The power that is transmitted to plate 2 (Fig. 4.15) causes: (1) near-field radiation from the effected region with the power, $W_{2,n}$; and (2) building of *eigen* vibrations in the plate and subsequently resonant sound transmission with the power, W_{2r}.

The relation between the velocity of excitation point, $v_{2,b}$, and the velocity of *eigen* vibrations, $v_{2,a}$, may be obtained by considering the energy balance of the power transmitted from plate 1 to plate 2 via the stud (Kristensen and Rindel, 1989):

$$W_{12} = \tilde{v}_{2,b}\,\mathrm{Re}\{Z_2\} = 2\pi f M_{s2} S < \tilde{v}_{2,a}^2 > \eta_2 \tag{4.87}$$

where η_2 is the total loss factor of plate 2 (the receiving plate). Eq. (4.87) may be combined with Eq. (4.86b) and the resonant radiated sound power, thus, becomes:

$$W_{2r} = < \tilde{v}_{2,a}^2 > \rho_0 S c \sigma_2 = \frac{\tilde{v}_{2,b}^2\,\mathrm{Re}\{Z_2\}\rho_0 c \sigma_2}{2\pi f M_{s2}\eta_2} \tag{4.88}$$

The sound radiation from near field, $W_{2,n}$, depends on the excitation type, as shown in Table 4.6. Accordingly, the total radiated sound power for a specific sound bridge may be written in the following form:

$$W_{2b} = W_{2n} + W_{2r} = W_{2n}\kappa \tag{4.89}$$

where κ is the total radiation factor of resonant transmission contribution. The factor, κ; for a point sound bridge is expressed as

$$\kappa_p = 1 + \frac{\pi\sigma_2 f_{c2}}{4\eta_2 f} \tag{4.90}$$

Correspondingly, this factor for line and edge connections may be found with the help of Table 4.7 as

$$\kappa_l = 1 + \frac{\sigma_2}{2\eta_2}\sqrt{\frac{f_{c2}}{f}}, \quad \kappa_e = 1 + \frac{\sigma_2}{4\eta_2}\sqrt{\frac{f_{c2}}{f}} \tag{4.91a, b}$$

If one, for instance, includes the resonant radiation the previous expressions of ΔR_M, the general expression of ΔR_M of rigid studs for the three excitation cases (point, line, edge) will becomes:

$$\text{Line source:}\quad \Delta R_M = 10\log f_{cl} + 10\log b - 10\log\kappa_l - 23.4 \tag{4.92}$$

$$\text{Point source:}\quad \Delta R_M = 20\log f_{cp} + 20\log e - 10\log\kappa_p - 44.8 \tag{4.93}$$

$$\text{Edge source:}\quad \Delta R_M = 10\log f_{cl} + 10\log(b_e) - 10\log\kappa_e - 20.4 \tag{4.94}$$

The importance of the resonant radiation would be that ΔR_M is reduced when frequencies approaches f_{c2} when the radiation factor grows; ΔR_M is here a negative value. For $f > f_{c2}$, $\sigma_2 \approx 1$ and $\kappa >> 1$, which implies that ΔR_M becomes little reduced. Note that Eqs. (4.92)-Eq. (4.94) can be used for all frequency range. However, the κ factor (resonant transmission) is only important at $f \geq f_c$. For frequencies $f < f_c$, $\sigma \approx 0$ (see Eq. (2.333)), and $\kappa \approx 1.0$ and subsequently these expressions are reduced to the previous derived expressions of ΔR_M; e.g., Eq. (4.93) is reduced to Eq. (4.79a). This is important result since these general

expressions requires quite lengthy calculations (for σ and η), if they are applied for all frequency regions.

The transmission loss of the double wall with mechanical connections for $f \geq f_c$ can be calculated, using the general expression for double connected walls:

$$R = R_{1+2} + \Delta R_M \tag{4.95}$$

where R_{1+2} is the transmission loss of the ideal double wall (unconnected double wall) and ΔR_M is the calculated adjustment.

In general, massive (heavy) double constructions are particularly sensitive for sound bridges because the critical frequency typically is low, which implies a considerable resonant sound transmission; some experimental results can be found in Vigran (1979).

Example 4.5: The wall with mechanical connections between the two leaves is constructed in the way shown in Fig. 4.16, where it is shown also the measured transmission loss. The density of the two gypsum boards is 800 kg/m^3 and that the critical frequencies for the two leaves are determined as f_{c1} = 3556 Hz and f_{c2} = 2462 Hz; and characteristic impedance of air, $\rho_0 c$ = 344(1.2)= 413 kg/m^2.s. The panels are directly attached to the framework along the entire length of the tree beams using screws. The total loss factor of plate 2 is assumed to be mainly due to material losses, $\eta \approx 1.8\%$. Draw the transmission loss curve.

Solution

$M_{s1} = 800(0.013) = 10.4\ \text{kg/m}^2$

$M_{s2} = 800(0.009) = 7.2\ \text{kg/m}^2$

The mass-spring-mass resonant frequency for the panel is found from Eq. (4.47b)

$$f_0 = \frac{344}{2\pi}\left[1.8\frac{1.2}{0.076}\left(\frac{1}{10.4} + \frac{1}{7.2}\right)\right]^{1/2} = 142\ \text{Hz}$$

The lowest order resonance across the air cavity is:

$$f_a = \frac{c}{2\pi d} = \frac{344}{2\pi(0.076)} = 721\,\text{Hz}$$

For frequencies $f < 0.5 f_c \approx 1250$ Hz, which lie in the mass-controlled region, the sound reduction index of each panels reads from Eq. (4.30).

$R_1(f) = 17.2 + 20\log f - 47.3$

$R_2(f) = 20.3 + 20\log f - 47.3$

Accordingly,

$R_1(f) + R_2(f) = 40\log f - 57.1$

The transmission loss, R, of the ideal double wall at frequencies: f_0 (=142) < f < (f_a = 721) is obtained from Eq. (4.51b)

$R(f)$ = 40log f –57.1 + 20log fd –29 = 60log f – 108.5

Then R (142 Hz) = 20.6 dB and R(721 Hz) = 63 dB

With this expression, the high frequency (the upper line) is drawn.

For $f < f_0$, the sound reduction index both panels is obtained from Eq. (4.49)

$R(f)$ = 24.9 + 20 log f – 47.3 , then R (100 Hz) = 17.6 dB.

With this expression, the low frequency (lower line) is drawn.

The case is line sound bridges, and so ΔR_M is determined according to Eq. (4.77):

$$\Delta R_M = 20\log\left[\frac{10.4\sqrt{2462}+7.2\sqrt{3556}}{10.4+7.2}\right]+10\log(0.6)-23.4=9 \text{ dB}$$

Accordingly, the transmission loss of the double wall as obtained from Eq. (4.64) becomes:

$R(f)$ = 24.9 + 20 log f – 47.3 + 9 = 20log f – 13.4 ($f_b < f < 0.5 f_{c2}$)

The cut-off frequency, $f_b \approx 240$ Hz, since: 20log 240 – 13.4 ≈ 60log 240 – 108.5

The results of $R(f)$ can be extended up to f_{c2} but the mass law yield good results up to $0.5 f_{c2}$.

For frequencies $f \geq f_c$:

The radiation efficiency of the gypsum board $\sigma \approx 1.0$, Eq. (2.333), and $\eta = 0.018$.

The total radiation factor is obtained from Eq. (4.91a):

$$\kappa_l = 10\log\left(1+\frac{\pi\sigma_2 f_{c2}}{4\eta_2 f}\right) = 10\log\left(1+\frac{\pi(1)2462}{4(0.018)f}\right) = 10\log(1+1.074\times10^5 f^{-1})$$

From Eq. (4.92), ΔR_M at higher frequencies becomes:

$$\Delta R_M(f) = 9 - 10\log(1+1.074\times10^5 f^{-1})$$

From Eq. (4.36), transmission loss for $f \geq f_c$ (damping-controlled region) may be written as

$$R_1(f) = 20\log 10.4 - 13\log 3556 + 33\log f + 10\log 0.018 - 48 = 33\log f - 91.3$$

$$R_2(f) = 20\log 7.2 - 13\log 2462 + 33\log f + 10\log 0.018 - 48 = 33\log f - 92.4$$

$$R_1(f) + R_2(f) = 66\log f - 183.7$$

The total transmission loss of double wall for $f > f_a$ is obtained from Eq. (4.53b)

$$R(f) = R_1(f) + R_2(f) + 6 = 66\log f - 177.7$$

The transmission loss of the wall, thus, becomes

$$R(f) = 66\log f - 177.7 + \Delta R_M(f) \quad (f \geq f_{c2})$$

R (2462) = 38.7 dB

For demonstration, if one uses, instead, the mass law, Eq. (4.64) together with the new expression of $\Delta R_M(f)$, then the result R (2462) = 38 dB, which is near to the above value, but in this case the increase in transmission loss will be slow compared with using the high frequency-transmission loss of the ideal double wall.

The results of transmission loss are plotted in Fig. 4.16. As can be seen, the agreement between measurements and prediction is reasonable. The prediction results may somewhat be improved if one calculates exactly σ and η.

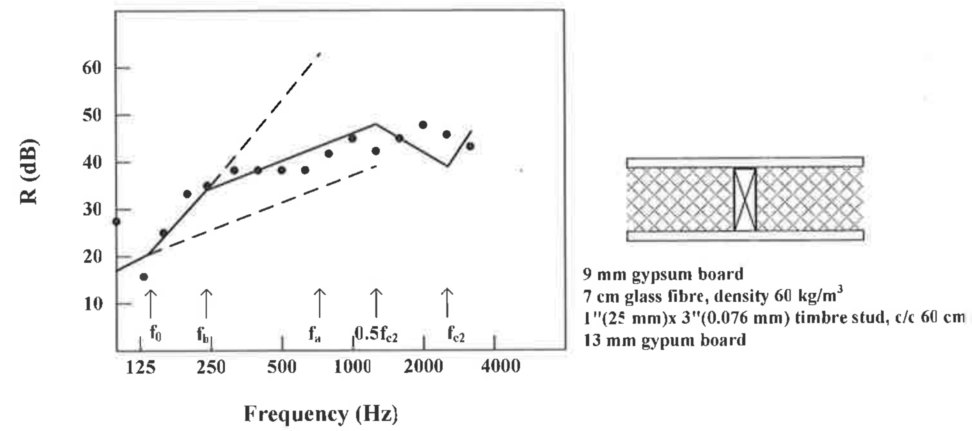


Fig. 4.16 A comparison between measured and calculated values for a 10 m² typical double construction of gypsum layers and common studs. (— —), theoretical predicted values for wall without connections; (●●●●), measured values; (——) predicted results for the construction with connections, Example 4.5.

4.4.5.2 Non-rigid studs

The above theoretical relations concern fully stiff (rigid) mechanical interconnections (e.g., wood studs or load-bearing steel studs). For interconnections that are not rigid (e.g., resilient steel rails, thin steel studs, light gauge non-load-bearing metal studs between two leaves) expressions can be derived to take account of the compliance of the connection. Consider the double construction shown in Fig. 4.15. Assume that the mechanical connection is elastic. The sound bridge, whether it is point shaped or line one is characterized by the impedance Z_b, which can, in general, be defined as

$$Z_b = \frac{F}{\tilde{v}_{1,b} - \tilde{v}_{2,b}} \tag{4.96}$$

where F is the force in the sound bridge that acts on the plates, $v_{1,b}$ and $v_{2,b}$ are the velocity of two plates at the sound bridge; Eq. (4.96) indicates that when the beam is rigid, then $Z_b \rightarrow \infty$. A bending wave that propagates in plate 1 with a velocity amplitude $v_{1,a}$, will have a reduced amplitude $v_{1,b}$ at the sound bridge because of the plate mobility Y_1 (also called mechanical admittance). In plate 2, the velocity out of the sound bridge, $v_{2,b}$, is determined by the plate admittance, Y_2; see also the mobility method in Chapter 11. These two situations are expressed as follows:

$$v_{1,b} = v_{1,a} - Y_1 F \text{ and } v_{2,b} = Y_2 F \tag{4.97a, b}$$

Subsequently, a new bending wave will generate in plate 2. The power that is transmitted from plate 1 to plate 2 via a stud can be determined as (Ver, 1971):

$$W_{12} = <\tilde{v}_{1,a}^2> \frac{|Z_b|^2 Y_2}{1 + 2\mathrm{Re}\{Z_b\}(Y_1 + Y_2) + |Z_b|^2 (Y_1 + Y_2)^2} \tag{4.98}$$

For a very stiff (rigid) sound bridge, $|Z_b| \rightarrow \infty$ and Eq. (4.98) is reduced to

$$W_{12} = <\tilde{v}_{1,a}^2> \frac{Y_2}{(Y_1 + Y_2)^2} \tag{4.99}$$

Lets introduce a coupling factor, γ, to describe the transmitted power via elastic and rigid sound bridge so that when $\gamma = 1$, the stud should be considered rigid. Subsequently, dividing Eq. (4.98) on Eq. (4.99), γ will take the following form:

$$\gamma = \frac{|Z_b|^2 (Y_1 + Y_2)^2}{1 + 2\mathrm{Re}\{Z_b\}(Y_1 + Y_2) + |Z_b|^2 (Y_1 + Y_2)^2} \tag{4.100}$$

For an elastic sound bridge, one may approximate Z_b as (Kristensen and Rindel, 1989)

$$Z_b \approx \frac{k_d}{j\omega} \tag{4.101}$$

where k_d is the dynamic stiffness of the sound bridge, which is assumed be elastic and the internal damping is often neglected (viscous damping). Thus, Eq. (4.100) is reduced to:

$$\gamma = \frac{k_d^2 (Y_1 + Y_2)^2}{\omega^2 + k_d^2 (Y_1 + Y_2)^2} \tag{4.102}$$

Note that if the sound bridge is considered viscoelastic (e.g. rubber), then the internal losses (damping) may also be expressed using a complex modulus E $(1+j)$. The coupling factor as expressed in Eq. (4.102) can now be solved with the help of the mobility formulas in Table 4.6 for each type of elastic sound bridge, i.e. point, line and edge connection. The results become:

$$\gamma_p = \left[1 + \left(\frac{8c^2 f M_m}{k_{dp} f_{cp}}\right)^2\right]^{-1} \tag{4.103}$$

$$\gamma_l = \left[1 + \frac{32\pi^2 c^2 f^3 M_m^2}{k_{dl}^2 f_{cl}}\right]^{-1} \tag{4.104}$$

$$\gamma_e = \left[1 + \frac{2\pi^2 c^2 f^3 M_m^2}{k_{de}^2 f_{cl}}\right]^{-1} \tag{4.105}$$

where $M_m = M_{s1}M_{s2} / (M_{s1}+M_{s2})$ and the subscripts p, l, and e denotes the point, line and edge connections, respectively. The frequencies, f_{cp} and f_{cl} are expressed by Eq. (4.79b) and Eq. (4.77b), respectively. The terms k_{dp} is the stiffness of stud with point connection to the plate (N/m), k_{dl} and k_{de} are the stiffness of stud (per unit length, L) with line and edge connection (N/m^2), respectively.

The dynamic stiffness of sound bridges for point and line connections, k_d, may be simply empirically calculated by choosing a value that give the best fit to experimental data for the resilient rails or steel studs. Theoretically, the spring constant or stiffness for a simply supported beam undergoing bending vibration is equal to $(48EI/L^3)$, and for clamped ends, $(192EI/L^3)$, where E is the elasticity modulus of the beam and L is the length. When source is bolted to the beam, angular motion at its free end will be hindered and this will result in increasing the spring stiffness of the resilient element by a factor of four. Instructively, for an elastic material with a thickness d, $k_d = E_d / d$ (N/m^3) where E_d is the dynamic stiffness (elasticity modulus) of the material; see also Chapter 11 and Sec. 5.4.1.

If one includes now the case of resilient studs in the previous expressions of ΔR_M, Eq. (4.92)-Eq. (4.94), the general expression of ΔR_M for the three excitation cases (point, line, edge) will become:

Line source: $$\Delta R_M = 10\log f_{cl} + 10\log b - 10\log \kappa_l - 10\log \gamma_p - 23.4 \tag{4.106}$$

Point source: $$\Delta R_M = 20\log f_{cp} + 20\log e - 10\log \kappa_p - 10\log \gamma_l - 44.8 \tag{4.107}$$

Edge source: $$\Delta R_M = 10\log f_{cl} + 10\log(b_e) - 10\log \kappa_e - 10\log \gamma_e - 20.4 \tag{4.108}$$

There expressions are general in the sense that they cover both rigid and elastic studs, below and above the coincidence frequency. If there are rigid studs in the double wall, then $\gamma = 1$, and if the resonant radiation is not considered (i.e., for $f < f_c$), then $\kappa \approx 1$, and so Eq. (4.106)-Eq. (4.108) are reduced to Eq. (4.77a), Eq. (4.79a), and Eq. (4.83), respectively. The transmission loss of the double wall can then be estimated using the general expression, Eq. (4.95).

4.4.5.3 Discussion of results and assumptions

The derivation of ΔR_M is based on the assumption that the transmitted forces via sound transmission bridges are only directed in perpendicular direction towards the partition leaves. This simplification still provides a useful and practical construction basis, as demonstrated in Example 4.5. Instructively, it can be shown that that if the partition leaves have higher bending stiffness (i.e. EI) than the interconnections (studs), then ΔR_M is determined by the bending transmission via studs, as done before. However, if the bending stiffness of the partition's leaves is substantially lower than the bending stiffness of interconnections, then ΔR_M is determined mainly by the longitudinal vibrations in the partition's leaf no.1 that are transmitted via the interconnections. The longitudinal vibrations refer to the movements of plate in the lateral direction. This implies that if it is desired to improve the sound insulation for a construction with rigid studs, the longitudinal transmission from the sender-room's leaf (leaf no. 1) must be observed. For instance, a 1″ (25.4 mm) thick timber stud has already a bending stiffness magnitude of about 700 Nm perpendicular to the fibre direction and perpendicular to the direction of vibration propagation in timber studs, while a 0.4″ (10 mm) thick chipboard (or particleboard: chips of wood compressed and glued into sheet form) has a bending stiffness of about 300 Nm irrespective of the propagation direction of bending waves, and the sheet metal in a typical thin sheet metal stud (thickness sheet metal plate is about 0.5 mm (0.02″)) has a bending stiffness magnitude of about 2 Nm. The low bending stiffness of the sheet metal is typical in relation to the bending waves that are propagated perpendicular to the thin sheet metal stud, while normal sheet metal studs have substantially higher bending stiffness for bending waves that are propagated parallel with the studs.

Furthermore, it is assumed that that neither empty cavities nor the attenuation of sound while passing through a porous material is considered. For the latter case (i.e., the influence of absorbent porosity), this effect can have an influence on the results at higher frequencies and one may check this by comparing with the results of Sec. 4.4.4. On the other hand, the influence of absorbent in the cavity is clear as the double partition that have absorbents in the cavity will have higher transmission loss than those partitions with empty cavity; see e.g., Fig. 4.17. It is, however, not necessary to fill out the whole cavity with absorbents in

order to obtain a relatively good sound insulation value, as the biggest improvement of the sound insulation is obtained with the first centimeters of the mineral wool filling. Apart from flow resistivity, the total absorbent volume and absorbent thickness are the most significant parameters for the choice of absorbent, which implies that the biggest absorbent volume and the thicker absorbent the better sound transmission loss is obtained.

Fig. 4.17 shows the transmission loss of different wall double constructions. As can be seen, the use of resilient mountings or studs contribute considerably to transmission loss, while rigid timber studs perform worse than a cavity wall with only absorption filling and is comparable with a cavity wall without studs and absorption inside. Consequently, when the interconnections in a connected double wall are made softer, a higher sound transmission loss is obtained. One can also see the obtained improvement due to the absorption in the cavity. In addition, a resilient interconnection between the stud or beam and the leaf will result in higher sound insulation. Such a situation has already been illustrated in Fig. 4.14, for which Eq. (4.78) predicts $\Delta R_M \approx 8$ dB whereas the actual value of $\Delta R_M \approx 13$ dB. Consequently, to obtain stiffeners that are weak towards bending is to make use of special studs that are made of sheet metals instead of massive timber studs. This practice is very common in plasterboard constructions.

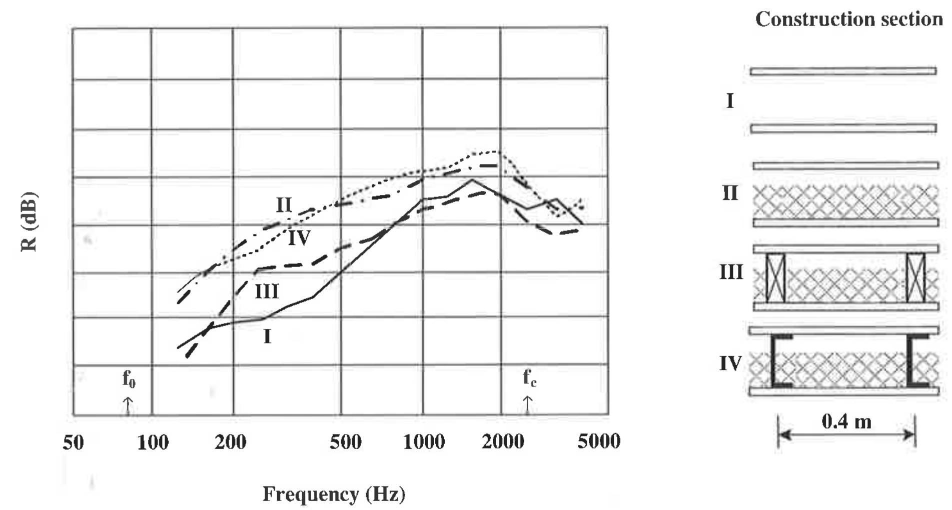


Fig. 4.17 Measured sound reduction index (transmission loss) of different wall double constructions. (I) double walls with only plasterboards; (II) same as (I) but with mineral wool in the cavity; (III) same as (II) but with rigid timber studs; (IV) same as (II) but with elastic steel beams (after Northwood, 1970).

To compare the line sound bridges with point ones, consider, e.g., a symmetrical plasterboard construction composed of 13 mm (0.5 in) thick plasterboards and with c/c-spacing for interconnections equal to 0.6 m. It can be obtained about 10 dB higher sound insulation value with point interconnections than with line interconnections. It can be shown experimentally that the single-sided point interconnection and double-sided point interconnection have equal transmission loss value.

4.4.6 The influence of stud configuration on sound insulation

The mechanical fixings or stiffeners, in form of studs or beams, between the two walls are essential for structural support and stability. On the other hand, they will create noise bridging transmission paths (structure-borne sound) that limit the maximum noise insulation obtained (equivalent to an electrical short circuit), as discussed earlier. However, it is possible, by implementing a suitable design, to eliminate, with a great extent, the insulation reduction by these connections. The distance between the stiffeners (i.e., studs or beams), D, has a specific ratio with respect to the wavelength at coincidence, λ_c (= c/f_c, where c is the sound speed in air, and f_c is the critical frequency of the leaves). By considering this ratio, as presented in Table 4.7, the stiffeners will not greatly reduce the sound insulation of the double wall. For lightweight double walls, it is often possible to fulfill these criteria. Take, for example, a typical plaster (gypsum) wall with resilient steel studs. Herein, the coincidence frequency is approx. 3150 Hz, which implies that $\lambda_c = 344/3150 = 0.11$ m. With a standards stud spacing of 60 cm (24 in), the ratio $D / \lambda_c \approx 5.5 > 5$; this is acceptable according to Table 4.7. If, on the other hand, the flexible steel studs are replaced by rigid timber studs, the criteria would not be fulfilled. Plaster walls with timber studs yield also, in general, worse insulation than similar walls with resilient steel studs (often in the range of 2 to 3 dB).

Table 4.7 Type of connection between two faces of double-leaf wall and the optimum ratio of stiffeners distance, D, and coincidence wavelength λ_c.

Connection type	D / λ_c
Resilient, parallel (standard)	> 5
Resilient, staggered	> 7
Rigid, parallel (standard)	> 10
Rigid, staggered	> 15

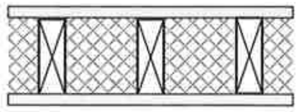
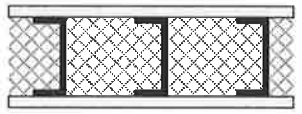
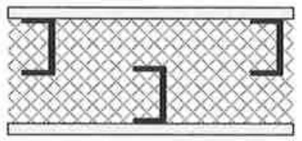
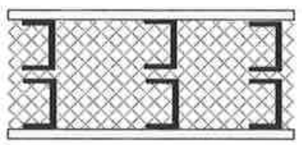
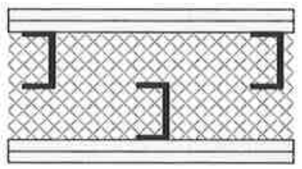
Table 4.8 compares different types of double wall configurations and possible improvement in their sound insulation. Note that the same conclusions drawn from the considering the sound reduction index, R_w, are valid also for the *STC*.

4.4.7 Double constructions in practice

As discussed earlier, when the gypsum board in a wall is solidly fastened to the wood studs on both sides, much of the sound is transmitted through the studs. Therefore, it is significant for the two wall surfaces to be supported independently from one another in order to control sound transmission. This can be done by fastening the gypsum board on each side of the wall to different lines of studs. In this context, the mechanical connection between the layers of wallboard can be reduced by the use of staggered wood studs, separate rows of wood studs, or a single row of wood studs with resilient metal furring strips to support the wallboard layers independently of each other. In general, to obtain higher R_w ratings, it is necessary to use a staggered-stud construction (refer to Table 4.3, Table 4.8 and Table 4.9), which provides better vibration isolation, and more layers of gypsum to add mass. A simple effective technique for double wall with wood studs can be the first stud and last stud are connected with gypsum boards via resilient channels to provide the necessary stiffness of the wall while the other in-between studs are unconnected with the panels. The idea is to provide stiffness without losing the high sound insulation. Table 4.9 offers some representative *STC* ($\approx R_w$) values for typical constructions. In case of two layers of gypsum board indicated in Table 4.9, the layers are assumed to be screwed together. As can be seen, the presence of the sound absorbing material will increase the sound insulation relative to the same wall without sound absorbing material. Best results are obtained when there are two layers of gypsum board on both sides of the wall in conjunction with resilient channels and absorption in the cavity. Technically, because of the mass-air-mass resonant frequency, f_0, for the small airspace between gypsum board layers due to resilient channels, the sound insulation value would be the same or lower than that for the same construction with a single layer. Note that walls with 16 mm board would be better than those with 13 mm board by a few decibels. Moreover, R_w values of 60 dB or more can be obtained if the air space is large enough and enough wallboard is used.

Resiliently mounted wall and ceiling gypsum board (drywall/ sheetrock/ wallboard) is fast becoming a building industry standard for reducing sound transfer between rooms in homes and offices.

Table 4.8 Different types of double wall configurations and their acoustical situation. The increase of R_w (dB) is equivalent to *STC* (point).

Double-wall cross-section	Properties and the acoustical situation
	Gypsum boards (drywall) with mechanical connections: Filling the cavity between vertical studs with standard glass fiber (or mineral wool) can improves the R_w rating by 3-6 dB. Using a heavier 12.2 kg/m^2 (2.5 lb/ft^2) mineral fiber acoustic insulation can improve it little further, adding up to 9 dB compared to a wall without insulation.
	A double wall has vertical wood or steel studs every 0.4 m (16 in) or 0.8 m (24 in); standard is 0.6 m (24 in). Gypsum board is screwed to the front and back of the stud. Regardless of how the cavity is filled with insulation, sound vibrates the gypsum board, which vibrates the stud, which vibrates the gypsum board on the other side, which vibrates the air to radiate sound in the room. Since wood (rigid stud) transmits sound well, a standard wood stud wall doesn't do well for soundproofing. A steel (flexible) stud with a "C" shape cross-section performs better. The "C" is resilient and absorbs sound, improving the R_w rating by up to 5 dB over wood
	Double wall with no mechanical connections: A staggered wall uses studs in a crossing (zigzag) pattern, one forward, one back, one forward, and one back. The forward ones form the wall for one side, and the back studs for the other side. The offset from the forward to the back studs is usually about 50 mm (2 in), so if the wall is built with 50 mm × 100 mm (2 in × 4 in), the total wall thickness is about 0.15 m (6 in). The staggered construction decouples the front wall from the back wall, preventing vibrations from traveling through the wall. The wider wall also has more space for insulation. The effect is to improve the wall by up to 10 dB
	A double wall with double studs separated by air gap. The outer surfaces are covered with thick gypsum board and the entire double-width cavity is filled with insulation. Like the staggered stud wall, the front and back of the wall don't share a common stud, preventing the transmission of vibrations through the stud. This may improve the R_w rating by about 15-20 dB compared to a single wall. Using a wider 0.15 m (6 in) stud instead of a 90 mm (3.5 in) stud only performs acoustically little.
	Adding another layer of gypsum board on each side of a staggered wall can improves the R_w rating by 8-10 dB. The gypsum board adds weight to the wall, making it harder for sound to vibrate. For the best effect, the second layer should be glued to the wall, not screwed on. The screws can provide a direct connection from the gypsum to the stud and give a flanking path along which vibrations can easily travel (short circuiting).

By isolating the mounting of the wall panels from the studding, (or joists), sound travel through the wall to the other side is greatly reduced (e.g., loud shouting is not heard). This is a real plus for apartments and rooms that should be quiet, such as bedrooms, for instance. Several methods to provide resilient mounting of these building wall and ceiling panels using drywall furring channel (DWFC) have been provided to the marketplace.

Table 4.9 Approximate *STC* ratings for walls with 13 mm gypsum board on both surfaces (after Quirt, 1985) [1].

Structural support	Layers of gypsum board on each wall surface					
	1+1		1+2		2 + 2	
	with sound absorbing material	without sound absorbing material	with sound absorbing material	without sound absorbing material	with sound absorbing material	without sound absorbing material
90 mm 24 gauge steel studs	45	39	49	44	53	50
38 × 89 mm wood studs with resilient steel, channels on one side	48	40	52	45	54	51
38 × 89 mm wood studs with resilient steel, channels on both sides	49	40	52	46	55	52
Staggered 38 × 89 mm wood studs	49	40	52	46	55	52
Double row of 38 × 89 mm wood studs with 25 mm gap between	57	46	60	52	63	57
150 mm load-bearing steel studs with resilient metal channels on one side	56	45	58	51	61	56

[1] Some deviations from the listed values may be expected because of variations in type of wallboard (e.g., fire-rated versus standard), wallboard attachment, type and thickness of absorptive material, and stiffness of steel studs or resilient channels.

The common internal partition used in single family homes with drywall attached directly to both sides of the wood studs has an R_w rating of about 33 dB. The addition of sound absorbing material in this wall increases its rating by about only 3 dB, because the sound energy is transmitted directly from one layer of wallboard to the other through the studs. The sound absorbing material in the cavity is of much less benefit than it would be if the layers were decoupled, in which case most of the sound would be transmitted through the air in the cavity.

Additionally, the use of resilient (rubber) fastening of linings increases the sound insulation if rubber clips are used. In practical situations, leaks and structure-borne connections between the face plates at edges of the partition usually limit the maximally achievable sound transmission loss at high frequencies to a range of 40-70 dB (Beranek and Vér, 1992). Acoustic sealing and caulking can help in reducing eventual leaks. The substitution of the generic components, e.g., shape of channel can influence sound insulation, as well. For example, a trapezoidal channel as that shown in Fig. 4.18 leads to better insulation than if the channel were rectangular. Usually, this relates to the effect of sound distribution via the channel itself.

Resilient channels used on one or both faces of single rows of stiff (rigid) studs (wood studs or load-bearing steel studs) also help to overcome peripheral transmission through header and sole (or bottom) plates. Adding resilient metal channels to one face of a single row of studs improves sound reduction considerably, allowing sound-absorbing material in the cavity to be effective, as evidenced by Table 4.9. In general, with asymmetrical (1× 2) constructions, there is should be no problem of which side the channels were placed on. Load-bearing steel studs behave in much the same way as do wood studs for their rigidity, but light gauge non-load-bearing metal studs and resilient thin steel rails are sufficiently flexible to reduce sound transmission through the material (Fig. 4.17), and can provide about the same sound reduction as wood-stud walls with resilient metal channels on one face. Usually, non-load-bearing steel studs are resilient enough to provide adequate mechanical decoupling between the layers of a double wall. It is noteworthy to indicate although adding weight will generally increase the transmission loss of a wall, adding a layer in the wrong place can reduce the effectiveness of the airspace and thus lower the transmission loss. For example, adding a layer of gypsum board to the inner face of one row of studs in the middle of the double stud wall can lower the sound insulation. Moreover, adding a layer of gypsum board to the inner face of both sets of studs can also reduce sound insulation.

Metal framing reduces sound transmission significantly better than equally dimensioned wood, in the same way as wider spacing between framing members, regardless of material.

Effect of the air space

In designing a wall, the importance of a large airspace should be remembered (refer to Table 4.4). The thicker the air space between the boards, the more flexible it is and thus it transmits less vibration to the other half of the wall, improving its sound insulation. A thin air space is enough for heavy "massive structured" double walls, but in the case of light double-framed board walls, the

Additionally, the use of resilient (rubber) fastening of linings increases the sound insulation if rubber clips are used. In practical situations, leaks and structure-borne connections between the face plates at edges of the partition usually limit the maximally achievable sound transmission loss at high frequencies to a range of 40-70 dB (Beranek and Vér, 1992). Acoustic sealing and caulking can help in reducing eventual leaks. The substitution of the generic components, e.g., shape of channel can influence sound insulation, as well. For example, a trapezoidal channel as that shown in Fig. 4.18 leads to better insulation than if the channel were rectangular. Usually, this relates to the effect of sound distribution via the channel itself.

Resilient channels used on one or both faces of single rows of stiff (rigid) studs (wood studs or load-bearing steel studs) also help to overcome peripheral transmission through header and sole (or bottom) plates. Adding resilient metal channels to one face of a single row of studs improves sound reduction considerably, allowing sound-absorbing material in the cavity to be effective, as evidenced by Table 4.9. In general, with asymmetrical (1× 2) constructions, there is should be no problem of which side the channels were placed on. Load-bearing steel studs behave in much the same way as do wood studs for their rigidity, but light gauge non-load-bearing metal studs and resilient thin steel rails are sufficiently flexible to reduce sound transmission through the material (Fig. 4.17), and can provide about the same sound reduction as wood-stud walls with resilient metal channels on one face. Usually, non-load-bearing steel studs are resilient enough to provide adequate mechanical decoupling between the layers of a double wall. It is noteworthy to indicate although adding weight will generally increase the transmission loss of a wall, adding a layer in the wrong place can reduce the effectiveness of the airspace and thus lower the transmission loss. For example, adding a layer of gypsum board to the inner face of one row of studs in the middle of the double stud wall can lower the sound insulation. Moreover, adding a layer of gypsum board to the inner face of both sets of studs can also reduce sound insulation.

Metal framing reduces sound transmission significantly better than equally dimensioned wood, in the same way as wider spacing between framing members, regardless of material.

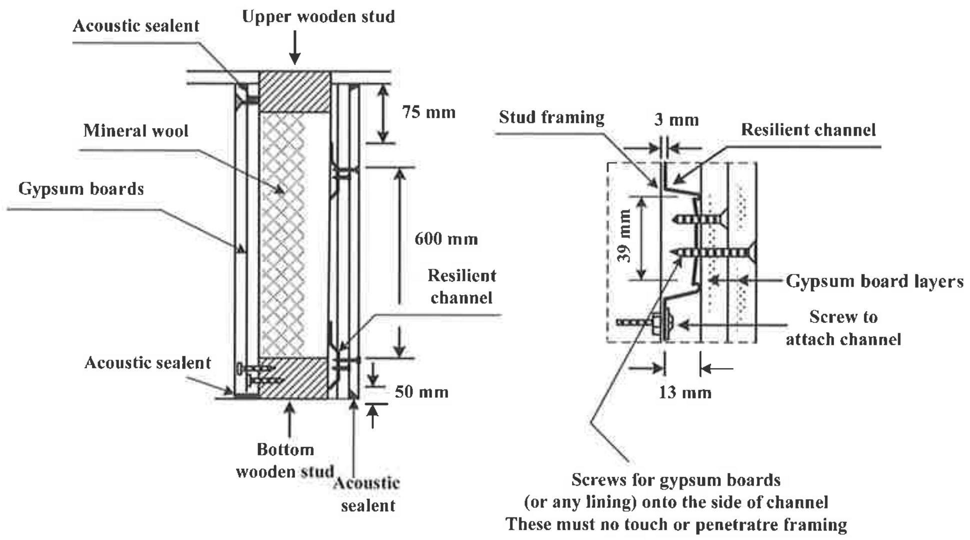
Effect of the air space

In designing a wall, the importance of a large airspace should be remembered (refer to Table 4.4). The thicker the air space between the boards, the more flexible it is and thus it transmits less vibration to the other half of the wall, improving its sound insulation. A thin air space is enough for heavy "massive structured" double walls, but in the case of light double-framed board walls, the

Consequently, rigid mechanical connections are the acoustical equivalent of an electrical short circuit or a sound (or thermal if heat transmission is considered too) bridge in an insulated wall and should be avoided. Further related issues are presented below.

Isolation of wall lining by resilient channels

Fig. 4.18 illustrates common methods to isolate wall linings, e.g. gypsum layers, from the stud framing. As can be seen in the left-hand figure, the acoustic sealant used at the ends of the panels is important to reduce the effect of sound leakage. Without a sealant, a deterioration of sound insulation will appears at low frequencies; commonly at $f > 100$ Hz and increases dramatically with increasing frequency and can reach up to 20 dB at $f > 1500$ Hz.



Isolating wall linings in a double wall

Details of channel set-up

Fig. 4.18 Acoustic details of isolating wall linings from the framing in double partitions. Gypsum material naturally has a high bulk density, and therefore its weight gives naturally good sound reduction at low frequencies. Coupled with soft mineral wool sealed - in plastic bags and fitted to the rear of the ceiling or wall panels, good sound reduction from room to room is achieved throughout the frequency range.

Additionally, the use of resilient (rubber) fastening of linings increases the sound insulation if rubber clips are used. In practical situations, leaks and structure-borne connections between the face plates at edges of the partition usually limit the maximally achievable sound transmission loss at high frequencies to a range of 40-70 dB (Beranek and Vér, 1992). Acoustic sealing and caulking can help in reducing eventual leaks. The substitution of the generic components, e.g., shape of channel can influence sound insulation, as well. For example, a trapezoidal channel as that shown in Fig. 4.18 leads to better insulation than if the channel were rectangular. Usually, this relates to the effect of sound distribution via the channel itself.

Resilient channels used on one or both faces of single rows of stiff (rigid) studs (wood studs or load-bearing steel studs) also help to overcome peripheral transmission through header and sole (or bottom) plates. Adding resilient metal channels to one face of a single row of studs improves sound reduction considerably, allowing sound-absorbing material in the cavity to be effective, as evidenced by Table 4.9. In general, with asymmetrical (1× 2) constructions, there is should be no problem of which side the channels were placed on. Load-bearing steel studs behave in much the same way as do wood studs for their rigidity, but light gauge non-load-bearing metal studs and resilient thin steel rails are sufficiently flexible to reduce sound transmission through the material (Fig. 4.17), and can provide about the same sound reduction as wood-stud walls with resilient metal channels on one face. Usually, non-load-bearing steel studs are resilient enough to provide adequate mechanical decoupling between the layers of a double wall. It is noteworthy to indicate although adding weight will generally increase the transmission loss of a wall, adding a layer in the wrong place can reduce the effectiveness of the airspace and thus lower the transmission loss. For example, adding a layer of gypsum board to the inner face of one row of studs in the middle of the double stud wall can lower the sound insulation. Moreover, adding a layer of gypsum board to the inner face of both sets of studs can also reduce sound insulation.

Metal framing reduces sound transmission significantly better than equally dimensioned wood, in the same way as wider spacing between framing members, regardless of material.

Effect of the air space

In designing a wall, the importance of a large airspace should be remembered (refer to Table 4.4). The thicker the air space between the boards, the more flexible it is and thus it transmits less vibration to the other half of the wall, improving its sound insulation. A thin air space is enough for heavy "massive structured" double walls, but in the case of light double-framed board walls, the

thickness of the air space must be at least 145 mm in order to fulfill the sound insulation requirement for walls between apartments. At high frequencies, standing waves are formed in the air space of a double wall, reducing its sound insulation. The effect of standing waves is reduced by installing sound-absorbent material, such as mineral wool or wood fibre insulation material, in the air space, as discussed before. The air space can be completely filled with light absorbing material, so for example wood fibre insulation can also be installed by blasting. The sound insulation of a wall can be improved by 5-6 dB on average by filling the air space with sound-absorbent material. The softer the absorbing material, the greater an improvement can be achieved. For example, when using soft mineral wool, the improvement effect can be 8-10 dB on average.

As indicated earlier, uniform board sheeting should not be installed in the air space of a double wall because in that case, the double wall becomes a triple wall, the sound insulation of which is lower than that of a double wall of equal mass and thickness, as indicated earlier. The impaired sound insulation of a triple wall is attributed to the generated several vibration subsystems, which can rise the mass-spring-mass resonance frequency, f_0, of the wall.

Filling the airspace with gases

By filling the cavity with porous sound-absorbing material, high sound transmission loss can be obtained. Alternatively, filling the airspace with a gas (e.g., SF_6 or CO_2) that has about 50% lower speed of sound than air can have the same effect as the sound-absorbing material (Gösele et al., 1982; Ertel and Möser, 1984). Using a light gas such as helium, which has about three times higher speed of sound than air (refer to table 2.4) also improves sound transmission loss to the same extent as a heavy gas fill. In this case, the improvement is due to the higher speed of sound in the gas fill, which makes it easier to push the gas tangentially than to compress it. However, this beneficial effect can be exploited to some types of partitions such as double windows, which are, in general, hermetically sealed and light transparent (Bernek ad Vér, 1992). This method can be a solution for the problem with glazing because there is no in-fill- could use transparent absorber.

Lightweight structures

When lightweight construction and high R_w (or *STC*) values are desired, double layer constructions must be used. For lightweight double partitions, it is practically difficult to produce constructions where the sealing with the associated attached constructions is adequate for normal montage in field. For lab test, the process of obtaining perfect sealing is somewhat painstaking, therefore one should, as a rule, expect to obtain a higher sound insulation value in the lab than in field. A typical deviation between 4 dB and 7 dB can thus be considered between a lab value and what can usually be obtained in practice with good sealing (e.g., paper machine felt between floor and ceiling strips and joists; caulking all attachments with adjoining constructions). It is entirely possible to construct a double partition that possesses so much high transmission loss but still remains relatively thin and lightweight; an example is shown in Fig. 4.19.

The sound insulation properties and acoustic behaviour of lightweight double partitions (e.g., wooden studs and gypsum boards) deviate markedly from solid structures (or massive structures, e.g. brick). For instance, flanking transmissions caused by lightweight structures often remain minor because gypsum wallboards have a low sound radiation coefficient, σ. Due to this, sounds caused by impacts on wall structures are usually not propagated beyond the neighbouring apartment. Similarly, joints between gypsum board structures differ acoustically from the rigid joints of solid structures. Joints between wood structures made using mechanical fasteners are flexible by nature and almost always have a seam that interrupts the continuity of the structure. Consequently, flexible joints with cutoffs contribute to reducing the effect of flanking sound transmission.

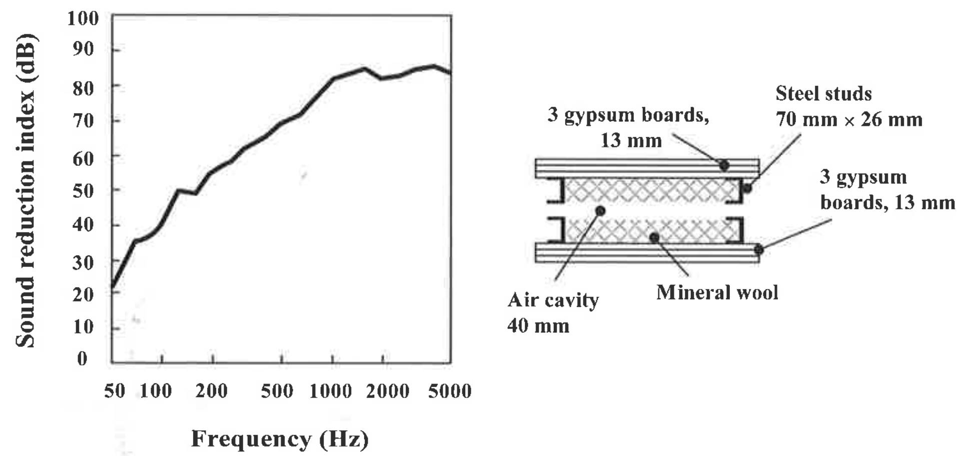


Fig. 4.19 Measured transmission loss for a wall composed of triple gypsum board layers (hard) on both sides and a double stud skeleton with 70 mm studs with mineral wool. R_w is 67 dB.

From the acoustical point of view, the absorption in the cavity is important in the lightweight double walls whereas the rigid connections have minor influence. In heavyweight double walls, the rigid connections are most important while the absorption in the cavity has minor influence. One can verify these arguments using the calculation statements mentioned in this Chapter; see also Chapter 5.

4.4.8 Practical guidelines on the sound insulation of double constructions

Based on the previous discussions, the following remarks are highlighted, as follows.

• Lightweight constructions can be very effective but introduce additional effects that must be appreciated if double layer designs are to be successful. Important factors, in addition to the masses of the component layers, are the depth of the air space, the use of sound absorbing materials within the air spaces, and the rigidity of the mechanical coupling between the layers. The ideal double layer assembly has no rigid mechanical connection between its two surfaces. However, the transmission loss of the ideal double leaf is limited mainly because the mechanical fixings between the two walls, which are essential for structural support and stability, will create noise bridging transmission paths that limit the maximum noise insulation obtained. This has the most effect when the noise insulation through the air cavity is comparatively high, which implies that the transmission through the fixings becomes significant. This usually occurs at mid to high frequencies (often above 500 Hz) and is more important for walls that have high transmission loss. Consequently, the key to good design is that the two panels must be acoustically and mechanically isolated from each other as much as possible.
• There is little sound attenuation at the resonant frequency, f_0. One should therefore try to lower it as low as possible, preferably, $f_0 < 100$ Hz.
• Absorption materials in the cavity should be used to prevent build up of reverberant energy and modal resonance.
• A break or separation between materials in the path of sound (the vibration path) can significantly reduce sound transmission.
• Leaves of double partition should be made of different thicknesses or materials to separate coincidence dips, especially important for glazing in windows where damping is low. Using laminated glass, with thin layers of suitable plastic between layers of glass, can reduce the effect of coincidence as the friction between the layers gives extra damping.
• A bigger air gap improves the transmission loss; this is useful for glazing. Typically, R_w in this case increases by 3 dB for each doubling of air layer thickness (acoustic isolation).

• Although more mass improves transmission loss, the coincidence can be problem. Consequently, the situation may be studied carefully.
• Separate studding should be used for both leaves for very high performance (mechanical isolation).
• Staggered studs should be used to reduce the effect of sound bridges.
• Resilient metal channels and ties should be used. Adding resilient metal channels to one face or both faces of single row of studs improves sound reduction noticeably, which also make sound-absorbing material in the cavity to be effective.
• Load-bearing steel studs behave almost as wood studs, but light gauge non-load-bearing metal studs and resilient thin steel rails are adequately elastic to reduce sound transmission through the material.
These same principles can be applied to floors and ceilings. A heavy false ceiling hung on springs can match the performance of a double wall.
• For concrete blocks, it is important to eliminate holes in mortar and blocks. Similarly it is important that glazing is properly sealed. For all constructions, good workmanship is the key to good insulation, for example gaps between floor boards will significant degrade partition performance.
• The case of orthotropic plates should analysed carefully. A typical example of orthotropic plates would be factory cladding and rib-stiffened panel. These have different stiffness in both directions, and therefore one obtains multiple coincidence frequencies; see Sec. 4.7.2. The coincidence region gets extended, and the isolation is poor unless a large amount of damping is present. In addition, the modal resonance frequencies are increased in frequency because of the panel shaping, leading to a decrease in the transmission loss. These are, in general, not good noise insulators.
• The sound absorbing material in the cavity is of much less benefit than it would be if the layers were decoupled, in which case most of the sound would be transmitted through the air in the cavity.
• Flanking needs to be considered, e.g., through lightweight glazing frames, and independently mounted glazing frames.
• Without a large inter-glazing spacing, triple glazing does not offer significant improvement in performance over double glazing. In many cases in practice, double glazing has been designed for thermal rather than acoustic reasons, and the transmission loss at some frequencies can be poor and even worse than single glazing and masking sounds at other frequencies reduced.
• Standing wave resonances between the layers of a double layer wall or floor occur at relatively high frequencies and the sound transmission losses can be further reduced by them. The negative effects of most of these resonances can be

reduced by the addition of sound absorbing material inside the cavities. For typical wall thicknesses (around 100 mm), the density and thickness of the sound absorbing material is not a very important factor. Increasing the thickness beyond about 75 mm has little effect on the R_w (or *STC*) rating, although, for floors or walls that are significantly thicker than normal, it becomes more important to use thicker layers of glass fibre. The type of glass fibre or mineral wool insulation normally used for thermal purposes absorbs sound well and is fairly adequate for use inside double layer walls as a sound absorbing material.

- Floors need to consider structure-borne (footfall) and airborne paths. Independent joists for ceiling and floor are too expensive, and so a resilient material may be used to vibration-isolate the two leaves. The resonant frequency, f_0, is usually very low for a floor, and so not a problem. Mineral wool should be placed between leaves (providing ventilation is not an issue). Floating floors are useful to prevent flanking transmission through the building structure, as discussed in Chapter 5.
- All wall partitions must continue to the true ceiling otherwise sound leaks can be a problem.
- There are, often, several other paths sound can follow apart from the direct path through the panel, *R*. These include air conditioning ducts, through ceiling spaces, around edge fixings, special openings, etc. Consequently, it is better to have a well-fitting light door than a loose-fitting heavy one. In practice, the transmission loss of a composite panel is dominated by it's weakest element; see Sec.4.6.
- The mass or weight of an assembly's membrane also contributes to sound control. For example, added sheets of gypsum board absorb more sound, and a cement block wall absorbs more sound than an empty frame wall.
- Materials with higher density and airflow resistance are better at reducing sound transmission. Likewise materials with higher density and airflow resistance are better at reducing sound transmission.

Example 4.6: In the plant room of an industrial building, there is a compressor which operates at 1500 revolution / minute. The machine yields 4 pressure impulses per each revolution. The neighboring room is used for the personal. The partition between the two rooms is composed of 13 mm plasterboards (surface density 10 kg/m^2) on both sides, which are connected by 70 mm timbre studs, c/c 600 mm. Why the personal complain about the high noise coming from the machine room? Make the necessary modifications.

Solution

The disturbance or excitation frequency from the machine f =1500(4)/60 = 100 Hz. The mass-spring-mass resonant frequency for the double wall is found from Eq. (4.47a):

$$f_0 = \frac{344}{2\pi}\left[\frac{1.2}{0.07}\left(\frac{1}{10}+\frac{1}{10}\right)\right]^{1/2} = 101 \text{ Hz}$$

Consequently, the resonant frequency coincides with the disturbance frequency from the machine, which results in the strong wall vibrations and the subsequent noise radiation to the neighboring room.

As modification, one can either increase the height of studs (gap depth) or the density of the plasterboards. For instance, changing d to 140 mm, the resonant frequency becomes: f_0 = 72 Hz, and the same result would be obtained by increasing the mass density, M_s. Thus, with such modification, f_0 will not be within the limits of frequencies of building acoustics, suitable for human hearing and no longer match with the excitation frequency of the machine.

4.5 External Lining for Sound Insulation

A simple way to increase the airborne sound insulation for a heavy solid single wall (e.g., façades) or floor is to clad it with a radiation-reducing layer or so called separate board layer or external lining. This can be accomplished by a plasterboard or particle board supported on studs that are fastened (e.g. with nails) at the original wall with a mineral wall blanket in the cavity, as shown in Fig. 4.20. The construction is, acoustically, an asymmetrical double wall (different properties for two leaves). Since the surface density of the radiation-reducing layer (e.g. plasterboard), M_{s1}, is much lower than the wall's surface density M_{s2} itself (M_{s1}<<M_{s2}), the formula of system's resonant or *eigen* frequency, f_0 , Eq. (4.47a), may be simplified to:

$$f_0 \approx \frac{60}{\sqrt{M_{s1}d}} \tag{4.109}$$

Below this frequency, the radiation-reducing layer yields insignificant contribution to the insulation as the total surface density is not effected. Above f_0, the lightweight layer (or leaf) yields an improvement in insulation, ΔR_1, which increases rapidly with frequency according to:

$$\Delta R_1 = 40\log\left(\frac{f}{f_0}\right) \quad (f > f_0) \tag{4.110}$$

An upper limit for insulation improvement is reached when sound transmission via interconnections becomes dominant. This transmission is frequency-dependent and is thought to contribute totally to the original wall insulation. When the radiation-reducing panel is nailed on rigid studs (line connection) with c/c-distance, b (m), this contribution, ΔR_2, can be calculated as

$$\Delta R_2 = 10\log\left(\frac{\pi b}{2\lambda_c}\right) \qquad (4.111)$$

where λ_c is the panel wavelength at coincidence (m), $\lambda_c = c/f_c$. Alternatively, if the panel is fixed point-wise in N points/m^2 (number of points per unit area of the wall), the expression of maximum insulation improvement becomes:

$$\Delta R_2 = 10\log\left(\frac{\pi^3}{8N\lambda_c^2}\right) \qquad (4.112)$$

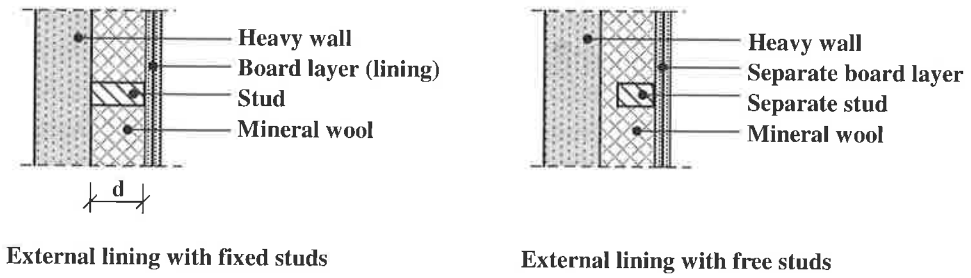


Fig. 4.20 The airborne sound insulation of a simple "massive structured" wall can be improved by doubling the structure by placing a radiation-reducing layer such as plasterboard whereby studs are fixed to or detached from the original construction.

Subsequently, the total sound reduction index improvement by additional layers for walls, ΔR_{tot}, is obtained as

$$\Delta R_{tot} = -10\log(10^{-\Delta R_1/10} + 10^{-\Delta R_2/10}) \qquad (4.113)$$

This implies that the minor value of both additional insulations becomes significant in design.

If the studs that are mounted separately from the wall (Fig. 4.20), there will be no theoretical upper limit for the insulation improvement, and so the sound reduction will be calculated according to Eq. (4.110); some results are shown in Fig. 4.21. As can be seen, the deteriorating effect of studs on the sound insulation

is clear, because fixed studs to the renovated wall are sound bridges, which offer an additional transmission path for sound. When studs are separated from the existed wall, there will be no mechanical interconnection between the leaf and the wall, thereby increasing the sound transmission reduction.

Example 4.7: In order to improve the sound insulation for a 15 cm thick concrete wall, an additional insulation has been performed. This contains of 45 mm timbre studs, c/c 600 mm and 13 mm thick plasterboard (surface density is 10 kg/m^2). The studs have been fixed firmly in the concrete and the space between them is filled with mineral wool. In order to improve the sound insulation further, a suggestion to separate the studs from the wall is put under discussion since the space allows for such arrangement. If the studs are relocated 50 mm from the concrete wall, how much would be the improvement? The mineral wool and plasterboards are still in use.

Solution

I. Before carrying out the extra improvement:

From Eq. (4.109), $f_0 = 60/\sqrt{0.045(10)} \approx 90$ Hz

From Eq. (4.110), $\Delta R_1 = 40\log(f/90) = 40\log f - 78.2$

From Example 4.4, the critical frequency of 13 mm plasterboard (gypsum board) is = 2618 Hz.

Thus, $\lambda_g = c/f_c = 344/2618 = 0.13$ m

From Eq. (4.111), $\Delta R_2 = 10\log(0.6\pi/\{2(0.13)\}) = 8.6$ dB.

Accordingly, the total sound improvement, Eq. (4.113) becomes:

$\Delta R_{tot}(f) = -10\log\{10^{7.82-4\log f} + 10^{-0.86}\}$ (E.1)

This function is plotted in Fig. 4.21.

II. After carrying out the extra improvement:

The cavity depth will be d = 0.045 m + 0.05 m = 0.095 m.

The resonant frequency for the new installation becomes

$f_0 = 60/\sqrt{0.095(10)} \approx 62$ Hz.

When the studs becomes free standing from the wall then $\Delta R_2 = 0$. Therefore,

$\Delta R_{tot}(f) = \Delta R_1 = 40\log(f/62)$ (E.2)

The difference between the two cases is calculated as: (E.2) – (E.1). The results are shown in the Fig. 4.21. As can be seen, there will be a great sound reduction when the studs are not fixed to the concrete wall. Moreover, the resonant frequency of the new system has been lowered, so it lies outside the interesting frequency limits of building acoustics, suitable for human hearing.

Renovation in practice

In addition to wooden studwork, doubling of existing massive structure can be carried out by constructing a separate wallboard with studs on top of the wall or with mineral wool put in-between by using resilient bars attached to the existing wall instead of the wooden studwork. In such cases, the improvement of the sound insulation of the wall is often not enough; due to flanking transmission, the floor and ceiling of the room will also have to be converted into double structures. The design of additional insulation is based on the frequencies at which the insulation must be increased. Especially, for a unit room (e.g. heating, etc) with low frequency sound the design is crucial due to the noise spectrum emitted by such a room. As revealed by Eq. (4.109)-Eq. (4.113), the important parameters that should be taken into account are: (1) the surface density of the layer and it is bending stiffness, (2) the distance between the wall and the layer and (3) fixation of the studs in the heavy wall (Fig. 4.20).

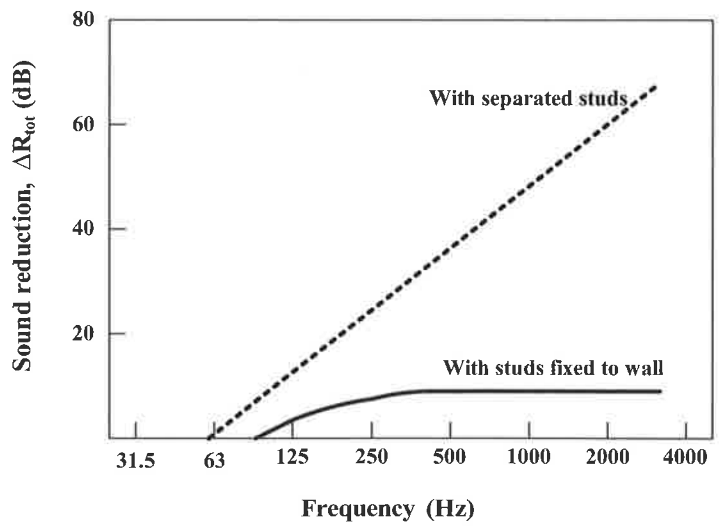


Fig. 4.21 Sound reduction improvement by mounting a radiation-reducing layer where the studs are either fixed or not fixed to the existed concrete construction. The prediction is based on Example 4.7.

Additional insulation is specially applicable and effective for heavy or massive wall. However, for lightweight walls (i.e. wallboard on studs), such a measure can lead to a deterioration in sound insulation due to the factors that are discussed previously. Nevertheless, the sound insulation of a double-framed wall can be improved by adding extra layers of plasterboard to the structure to increase the mass or by increasing the thickness of the air space. The air space can be thickened by removing the sheeting from one side of a double-frame wall before building the new additional wall. If, on the other hand, the sheeting removed from the existing wall serves as a stabilizing structure, the bracing capacity of the sheeting can be replaced by diagonal grid siding. The diagonal siding boards should not be spaced too dense, for example with a spacing c/c of 300 mm (Lahtela, 2005), because the air space in a double wall should not have a uniform laminar surface interrupting the air cavity that is filled with mineral wool. Renovation of some old wall structures to improve their sound insulation is shown in Fig. 4.22.

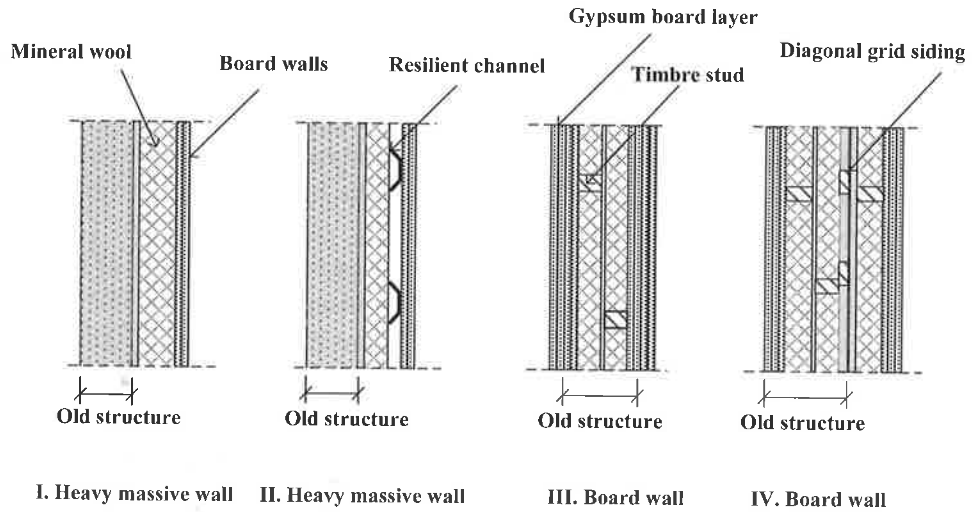


Fig. 4.22 Renovation of some old wall structures to improve their sound insulation. (I) Sound radiation reducing layer is built on an old heavy massive concrete or brick wall; (II) Sound radiation reducing layer with resilient bars is built on an old heavy massive concrete or brick wall; (III) Adding extra plasterboard layers on both sides of an old double framed wall increases the wall mass wall; (IV) Increasing the thickness of the air cavity of an old double framed wall by adding a new stud work such that the stabilising sheeting on one side of the double wall is replaced with diagonal grid sliding.

Seemingly minor construction details can affect the acoustic performance of a renovated double wall. For example, when gypsum wallboard is attached to steel furring or resilient channels, using screws that are too long will result in the screw contacting the face of e.g., the concrete masonry substrate, which in turn becomes an efficient path for sound and vibration transmission; see also Fig. 4.18.

4.6 Multiple and Composite Partitions

4.6.1 Multi-leaf partitions

Multi-leaf partitions or so-called multiple walls or lagging structures are those structures, which are composed of three or more leaves. As with the double partitions, a substantial increase in the sound insulation will no be achieved before the frequency exceeds the system's *eigen* or resonant frequency, f_0; a symmetric wall is assumed. Below this frequency, the insulation is determined by the total surface density (mass per unit area). Above the resonance frequency, the insulation increases very fast with frequency and the more leaves that compose the wall the faster increase in sound insulation becomes. Therefore, a multiple wall can be a solution if very high insulation is required and if there is an available sufficient space. If, on the other hand, the space is limited, a properly designed double partition yields, in general, as good insulation as a symmetrical multiple wall. Asymmetrical multiple constructions has been used with some success in some window constructions.

A general procedure to calculate the transmission loss for multi- leaf (layer) constructions may be found in Hamada and Tachibana (1985), Au and Byrne (1987), and Beranek and Vér (1992). This procedure is general in the sense that the layers can be orthotropic or isotropic, composite or single, filled with an absorbent or without it (only air) and can also be applied to a double construction. The method allows for considering the absorption properties of porous material in the cavity. In addition, the transmission loss is expressed as a continuous function that covers the frequency range. However, it is assumed that there are no mechanical connections between layers, and so the structure borne sound is neglected through the sound bridges. It can be shown that the method yields reasonable results below the critical frequency of the structure but not at and above the coincidence frequency of the structure. The method is based on a network technique so that one starts from the receiver side of the construction and end with the source side in which analytical and numerical expressions are

used. Alternatively, the general methods of analysing double constructions, as presented in this chapter, may be employed to analyse multi-leaf partitions.

4.6.2 Transmission loss of composite walls

A partition between two rooms consists, often, of various components with different sound reduction indices. The total power transmitted through a composite wall (elements in parallel, e.g., a window or door in the wall) is the sum of the power transmitted through each element, since the incident acoustic intensity is the same for all elements:

$$W_{tr} = \sum_j W_{tr,j} = \tau W_{in} = \tau S I_{in} = I_{in} \sum_j \tau_j S_j \tag{4.114}$$

The quantity $S = \sum S_j$ is the total surface area, and τ_j is the sound power transmission coefficient for each individual element. The overall sound power transmission coefficient for elements in parallel in a composite wall is obtained by determining an average transmission coefficient in a manner similar to finding the average absorption coefficient. Accordingly,

$$\tau = \frac{\sum \tau_j S_j}{S} = \frac{\tau_1 S_1 + \tau_2 S_2 + ...}{S_1 + S_2 + ...} \tag{4.115}$$

If the different components has sound reduction indices $R_1, R_2, \dots, R_n$ (dB) and areas $S_1, S_2, \dots, S_n$ (m^2), the total or overall sound reduction index (or transmission loss), R_{tot}, for the composite wall can be obtained by combining Eq. (4.115) and Eq. (4.114) and observing that the sound reduction index is expressed by Eq. (1.20). The result becomes:

$$R_{tot} = 10\log\left(\frac{S_1 + S_2 + ... + S_n}{S_1.10^{-\frac{R_1}{10}} + S_2.10^{-\frac{R_2}{10}} + ... + S_n.10^{-\frac{R_n}{10}}}\right) \tag{4.116}$$

The overall sound reduction index (transmission loss) can be considered as a special case of the field sound reduction index (refer to Chapter 3) if all sound transmission occurs via the components under concern since

$$R_{tot} = L_1 - L_2 - 10\log\left(\frac{A_2}{S}\right) \tag{4.117}$$

where L_1 and L_2 are the sound pressure level in the sender-room and receiver room, respectively, A_2 is the total absorption surface area in the receiver-room and S is the partition's area.

The calculated composite sound reduction index is only meaningful at a certain distance (> 0.5-1.0 m) from the wall. Immediately close to the wall, the sound reduction index for the respective component determines the sound pressure level of the receiver-room (refer also to Chapter 9). For instance, Eq. (4.117) can be used to calculate the sound pressure level in the receiving room from the external pressure level and R may also be taken as a single or (weighted number).

Generally, openings in a panel effect its sound insulation, because the sound power transmission coefficient for an opening is unity (all energy is transmitted through the opening); this effect is illustrated in the following Example 4.8. In general, if a partition is composed of elements with different sound insulation, the parts with lower sound insulation can considerably degrade the overall result. If the difference between insulation is relatively small (7 dB or less) there needs to be a comparatively large area of the lower insulation element before the overall sound insulation is significantly affected, as inspected from Eq. (4.116). A greater difference in sound insulation normally results in a larger reduction of overall insulation.

Example 4.8: A partition wall consists of a 3.7 m × 30 m concrete wall (R = 50 dB) with a 1.2 m × 1.8 m window (R = 25 dB), a 2.1 m × 1.0 m door (R = 30 dB), and an opening ($R \approx 0$), 0.012 m × 1 m under the door. Determine the overall transmission loss for the wall with the openings included. If the opening under the door is well-sealed, how much improvement is obtained? The total absorption of the room on the receiving side of the wall is 29 m sabins.

Solution

One can directly use Eq. (4.116) to calculate the total sound reduction index. For illustration, the transmission coefficient is calculated. The results where all the openings are included, are tabulated as follows.

Partition section	Dimension area, S (m^2)	R (dB)	$\tau_j = 10^{(-R/10)}$	$\tau_j \times S$ (m^2)
Wall	3.7 × 30 = 111	50	0.00001	0.00111
Window	1.2 × 1.8 = 2.16	25	0.00316	0.00683
Door	2.1 × 1.0 = 2.10	30	0.00100	0.00210
Leak	0.012 ×1 = 0.012	0	1.00000	0.01200
Total	= 115.3			= 0.02204

From Eq. (4.115), it follows that

$$\tau = \frac{0.02204}{115.3} = 0.00019$$

The transmission loss or sound reduction index for the composite wall is

$$R = 10\log\left(\frac{1}{\tau}\right) = 10\log\left(\frac{1}{0.00019}\right) = 37.2 \text{ dB}$$

Noise reduction, NR, is obtained from Eq. (3.5) and Eq. (4.117):

$$NR = L_1 - L_2 = R + 10\log\left(\frac{A_2}{S}\right) = 37.2 + 10\log\left(\frac{29}{115.3}\right) = 31.2\,\text{dB}$$

If the gap under the door is properly sealed, it follows that:

$$\tau = \frac{0.010052}{115.3} = 8.72\times10^{-5} \text{ and } R = 10\log\left(\frac{1}{8.72\times10^{-5}}\right) = 40.6 \text{ dB}$$

Accordingly, the new NR after treatment becomes

$$NR = R + 10\log\left(\frac{A_2}{S}\right) = 40.6 + 10\log\left(\frac{29}{115.3}\right) = 34.6$$

Improvement = 34.6 – 31.2 = 3.4 dB

The NR will, thus, increase 3.4 dB after performing acoustical treatment on the gap under the door, which is a significant increment. Consequently, if the noise reduction for a wall is to be effective, any openings must be as small as possible or completely eliminated, if practical.

4.7 Sound Transmission Loss of Inhomogeneous and Orthotropic Plates

In this section, sound transmission loss of inhomogeneous plates (consist of more than one material) as well as and orthotropic plates (having different bending stiffness in each direction) of some building configurations are investigated.

4.7.1 Composite panel of two laminated layers

Panels composed of two or more laminated solid layers or ply are often used as partitions for enclosures and other acoustic structures. In principle, layers combined with each other will render the composed panel inhomogeneous, which requires then a special analytical treatment. Laminating layers increases the sound insulation due to increases in the resulted mass and sometimes the damping if the attached layer has higher values of damping factor. Typical examples are an aluminum plate bonded to a rubber sheet. If the layers are bonded at the interface with no air space, as shown in Fig. 4.23, then the composite panel bends about an overall neutral axis due to bending stress induced by the airborne sound wave. The situation, in general, is treated as a single panel. However, the bending stiffness, B, surface density, M_s, and loss factor (or damping coefficient), η, has to be modified with respect to multi-ply laminated panel.

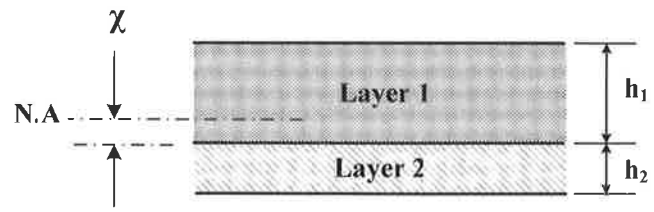


Fig. 4.23 A laminated panel composed of two solid plates in bending.

Consider the composite plate shown in Fig. 4.23, let the distance from the interface between the two layers to the overall neutral axis be, χ, positive toward material 1 side, the location of the neutral axis for the composite panel is found from the following expression:

$$\chi = \frac{E_1 h_1^2 - E_2 h_2^2}{2(E_1 h_1 + E_2 h_2)} \tag{4.118}$$

where E_1 and E_2 are Young's modulus of plate 1 and 2, respectively, and M_{s1} and M_{s2} are their surface densities.

The sound reduction index for Region II, the mass-controlled region, may be determined from Eq. (4.28) or Eq. (4.30), where surface density, M_s, for the layered panel is given by the following expression:

$$M_s = \rho_1 h_1 + \rho_2 h_2 \tag{4.119}$$

The critical or wave coincidence frequency for the layered panel may be found from the general expression of critical frequency:

$$f_c = \frac{c^2}{2\pi}\left(\frac{M_s}{B}\right)^{1/2} \tag{4.120}$$

where c is the speed of sound in the air around the panel, and B is the flexural rigidity or bending stiffness of the panel. For the given composite plate shown in Fig. 4.23, B is given by the following expression:

$$B = \frac{E_1 h_1^3}{12(1-\mu_1^2)}\{1+3(1-2\chi/h_1)^2\} + \frac{E_2 h_2^3}{12(1-\mu_2^2)}\{1+3(1+2\chi/h_2)^2\} \tag{4.121}$$

Note that the algebraic sign for χ must be maintained in Eq. (4.121) so that the quantity χ is positive when the overall neutral axis is on the layer 1 side of the interface.

The sound reduction index (transmission loss) for a layered panel in damping-controlled Region may be determined from Eq. (4.34) or Eq. (4.36), with the overall damping coefficient calculated from the following expression (Cremer et al., 1988; Barron, 2003):

$$\eta = \frac{(\eta_1 E_1 h_1 + \eta_2 E_2 h_2)(h_1 + h_2)^2}{E_1 h_1^3\{1+3(1-2\chi/h_1)^2\} + E_2 h_2^3\{1+3(1+2\chi/h_2)^2\}} \tag{4.122}$$

In case that there are more than two layers, then one can take two adjacent layers and calculate their overall B, M_s and η so that the two layers are reduced to one, which is combined with the remaining layers and so on, using the same procedure as above.

Example 4.9: An aluminum plate (type 2014) has a thickness of 1.8 mm (0.07 in) and is bonded to a hard rubber sheet having a thickness of 4.6 mm (0.18 in). The panel dimensions are 450 mm (17.7 in) by 700 mm (27.6 in). The air around the panel is at 21°C (70°F), for which the density and speed of sound are 1.20 kg/m^3 (0.075 lb$_m$/ft^3) and 343.8 m/s (1128 ft/sec), respectively. Determine the sound reduction index (transmission loss) for the panel at (a) 500 Hz and (c) 8 kHz. Check also if the rubber layer that is attached to the aluminum sheet is of advantage with respect to sound attenuation. Consider that the total energy losses are only due to material damping since the plate is not coupled to other plates (coupling loss factor = 0) and the radiation damping is often neglected.

Solution

Let the aluminum and rubber plates be layer 1 and 2, respectively. The properties of the aluminum (subscript 1) and rubber (subscript 2) are selected from Table A (Appendix) as follows:

ρ_1= 2800 kg/m^3, ρ_2 = 950 kg/m^3

E_1 = 65.2 GPa, E_2 = 1.9 GPa

μ_1= 0.33, μ_2= 0.40, η_1= 0.001, η_2 = 0.080

The panel $E = E/(1-\mu^2)$, so $E_1 \approx$ 73.1 GPa and $E_2 \approx$ 2.30 GPa

The surface density for the composite panel is found from Eq. (4.119)

M_s= 2800(0.0018) + 950(0.0046) = 9.41 kg/m^2

The location of the neutral axis for the composite panel is found from Eq. (4.118)

$$\chi = \frac{73.1(0.0018)^2 - 2.30(0.0046)^2}{2\{73.1(0.0018) + 2.30(0.0046)\}} = 0.00066 \text{ m}$$

The bending stiffness, B, for the composite panel is found from Eq. (4.121):

$$1 + 3(1 - 2\chi/h_1)^2 = 1 + 3\{1 - (2(0.00066)/0.0018)\}^2 = 1.21$$

$$1 + 3(1 + 2\chi/h_2)^2 = 1 + 3\{1 + (2(0.00066)/0.0046)\}^2 = 5.98$$

$$B = \frac{73.1(10^9)(0.0018)^3(1.21)}{12(1-0.33^2)} + \frac{2.3(10^9)(0.0046)^3(5.98)}{12(1-0.4^2)} = 181.05 \text{ N.m}$$

The critical or wave-coincidence frequency for the composite panel is found from Eq. (4.66):

$$f_c = \frac{(343.8)^2}{2\pi}\left(\frac{9.41}{181.05}\right)^{1/2} = 4274 \text{ Hz}$$

If the panel were constructed of aluminium only (layer 1), the critical frequency would be found from Eq. (4.120):

$$f_{c1} = \frac{(343.8)^2}{2\pi}\sqrt{\frac{12(2800)}{73.1(10^9)(0.0018)^2}} = 7061 \text{ Hz}$$

(a) For a frequency of 500 Hz. This frequency is less than the critical frequency, so the panel behaviour falls in Region II, the mass-controlled region. The transmission loss may be found from Eq. (4.30) for the composite panel:

$$R = 20\log 9.41 + 20\log 500 - 47.3 = 26.2 \text{ dB}$$

For a single aluminium ply, the transmission loss reads

$$R = 20\log 5.04 + 20\log 500 - 47.3 = 20.7 \text{ dB}$$

The addition of the mass of the rubber sheet increases the transmission loss by about 5.5 dB.

(b) For a frequency of 8 kHz. This frequency is greater than the critical frequency and so the panel behaviour falls in Region III, the damping-controlled region. The composite panel damping factor is found from Eq. (4.122):

$$\eta = \frac{\{0.001(73.1)(0.0018) + 0.08(2.3)(0.0046)\}(0.0018 + 0.0046)^2}{73.1(0.0018)^3(1.21) + 2.3(0.0046)^3(5.98)} = 0.02$$

Using Eq. (4.36), the transmission loss at the critical frequency for normal incidence reads:

$$R = 20\log 9.41 - 13\log 4274 + 33\log 8000 + 10\log 0.02 - 48 = 36.1 \text{ dB}$$

For a single layer of aluminium, the transmission loss for the aluminium alone at 8000 Hz is as follows:

$$R = 20\log 5.04 - 13\log 7061 + 33\log 8000 + 10\log 0.001 - 48 = 14.8 \text{ dB}$$

The addition of the rubber layer increases the transmission loss at 8 kHz by about 21 dB.

4.7.2 Orthotropic plates

Sound transmission in orthotropic panels differs from that of isotropic panels because in orthotropic plates, the bending stiffness varies in directions. The difference in bending stiffness for plane plate may result from the anisotropy of the plate material such as wood caused by grain orientation or from construction of the plate such as corrugation, ribs, cuts and so on. Consequently, the speed of free bending waves is different for these two directions and the orthotropic panels have two coincidence frequency. Panels may have ribs attached to increase the stiffness of the panel and to reduce imposed static stress levels for a given applied load. Note that the increase in bending stiffness caused by corrugations, ribs, and stiffeners often decreases the transmission loss. If, however, the bending stiffness of the plate decrease, such if one makes partial-depth saw cuts, the transmission loss would increase.

Rib-stiffened panel

Let's consider a typical case of orthotropic plates, the rib-stiffened panel shown in Fig. 4.24, which has a stiffness that is different in the direction parallel to the ribs (the stiffest direction) than in the direction perpendicular to the ribs. This difference in stiffness has an influence on the transmission loss for the panel (Maidanik, 1962).

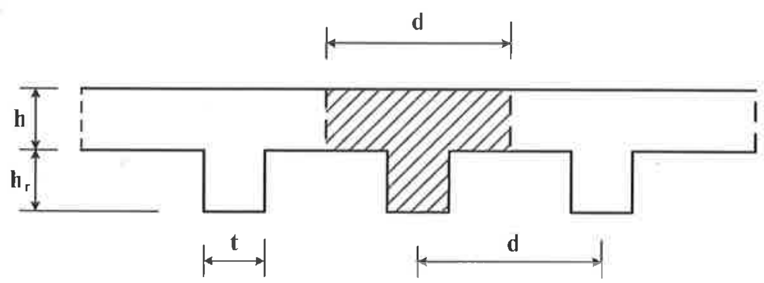


Fig. 4.24 Dimensions for a rib-stiffened panel. d is c/c distance (spacing) between ribs, and t is the thickness of the rib.

The sound reduction index for Region II, the mass-controlled region ($f < f_{c1}$), may be determined from Eq. (4.28) or its approximation Eq. (4.30), where the surface density, M_s, is expressed as

$$M_s = \rho h\{1 + (h_r/h)(t/d)\} \tag{4.123}$$

where ρ is the density of the panel. Usually, there are two different wave coincidences or critical frequencies for an orthotropic plate, such as a rib-stiffened panel, corresponding to the different stiffness of the panel. The two critical frequencies are given by expressions similar to Eq. (4.120):

$$f_{c1} = \frac{c^2}{2\pi}\left(\frac{M_s}{B_1}\right)^{1/2}, \quad f_{c2} = \frac{c^2}{2\pi}\left(\frac{M_s}{B_2}\right)^{1/2} \tag{4.124a, b}$$

where B_1 and B_2 are the bending stiffness for the stiffest direction and the direction perpendicular to this, respectively, which are given by the following expressions (Ugural, 1999):

$$B_1 = EI/d, \quad B_2 = \frac{Eh^3}{12\{(1-(t/d)) + (t/d)/(1+(h_r/h))^3\}} \tag{4.125a, b}$$

where I is the moment of inertia about the neutral axis of the T-section shown shaded in Fig. 4.24, d is the centre -to- centre (c/c) spacing of the ribs, and E is the Young's modulus of the plate: typical E divided by $(1-\mu^2)$, where μ is the Poisson's ratio; as in acoustic applications, E is the dynamic modulus of elasticity; see Example 4.10.

For the intermediate frequency range, $f_{c1} < f < f_{c2}$, the transmission loss may be calculated from the following expression (Beranek and Ve´r, 1992):

$$R = R_n(f_{c1}) + 10\log(\eta) + 30\log(f/f_{c1}) - 40\log\{\ln(4f/f_{c1})\} + 10\log\{2\pi^3(f_{c2}/f_{c1})^{1/2}\} \tag{4.126}$$

The quantity R_n (f_{c1}) is the transmission loss for normal incidence at the critical first frequency (lower critical frequency), f_{c1}, expressed by Eq. (4.35).

For the high-frequency range, $f > f_{c2}$, the transmission loss may be found from the following expression:

$$R = R_n(f_{c2}) + 10\log(\eta) + 30\log(f/f_{c2}) - 2 \tag{4.127}$$

The lowest *eigen* frequency for a homogenous single plate is expressed in Eq. (4.22). However, for an orthotropic plate this frequency would read as

$$f_{11} = \frac{c^2}{4}\left(\frac{1}{f_{c1}a} + \frac{1}{f_{c1}b^2}\right) \tag{4.128}$$

where a and b denotes plate dimension such that f_{c1} is calculated with respect to a (the stiffest direction) and f_{c2} to b (direction perpendicular to it).

The radiation factor or efficiency in the frequency range $f_{c1} < f < f_{c2}$ can be expressed as (Heckl, 1960):

$$\sigma \cong \frac{1}{\pi^2}\sqrt{\frac{f_{c1}}{f_{c2}}}\left(\ln\frac{4f}{f_{c1}}\right)^2 \tag{4.129}$$

Other orthotropic plates

The above expressions may well be generalised to other typical orthotropic plates, except for the expressions of M_s and B, which can be different according to the type of plate. For instance, the bending stiffness (Nm) of corrugated plates reads (Cremer et al., 1988):

$$B_1 = B_2(s/s'), \quad B_2 = Eh^3/12 \tag{4.130a, b}$$

where s is distance between corrugations along surface, s' is the distance along a straight line, h is the thickness of the corrugated plate, E is the Young' modulus of elasticity of plate: E divided by $(1-\mu^2)$. The surface density of thin corrugated plate reads: ρh. In some literature x and y are interchanged for 1 and 2, respectively.

Example 4.10: An oak wood sheet, 1.31 m (51.2 in) by 2.33 m (91.7 in) with a thickness of 12.7 mm (0.50 in), has oak wood ribs attached. The dimensions of the ribs are 23.1 mm (0.91 in) high and 20.3 mm (0.8 in) thick. The ribs are spaced 90.6 mm (3.6 in) apart on centers and are oriented parallel to the long dimension of the sheet. Air at 25°C (77°F) and 101.3 kPa (14.7 psia) is on both sides of the panel for which c = 346.3 m/s and ρ_0 = 1.184 kg/m³. Determine the sound reduction index of the panel for a frequency of 1000 Hz. Compare the results with the same panel but without ribs. Te damping coefficient, η, is mainly due to internal material losses.

Solution

Consider the T-section to consist of two rectangular parts 1 and 2 as the illustration. Let C(x_C, y_C) be the centroid of the T section. Let A_i be the area and x_i, y_i the coordinates of the centroid of the i^{th} part. The y coordinate of the centroid is obtained as

$y_C = \sum A_i y_i / \sum A_i$

= [(23.1×20.3) × 11.55 + (90.6×12.7) × 29.45] / [(23.1× 20.3) + (90.6×12.7)]

= 24.267 mm

The moment of inertia I^C_{xx} of the T-section about an axis parallel to the x axis through C would be

$$I^C_{xx} = \sum[(b_i h_i^3)/12 + (b_i h_i)(y_C - y_i)^2]$$
$$= 20.3(23.1)^3/12 + 20.3(23.1)(24.267-11.55)^2 + 90.6(12.7)^3/12 +$$
$$+ 90.6(12.7)(24.267-29.45)^2$$
$$= 1.431\times10^5 \text{ mm}^4$$

The bending stiffness of the stiffened panel in the direction parallel to the ribs, B_1, is found from Eq. (4.125a) and the properties of oak wood material are found in Table A (Appendix).

$$B_1 = 11.0\times10^9(1.341\times10^5\times10^{-12})/0.0906(1-0.15^2) = 1.666\times10^4 \text{ Nm.}$$

The bending stiffness in the direction perpendicular to the ribs, B_2, is found from Eq. (4.125b):

$$B_2 = \frac{11.0\times10^9(0.0127)^3}{12\{(1-(0.0203/0.0906)) + (0.0203/0.0906)/(1+(0.0231/0.0127))^3\}(1-0.15^2)}$$
$$= 2436 \text{ Nm}$$

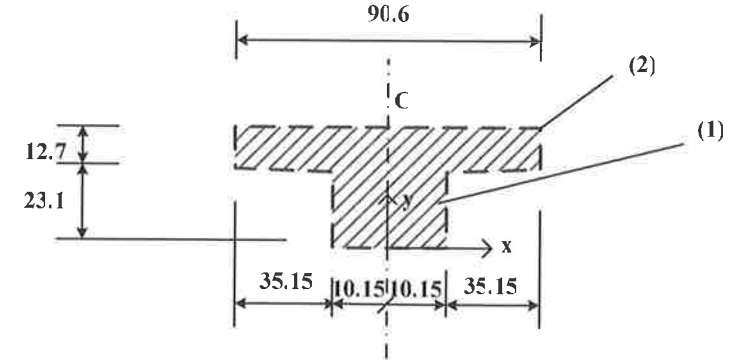


Illustration of Example 4.10 Calculation of moment of inertia of T-section.

The surface mass or mass per unit surface area for the rib-stiffened panel may be found using Eq. (4.123):

$$M_s = 770(0.0127)\{1+(0.0231/0.0127)(0.0203/0.0906)\} = 13.764 \text{ kg/m}^2$$

The two critical frequencies may be determined from Eqs. (4.124a, b)

$$f_{c1} = \frac{(346.3)^2}{2\pi}\left(\frac{13.764}{16{,}660}\right)^{1/2} = 549\,\text{Hz};\; f_{c2} = \frac{(346.3)^2}{2\pi}\left(\frac{13.764}{2436}\right)^{1/2} = 1435\,\text{Hz}$$

For the rib-stiffened panel, the frequency of 1000 Hz falls in the intermediate region. Subsequently, the transmission loss may be found from Eq. (4.126). The transmission loss for normal incidence at the lower critical frequency is calculated from Eq. (4.35):

$$R_n(f_{c1}) = 10\log\left(1+\left(\frac{\pi(13.764)(549)}{410}\right)^2\right) = 35.3 \text{ dB}$$

The transmission loss for the rib-stiffened panel at 1000 Hz will now read:

$$R = 35.3 + 10\log(0.008) + 30\log(1000/549) - 40\log\{\ln(4(1000)/549)\} +$$
$$+10\log\{2\pi^3(1435/549)^{1/2}\} = 30.2\,\text{dB}$$

For the panel without the ribs, the surface mass is as follows:

$$M_s = \rho h = 770(0.0127) = 9.78\,\text{kg/m}^2$$

The critical frequency for the panel without stiffening ribs is determined from the general expression, Eq. (2.331).

$$f_c = \frac{346.3^2}{2\pi}\left(\frac{12(9.78)(1-0.15^2)}{11(10^9)(0.0127)^3}\right)^{1/2} = 1363 \text{ Hz}$$

For the oak wood panel without stiffening ribs, the frequency of 1000 Hz falls, theoretically, in the transmission loss Region II, the mass-controlled region, Eq. (4.30):

$$R = 20\log 9.78 + 20\log 1000 - 47.3 = 32.5\,\text{dB}$$

Note that this result is very approximate since the field-incidence mass law is valid up to $0.5f_c$. As can be seen, the stiffening of the plate contributes to deteriorating the transmission loss.

4.7.3 Sandwich plates

Structural members made of two stiff, strong skins separated by a lightweight core are known as sandwich panels, as shown in Fig. 4.25. The separation of the skins, which actually carry the load, by a low density core, increases the stiffness of the panel with little increase in weight producing an efficient structure. In buildings constructions as well as aircraft/vehicles structures, a honeycomb sandwich panels in which the core resembles the honeycomb are often used. Often, honeycomb panels consist of a resin-impregnated paper, plastic or metal foil core sandwiched between very thin metal or fibre-reinforced plastic faceplates (skins); refer also to Table 4.10. When the core material in a sandwich construction has a certain degree of stiffness in bending and extension, energy is dissipated in the core by the flexural vibrations of the plate. Sandwich structures without the second thin panel are also frequent. They exist typically in pairs so that both absorbing faces point inside the cavity.

The sandwich panel is probably the most difficult wall type to model. The core material transmits shear forces, like in the case of thick walls. While the core keeps the skins at equal distance apart from each other thereby increasing the stiffness, it also bears most of the shear loading. For the pure shear waves (in the absence of bending forces), the propagation velocity can be written as (see Eq. (2.134))

$$c_s = \left(\frac{Gh}{M_s}\right)^{1/2}, \quad G = \frac{E_c}{2(1+\mu)} \qquad (4.131a, b)$$

where G is the shear modulus, h the total plate thickness, M_s the total surface density (mass per unit area), and E_c the elasticity modulus of the core material. In the shear wave, there is no elongation or shortening of the skins and shearing in the core material is perpendicular to the propagation direction.

The total surface density of a honeycomb plate, M_s, is calculate as

$$M_s = M_{sc} + 2M_{sf} = \rho_c h_c + 2\rho_f h_f \qquad (4.132)$$

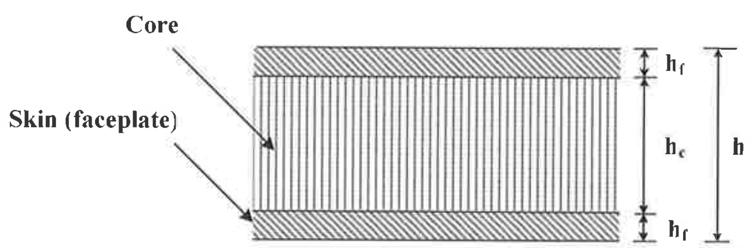


Fig. 4.25 Dimensions of sandwich plate (or honeycomb panel). In sandwich plate, bending and shear waves are generated in the elastic core, along with bending waves in the skin and dilatation vibrations in the elastic core.

Indices c and f refer to the core material and the faceplates or skins, respectively; it here assumed two identical skins, otherwise the surface density of each skin will be calculated separately.

Combined bending stiffness of a honeycomb plate is determined as (Ver and Homer, 1971):

$$B = EI = \frac{1}{2}E_f h_f (h_f + h_c)^2 \qquad (4.133)$$

It is assumed here that the core material is incompressible to keep the skins at equal distance.

Bending waves appear in the core material and the propagation velocity of the overall cross-section of bending wave in the absence of shear distortion, c_B, is expressed by the general expression, Eq. (2.153) with the stiffness and density expressed above. The result may be simplified assuming that h_f << h_c, which results in the following expression:

$$c_B \approx \left(\frac{\omega^2 E_f h_c^2 h_f}{2M_s(1-\mu^2)}\right)^{1/4} \qquad (4.134)$$

Moreover, bending waves appear in each of the skins and for the propagation velocity, c_{Bf}, of these waves, only half of the core material mass, M_{sc}, is added to the mass of skins, M_{sf}, as inspected by Eq. (4.133). Accordingly, the bending wave velocity for the faceplate becomes:

$$c_{Bf} = \left(\frac{\omega^2 E_f h_f^3}{6M_s(1-\mu^2)}\right)^{1/4} \qquad (4.135)$$

The relationship between pure bending (in the absence of shear distortion), pure shear (in the absence of transverse bending force), and transverse waves in a honeycomb panel may be written as a 6-grade equation (Kurtze and Watters, 1959):

$$\left(\frac{c_s}{c_B}\right)^4 c_{B,eff}^6 + c_s^2 c_{B,eff}^4 - c_s^4 c_{B,eff}^2 - c_s^2 c_{Bf}^4 = 0 \qquad (4.136)$$

where $c_{B,eff}$ is the effective bending wave velocity. For most practical constructions, $c_B >> c_{Bf}$, and so Eq. (4.136) may be solved as

$$c_{B,eff} \approx \left(\frac{1}{c_B^3} + \frac{1}{c_{Bf}^3 + c_s^3}\right)^{-1/3} \qquad (4.137)$$

The coincidence ($f = f_c$) occurs when $c_{B,eff} = c$ (air), which plays an important role for sound transmission, and $c_{B,eff}$ accounts for the panel flexure that influences the transmission loss in the sandwich panel. Inspection of the latter expression reveals that the value of $c_{B,eff}$ approaches the value of c_{Bf} near coincidence. Further, f_c lies between the lowest critical frequency that corresponds to pure bending and the highest critical frequency for the faceplates alone.

The important parameter for sound transmission is the ratio between bending wave velocity and sound speed in air, as expressed by Eq. (2.276). Consequently, the coincidence frequency of the sandwich plate is rewritten as

$$f_c = \frac{c^2 f}{c^2_{B,eff}} \tag{4.138}$$

This result affect the radiation factor, σ, (if plate is considered finite), which doesn't decrease quickly towards low frequencies as the case for homogenous plates. Subsequently, the resonant transmission plays more important role for sound transmission than for the case of homogenous plates. Table 4.10 offers material properties for a number of core materials for sandwich constructions.
For $f > f_c$, the transmission loss may be approximated as

$$R \approx R_n + 10\log\eta + 10\log\{(f/f_c) - 1\} - \Delta R_s - 2 \tag{4.139}$$

where R_n is transmission loss for normal incidence, Eq. (4.27), f_c is expressed by Eq. (4.138), and ΔR_h for sandwich plate is between 0 and 3 dB: ΔR_s = 0 dB for pure bending waves and ΔR_s = 3 dB for pure shear waves.

The effect of dilatation resonance

Honeycomb panels contain cores that are very stiff in compression normal to the plane of the panel. As such cores are compressible materials (e.g., plastic foams, Table 4.10) dilatational resonance occurs between the masses of skins due to the compressional stiffness of the core; the masses oscillate with the core material in between as a spring. This phenomenon is analogous to the resonant frequency, f_0, of double walls (mass-air-mass), but tends to occur practically at higher frequencies (Fahy, 1985). This leads to deterioration of sound insulation of sandwich plates. This is clearly shown in Fig. 4.26. Note that the mass low represents an upper limit for the practical obtained sound insulation of sandwich plates and that small changes in the core material parameters can imply drastic influence on the sound insulation.

Dilation resonance occurs at the frequency where the combined stiffness impedance of the faceplate and that of the enclosed air equals the mass impedance of the plate. Dilatation resonance frequency can be calculated as (Ver and Holmer, 1971):

$$f_{dil} = \frac{1}{2\pi}\left(\frac{4E_{ct}}{h_c(2M_{sf} + M_{sc}/3)}\right)^{1/2}, \quad E_{ct} = \frac{E_c}{3(1-2\mu)} \tag{4.140a, b}$$

where E_c is core material elasticity modulus, and μ the Poison's ratio. It is obvious that μ has big influence on f_{dil}; a rubber has $\mu \approx 0.5$, which implies that

f_{dil} becomes very high. Obviously, the dilatational resonant frequency depends only on the type of core material and mass density.

Table 4.10 Material properties for a number of core materials of sandwich constructions. *E* and *G* only indicate stress and shear normal to the panel plane (after Kristensen and Rindel, 1989).

Material	ρ (kg/m^3)	E ($\times 10^6$ N/m^2)	G ($\times 10^6$ N/m^2)
Polyurethane foam	40	14	3
Polyurethane foam	100	30	18
Polystyrene foam	20	10	6
Polystyrene foam	160		20
fenol foam	60	50	10
Silicone rubber			0.02
Natural rubber			1
Urethane rubber			10
Honeycomb with framework of			
Paper	20		40
Plastic	30	100	20
Plastic	120	700	140
Aluminium	20		100
Aluminium	120	2000	500
Plates of mineral wool (fibre ≠ layer plane)			
Rock wool	200	0.4-0.7	
Glass wool	120	0.3-0.5	9
Lamina plates of mineral wool (fibre ⊥ layer plane)			
Glass wool	120	< 6.5	1.15
Glass wool	80	< 1.9	0.46
Glass wool	45	< 1.2	0.17

Design considerations

The sandwich construction must be designed such that the shear wave (S-wave) speed remains below the speed of sound in air ($c_s << c$) as this results in high critical frequency and low contribution from the resonant transmission at higher frequencies (no trace coincidence occurs). This implies that the core material must have low G, as inspected from Eq. (4.131a). In this case, honeycomb panel will exhibit almost mass-controlled transmission behaviour over the whole frequency range, as shown in Fig. 4.27. However, in practice, the stiffness, G, is often large and one would obtain instead $c_s \approx c$ or possibly $c_s >> c$, which result in a critical frequency that usually falls in the middle of the audio frequency range, which is not favoured as discussed before. This is why ordinary sandwich panels are very poor sound barriers because of their low mass and high bending stiffness. Fig. 4.27 shows measured results for a sandwich plate. The best results are obtained when $c_s < c$. When a stiffer core plate is used, a low value of f_c is obtained thereby yielding lower sound insulation in which the resonant transmission becomes dominate.

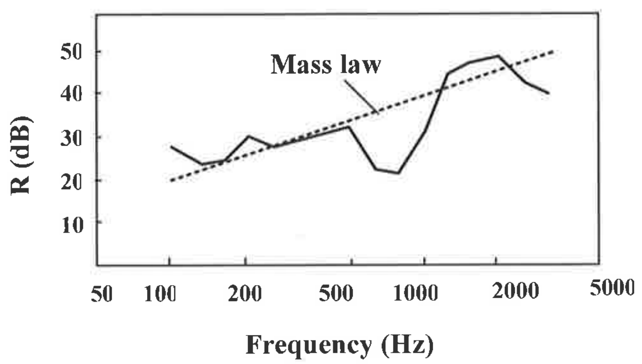


Fig. 4.26 Measured transmission loss of a sandwich plate consists of 13 mm gypsum boards with a core of 55 m polyurethane foam, compared with the mass low. The dilatation resonant frequency is about 730 Hz (after Homb et al., 1983).

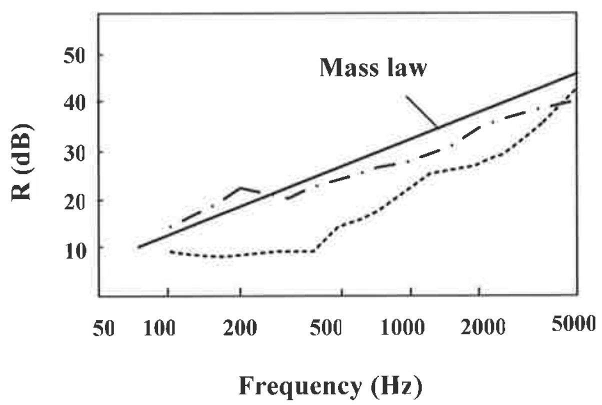


Fig. 4.27 Measured sound reduction index of honey comb sandwich panels. — • — •, Low core shear stiffness: $c_s < c$, and f_c high; •••••, High core shear stiffness, $c_s > c$, and f_c low (after Fahy, 1985).

A sandwich panel contains three adhesively connected material layers; two thin panels with one poroelastic layer between them. Adhesive bonding makes the sandwich panel very stiff compared with a thin panel with the same mass. This is also the reason for their popularity. In order for the sandwich to function correctly the adhesive layers between the skins and the core must be able to transfer the loads, and thereby be as least as strong as the core material. Without a proper bond, the three entities work as separate beams/plates and the stiffness is lost. This is why proper core/skin bonding is so critical. According to Moore and Lyon (1991), asymmetric and symmetric modes of the panel can be distinguished, both of which can produce separate coincidence effects resulting in a strong reduction of R. During asymmetric motion, the thickness of the core material changes during compression and expansion, while during symmetric motion, the panels are in phase and the thickness of the core is constant. In the case of asymmetric modes, the core can act as a spring.

The dilatational resonance is a result of core compressibility, which implies that the compressibility of core material is not desired for the acoustic design of sandwich plates. It can be located even at middle frequencies when thick rigid cores are used. Dilation resonance, leads to further deterioration of the sound transmission loss of sandwich panels. Therefore, the dilatational resonant frequency may also be checked and one should try to lower it or move it to upper frequency range, preferably outside the interesting frequency range of building acoustics.

Sound transmission loss for sandwich panels can be substantially higher than for homogeneous panels of the same mass per unit area, provided that such plates favor the propagation of the free shear waves (with frequency-independent propagation speed) rather than free bending waves for which the propagation speed increases with increasing frequency. Moreover, the increase in stiffness results in the coincidence effect occurring at markedly lower frequencies than that of a thin panel with the same mass (Jones, 1981). Consequently, the best design is a compromise between structural and acoustic considerations since if the shear stiffness is made too low to increase sound insulation, the static stiffness and structural stability of the honeycomb construction may be unsatisfactory.

4.8 Sound Transmission through a Finite-Size Panel

For most building acoustic applications, where the first resonance frequency of typical plate-like partitions is well below the frequency range of interest and the lateral plate dimensions is much greater than the bending wavelength of the forced vibration, the expressions that are strictly valid only for infinitely large panels can be used to predict the sound transmission loss of finite panels. However, in many industrial applications, the finite size of the panel must be taken into account. Consider an infinite plate, excited over an area. When the airborne sound field in the room excites the bending wave field in the plate inside the room, this excited field will be strongly correlated to the forcing airborne field. This bending wave field is called forced. Due to excitation, a bending wave field will also be generated in the other side of the plate (the receiver room). This field consists of waves propagating at the phase speed of free bending wave. At high frequencies, theses waves will emerge from the boundaries of the excited area towards the unexcited part of the plate, as discussed in Chapter 2. In reality, no plates are infinite, and when the sound-forced bending waves in finite panels encounter the edges of the plate, they are reflected and propagate in the plate as free bending waves. After repeated reflections, mode shapes or *eigen* vibrations with *eigen* frequencies are built up. If the plate is not too large and damping is no too high, the reflections will give rise to a field where the amplitude does not vary very much over the surface, a resonant field. The sum of the incident forced bending wave and the generated free bending wave satisfies the particular plate edge condition (e.g., zero displacement and angular displacement at a clamped edge). Consequently, the sound-forced bending waves continuously feed free bending wave energy into the finite panel and build up a reverberant, free-bending wave field, which radiates sound into air, called resonant radiation. The mean-square vibration velocity of this free bending wave field, $<v_r^2>$ can be obtained using a power balance for the finite plate. The power introduced into the finite plate at the edges equals the power lost by the plate owing to the total damping losses in the plate, which are composed of viscous damping losses in the plate material (internal losses), energy flow into connected structures (boundary losses), and sound radiation into air (radiation losses); see Sec. 4.10. To put these facts in a mathematical form, the total transmitted (or radiated) sound power, W (watts), is the sum of sound power of forced radiation, W_f, and sound power of resonant radiation, W_r, radiated by the finite panel:

$$W = W_f + W_r = \rho_0 cS(<\tilde{v}_f^2>\sigma_d + <\tilde{v}_r^2>\sigma) \tag{4.141}$$

where $<\tilde{v}_f^2>$ is the mean-square vibration velocity of the sound-forced bending waves (average in space and time), σ_d is the radiation efficiency of the forced waves, S is the surface area of the panel, and σ is the radiation efficiency of the free bending waves (resonant radiation); the expressions of σ is found in Sec. 2.15.3.

The radiation factor (or efficiency) of the forced bending wave, σ_d, is generally a function of the incident angle, φ (oblique incidence) as well as plate

area and can be smaller or larger than unity; see also Sewell (1970) and Timmel (1991). For typical applications in building acoustics, a random incidence (diffuse sound field) is assumed to occur in the room. In this case, the radiation factor, σ_d, is of the forced bending wave may be expressed as

$$\sigma_d = \int_0^{\pi/2} \sigma_L(\varphi)d\varphi \tag{4.142}$$

where $\sigma_L(\phi)$ is the radiation factor with respect to incident angle φ, which may be taken as (Ljunggren, 1991):

$$\sigma_L(\phi) = \frac{k}{0.5L\pi} \int_{-k}^{+k} \frac{\sin^2\{0.5L(k_x - k_0)\}}{(k_x - k_0)^2 \sqrt{k^2 - k_x^2}} dk_x \tag{4.143}$$

where k is the wave number in air, $k_0 = k \sin\varphi$, is the trace wavenumber of the exciting pressure along the plate, L is the excited length of the plate, which may be taken as $L = (2S/\pi)^{1/2}$ as the mean projected trace length (mean value of $\sqrt{L^2}$); Kosten (1960) suggested that L is the mean free path and it reads $L = \pi S / U$, where S is the area of the excited part of the plate, and U is the perimeter of the radiating area S, $U = 2(l_x + l_y)$, where l_x and l_y are the dimensions of the excited area of the plate, S. The two expressions of L are equal for square plate and the result of σ_d using expressions are approximately equal.
For most applications in building acoustics, the radiation efficiency for the random incidence (diffuse) of sound-forced excitation, σ_d, may be approximated as

$$\sigma_d \approx 0.5\{0.2 + \ln(k\sqrt{S})\} \tag{4.144}$$

where k is the wave number in air ($k = 2\pi f / c$). Eq. (4.144) is theoretically valid for $k\sqrt{S} > 1$, but it should be for $k\sqrt{S} > 1.8$, and for $1 < k\sqrt{S} < 1.8$, the result from Eq.(4.144) should be multiplied by factor 2, if the results are to be approximately comparable with Eq. (4.142) and Eq. (2.143). In addition, Eq. (4.144) concerns a finite square plate with free moving edge (Sato, 1973). However, it can be used as an approximation for rectangular plates too. It is worthwhile to indicate that the corresponding radiation efficiency for oblique-incidence of sound-forced waves in infinite plate is $\sigma_L(\phi) = 1/\cos\phi$; this expression is also for finite plates but it represent one of two expressions from which the minimum value is taken.

As indicated above, when a panel is mechanically excited, most of the energy is produced by resonant panel modes irrespective of excitation frequency. If a panel is acoustically excited by incidence, its vibrational response comprises both a forced vibrational response at the excitation frequency and a resonant response at all relevant natural frequencies which are excited by the interaction of the forced bending waves with the panel boundaries, as mentioned earlier. Fig. 4.28 shows the typical behaviour of transmission loss of finite plate. The transmission loss through the plate in the mass region and forward is characterized by being composed of forced and resonant (reverberant free bending wave) transmission. The forced transmission is the transmission that caused by the sound-forced waves that are generated in a plate due to the incidence of airborne sound waves. The airborne sound waves will excite the *eigen* vibrations (free bending waves) that characterize each plates, which in turn radiates sound into air in the receiver room. Since below the critical frequency f_c, the radiation is very low and can be neglected, the resonant transmission is of lower importance. Above f_c ($\sigma \geq 1$), the sound radiation is efficient and forced transmission is of lower importance. Note that since the radiation efficiency is low ($\sigma << 1$) below the critical frequency of the panel ($f << f_c$), the vibration response of the panel is controlled by the free bending waves (i.e., $<v_r^2> >> <v_f^2>$) but the resulted sound radiation is controlled by the less radiating forced waves, as shown in Fig. 4.28. However, in the vicinity of the coincidence region both parts of transmission should be taken into consideration. Note that it is not the incident sound power but the mean-square incident sound pressure on the source side that is forcing the panel to vibrate. Consequently, the sound transmission loss of a finite partition may be defined as (Rindel, 1975; Sato, 1973):

$$R = 10\log\left(\frac{E_1}{E_2}\frac{S}{A}\right) = 10\log\left(\frac{<\tilde{p}_1^2> S}{4\rho_0 c W_t}\right) \quad (4.145)$$

where E_1 is the energy in the source room, Eq. (2.274b): $E_1 = <\tilde{p}_1^2> V/\rho_0 c^2$, $<\tilde{p}_1^2>$ is the average square sound pressure in time and space over the whole source room, E_2 is the energy in the receiver room, found by combining Eq. (2.279), Eq. (2.274a), and Eq. (2.255): $E_2 = 4W_t V/cA$, S is the area of the panel (one side), A is the total absorption in the receiver room, and W_t is the radiated sound power from the receiver side of the panel (transmitted sound power) owing to the velocity of the plate, Eq. (2.255).

4.8.1 Forced transmission

The wall impedance, Z_w, is the difference in sound pressure across the plate divided by the velocity of the plate. Consequently, from the wall impedance the vibration velocity, v_f, of the forced-sound waves in a plate may be obtained by knowing the sound pressure, p_1, in the source room (in front of the partition) as (Cremer et al., 1988):

$$<\tilde{v}_f^2> \approx \frac{2<\tilde{p}_1^2>}{|Z_w|^2} \quad (4.146)$$

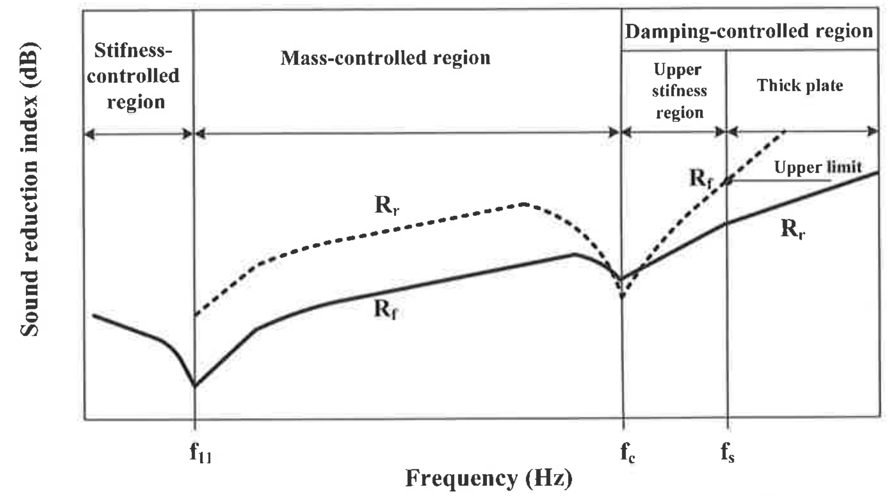


Fig. 4.28 Typical transmission loss for a massive homogenous finite plate is divided into forced and resonant parts, which is to be added logarithmically. The terms R_f and R_r denote the forced and resonant transmission losses, respectively. The dotted line refers to the secondary importance for the total sound reduction index of the plate. The frequency f_s is the limit between thin and thick plate.

Substituting Eq. (4.146) into Eq. (4.145) and knowing from Eq. (4.141) that the radiated or transmitted power for forced transmission is: $W_{2t} = \rho c S <\tilde{v}_f^2> \sigma_d$ (index 2 refers to the receiver side of plate) will lead to general expression for forced transmission loss expression, R_f:

$$R_f = 10\log\left(\frac{<\tilde{p}_1^2> S}{4\rho_0 c W_{2t}}\right) = 20\log\left|\frac{Z_w}{2\rho_0 c}\right| - 10\log(2\sigma_d) \quad (4.147)$$

The finite wall impedance may be expressed as (Kristensen and Rindel, 1989):

$$Z_w = j\omega M_s\left[1-\left(\frac{f_{11}}{f}\right)^2\right]\left[1-\left(\frac{f}{f_c}\right)^2\right] + \eta\omega M_s \quad (f_{11} << f_c) \quad (4.148)$$

where f_{11} is the lowest *eigen* frequency, Eq. (4.22), and η is the total loss factor.

One may simplify the expressions of transmission loss with respect to each region by approximating the physical situation of typical building constructions

(η << 1). By knowing that the forced response at $f_{11} < f < f_c$ is controlled mainly by the mass, the wall impedance may be approximated as

$$Z_w \approx j\omega M_s \tag{4.149}$$

Substituting Eq. (4.149) into Eq. (4.147) would result in

$$R_f \approx R_0 - 10\log(2\sigma_d) \; (f_{11} < f < f_c) \tag{4.150}$$

The quantity R_0 is the approximated sound reduction index for normal-incidence sound field or mass law ($R_0 \approx R_n$): $R_0 = 20 \log (\pi f M_s / \rho_0 c)$. In practice, the most important part of sound reduction index for the forced part is Eq. (4.150). Eq. (4.150) implies that the sound transmission loss of finite panels can be larger than the normal-incidence mass law if the size of the panel is small and at low frequency.

4.8.2 Resonant transmission

As the plate is considered finite, the free bending waves radiated from the plate boundaries will cause a reverberant field in the sending room. This case has been analysed in Sec. 2.13.5 and Eq. (2.286) expresses the resonant transmission loss. Eq. (2.286) can also be obtained by knowing that the relation between the velocity of the reverberant field (*eigen* vibrations) is expressed by Eq. (2.278), which may be rewritten as

$$<\tilde{v}_r^2> \approx \frac{<\tilde{p}_1^2>}{(\omega M_s)^2}\frac{\pi\sigma f_c}{2\eta f} \tag{4.151}$$

The resonant radiated sound power is given in Eq. (4.141) as: $W_{2r} = <\tilde{v}_r^2> \rho c S \sigma$. Consequently, substituting Eq. (4.151) into Eq. (4.145) leads to the resonant sound transmission loss:

$$R_r = 10\log\left(\frac{<\tilde{p}_1^2> S}{4\rho c W_{2r}}\right) = R_0 - 10\log\left(\frac{\pi\sigma^2 f_c}{2\eta f}\right) \tag{4.152}$$

where σ is the radiation factor for the *eigen* vibrations (or free bending waves) (Sec. 2.15.3). Eq. (4.152) is approximately equal Eq. (4.36) for infinite plates provided that σ=1 in Eq. (4.152).

As discussed in Sec. 4.2.4, in many cases in practices, the walls that separate the two apartments are load-bearing (e.g., concrete) while the internal walls in the apartments are made of lightweight materials (plasterboard walls) that are not load-bearing. This implies the excited part of the plate, S, becomes less than the area of the whole plate S_{tot}. In this case, the loss of vibration energy in the whole plate, S_{tot}, which extends vertically or horizontally and is attached to lightweight partitions on both sides, should be taken into account. Accordingly,

$$R_r = 10\log\left(\frac{<\tilde{p}_1^2> S}{4\rho c W_{2r}}\right) = R_0 - 10\log\left(\frac{\pi\sigma^2 f_c}{2\eta f}\right) - 10\log\left(\frac{S}{S_{tot}}\right) \tag{4.153}$$

When $S = S_{tot}$, Eq. (4.153) reduces to Eq. (4.152). Eq. (4.153) indicates that the resonant transmission depends directly on the loss factor η, which determines how strong the *eigen* vibrations (modes) are excited.

For many building constructions, especially thin plates, the resonant transmission is often meaningless for $f < f_c$ but it dominates for $f > f_c$ in which $\sigma \geq 1$.

Consideration of thick plate

For thin plate, the bending wave velocity is related to f_c by:

$$c_B = c\sqrt{f / f_c} \tag{4.154}$$

At coincidence, $f = f_c$, and $c_B = c$. Eq. (4.154) is correct within 10% when $h > \lambda_B/6$ (the condition of thin plates) where λ_B is the wavelength of the bending waves and h is the panel thickness.

The frequency f_s at which the thin plate becomes thick (Fig. 4.28) may be derived by setting $\lambda_B = 6h$ (the limit between thin and thick plate) and $f = f_s$ in Eq. (4.154), the result becomes:

$$f_s = \frac{1}{f_c}\left(\frac{c}{6h}\right)^2 \tag{4.155}$$

where c is sound speed in air. Considers different construction materials of thickness 200 mm: for concrete, $f_s \approx 10 f_c$, for steel $f_s \approx 22 f_c$, for brick $f_s \approx 7 f_c$ and lightweight concrete, $f_s \approx 2.5 f_c$. This means that the sound transmission at higher frequencies through thick plates made of brick and lightweight concrete cannot occur as pure bending waves but as a combination of pure bending waves and other wave types such as shear waves, as discussed in Chapter 2. The transmission of sound bending waves in thick plates at high frequencies occurs slower than that given by Eq. (4.153). Subsequently, the effective critical frequency, f_c^*, of thick plates becomes quite higher than f_c for $f > f_s$. The corrected f_c for thick plate may be expressed using Eq. (4.155) and a correction factor (Cremer et al., 1988) as

$$f_c^* = f_c\left(7.35\frac{h^2 f f_c}{c^2} + \left[\left(7.35\frac{h^2 f f_c}{c^2}\right)^2 + 1\right]^{1/2}\right) \approx f_c\left(\frac{f}{5f_s} + \left[\left(\frac{f}{5f_s}\right)^2 + 1\right]^{1/2}\right) \tag{4.156}$$

For instance, the resonant transmission loss, Eq. (4.153), may be modified to include the case of thick plate (higher frequencies, $f > f_s$) as

$$R_r = R_0 - 10\log\left(\frac{\pi\sigma^2 f_c^*}{2\eta f}\right) - 10\log\left(\frac{S}{S_{tot}}\right) \qquad (4.157)$$

For $f << f_s$, $f_c^* \approx f_c$ and it would be no need for correction.

4.8.3 Total sound transmission loss

The total transmission loss of plate is described as the logarithmic addition of forced and resonant transmission as

$$R = 10\log\left(\frac{W_1}{W_{2f} + W_{2r}}\right) = -10\log(10^{-R_f/10} + 10^{-R_r/10}) \qquad (4.158)$$

where W_1 and W_2 are the incident sound power on the panel and the transmitted one, respectively. The expression for forced transmission, R_f, is obtained from Eq. (4.147) whilst the general expression for resonant transmission, R_r, is read from Eq. (4.157). The general expression for $f_{11} << f_c$ becomes:

$$R = R_0 - 10\log\left[\frac{2\sigma_d}{\{1-(f_{11}/f)^2\}^2\{1-\{f/f_c\}^2\}^2 + \eta_{eq}^2} + \frac{\pi\sigma^2 f_c^*}{2\eta f}\frac{S}{S_{tot}}\right] \qquad (4.159)$$

where η_{eq} is the equivalent loss factor for the 1/3 octave at resonance, which reads: $(\eta^2 + 0.1\eta)^{1/2}$, and f_c^* is the critical frequency corrected for thick plate.

The solution may be divided according to the frequency, and the following expressions are obtained for most typical applications:

$$R = R_0 - 10\log(2\sigma_d) + 20\log\{1-(f/f_c)^2\} \quad (f_{11} < f < f_c) \qquad (4.160)$$

$$R = R_0 - 10\log\left[\frac{2\sigma_d}{\eta_{eq}^2} + \frac{\pi f_c}{2\eta f}\frac{S}{S_{tot}}\left[(2\pi f_c/c)^{1/2} l_x(0.5 - 0.15(l_x/l_y))\right]^2\right] (f = f_c) \qquad (4.161)$$

$$R = R_0 + 10\log(f/f_c^*) + 10\log\eta + 10\log(1-(f/f_c)) - \Delta R_s - 10\log S/S_{tot} - 2 \quad (f > f_c; f > f^*{}_c) \qquad (4.162)$$

The correction ΔR_s is for thick plate, which reads according to Eq. (4.169) as

$$\Delta R_s = 10\log\left[(f/5f_s) + \sqrt{(f/5f_s)^2 + 1}\right] \qquad (4.163)$$

where λ_c is the wavelength at coincidence, $\lambda_c = c/f_c$, l_x and l_y ($l_x \le l_y$) are the dimensions of the plate. For $f < f_c^*$, $\Delta R_s \approx 0$, with respect to Eq. (4.162). The sound radiation factor due to free bending waves, σ, may be calculated from Eq.

(2.339)-Eq. (2.344). Further, the radiation efficiency at $f = f_c$ is obtained from Eq. (2.343b).

Limitations

The previous expressions work well for constructions that have high f_c, much greater than f_{11}. When f_{11} approaches or becomes fairly less than f_c, the results will not be valid as in the case of massive constructions with relatively large thickness. For example, a concrete plate with area 4 m^2, thickness 0.2 m, will have $f_c \approx 100$ Hz and $f_{11} \approx 145$ Hz. In this case, the previous derived expressions of transmission loss of finite plate will not yield valid results. One may in this case either use the expressions of infinite plate if the area of plate is not too small in size, or use the model reviewed in Sec. 4.8.4. At any rate, transmission loss of massive constructions (e.g., steel and concrete) with relatively small thickness can still be predicted using the previous expressions provided that $f_{11} << f_c$. Unfortunately, there is no simplified and comprehensive model for finite plates. Apart from its limitations, the advantage of the previous model is that it is simple to calculate and it covers the whole frequency range under interest.

Discussion

Inspection of Eq. (4.160)-Eq. (4.163) indicate that the resonant transmission becomes dominant for $f > f_c$ while the forced one is dominant for $f < f_c$. However, at coincidence region, the contribution of both parts can be important. Moreover, the effect of damping and the radiation efficiency of free bending waves (*eigen* vibrations) in the mass-controlled region are of secondary importance. Subsequently, the damping doesn't control the forced response of the plate at least for typical building constructions in which the damping factor is usually very small ($\eta < 0.1$). However, the resonance response of the plate is controlled by the damping, as indicated by Eq. (4.162). Moreover, the damping is also of secondary importance for the stiffness-controlled region. For many cases in practice, the low stiffness region whereby the plate vibrates as a stiff membrane lies typically below 10-50 Hz and as such it is not within the frequency region of building acoustics.

In general, the expressions of transmission loss of finite plates are only important when the size of plate becomes small, which is not commonly encountered in building acoustic application, with the exception of small windows or similar, as discussed later on.

4.8.4 Literature review

A complete analytical model for predicting the sound reduction index of a single plate has been developed by Kernen and Hassan (2005). The analytical model is valid for a wide frequency range, both below, above and at the critical frequency. Special interest is paid to the area dependency of the sound reduction index. In the model, the expressions for forced and resonant sound transmission read:

$$R_f = -10\lg\left[\left|\frac{2\rho_0 c\omega}{B}\right|^2 \frac{k}{\pi L}\int_0^{\frac{\pi}{2}}\int_{-k}^{+k}\frac{\left|-e^{j(k_0+k_x)L/2}+e^{-j(k_0+k_x)L/2}\right|^2}{\sqrt{k^2-k_x^2}\left|(k_x^4-k_B^4)(k_0+k_x)\right|^2}dk_x \sin\varphi d\varphi\right] \tag{4.164}$$

$$R_r = -10\lg\left[\frac{8k_B}{\eta}\frac{US}{LS_{tot}}\left(\frac{\rho_0 c}{M_s\omega}\right)^2\langle\cos\psi\rangle\sigma \int_0^{\frac{\pi}{2}}\left[\frac{\sin[(k_0-k_B)L/2]e^{-jk_B L/2}}{(k_0-k_B)}+\frac{\sin[(k_0+jk_B)L/2]e^{-k_B L/2}}{(jk_0-k_B)}\right]^2 \sin\varphi d\theta\right] \tag{4.165}$$

where k is the wave number in air, $k_0 = k\sin\theta$, is the trace wavenumber of the exciting pressure along the plate, L is the waves mean free path, $L = \pi S / U$, where S is the area of the excited part of the plate, and U is the perimeter of the radiating area of the plate, $U = 2(l_x+l_y)$, where l_x and l_y ($l_x \le l_y$) are the dimensions of the excited part of the plate ($S= l_x+l_y$), φ is the angle of incidence, ψ is the angle between the direction of free waves and boundary normal, $\langle\cos(\psi)\rangle$ is the mean value of $\cos(\psi)$, which reads as

$$\langle\cos\psi\rangle = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\left[1-(k_0\sin\phi/k_B)^2\right]^{\frac{1}{2}}d\phi \tag{4.166}$$

where ϕ is the angle of forcing waves. The forced radiation factor, σ_{dL} is imbedded in Eq. (4.164). The total transmission loss is obtained using Eq. (4.158).

The integration range in the latter two expressions is from 0 to $\pi/2$ for diffuse field. However, in practice, the integration may be limited to 78° instead of 90° (Cremer et al., 1988) to reduce the deviation between theoretical and laboratory measurements, as discussed later on.

Other models

Another models of finite plates may be found in EN 12354-1(2000), Josse and Lamure (1964), Ljunggren (1991), and Swell (1970). The last two models have problems in the coincidence region. The EN model is basically the model of Josse and Lamure with small modifications.

4.8.5 Dependence of sound transmission loss on panel area

Fig. 4.29 shows the transmission loss calculated using expression for finite and infinite plate expressions. As can be seen, the simplified model described by Eq. (4.159)-Eq. (4.163) yields good agreement with measurements and with the more detailed model, Eq. (4.164)-Eq. (4.166). The predicted sound transmission loss of finite panel is little higher than the predicted transmission loss of infinite panels for frequencies up to (1/4) f_c and then the differences become smaller especially near f_c. At $f = f_c$, the difference between two cases is about 10 dB. This is quite high difference between two cases, which should be considered by the designer. Although expressions of finite plate yield better agreement than the expressions of infinite plates, nevertheless, approximating the sound transmission in finite plate using the model of infinite plates has given reasonable results.

The area dependency of the transmission loss may be further investigated with the help of Fig. 4.30. As can be seen, when the plate area becomes small, the sound reduction index increases. The random-incidence sound (diffuse sound field in the source room assuming waves incident from 0 to 90°) transmission loss for $f \ll f_c$ of partitions of approximately 4 m^2 surface area, which are typically used in laboratory measurements, yield predicted values that are about 5 dB below the normal-incidence mass law. Consequently, the field-incidence mass law defined in Eq. (4.28) for partitions of this size is theoretically justified. The random-incidence transmission loss of a finite panel for $f \ll f_c$ is obtained from Eq. (4.150) where the radiation efficiency of random incidence sound-forced excitation is given by Eq. (4.142) or the approximate expression, Eq. (4.144). The results of Fig. 4.29 and Fig. 4.30 demonstrates that for most building acoustics applications, it is sufficient to use infinite models alone to predict the sound transmission loss without the resort to finite models as most building panels are relatively large in size; see also Fig. 4.29.

Inspection of Eq. (4.159)-Eq. (4.163) shows that there is a strong area dependency of the resonant and forced transmission for low frequencies; for frequencies at and below f_c, the transmission loss increases with decreasing plate area for both forced and resonant transmission; see also Fig. 4.30. However, above the critical frequency, only the resonant part of the transmission is affected by the area size, and the transmission loss increases with decreasing area, too.

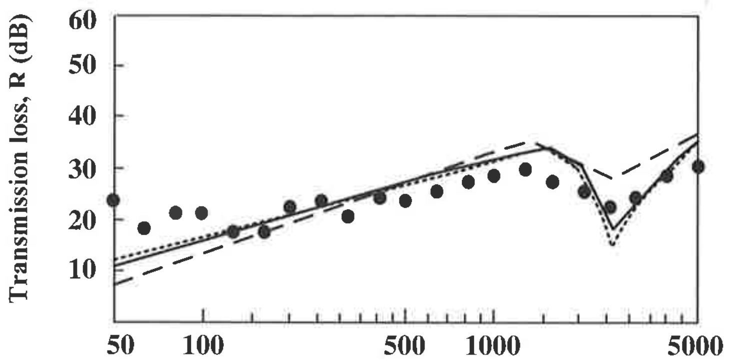


Fig. 4.29 The sound reduction index of a 16 mm, 4.4 m^2 chipboard. Young's modulus is E = 1.5 GPa and ρ = 626 kg/m^3, the loss factors = 4%. (●●●), Measured results; (——), Calculated according to Eq. (4.164)-Eq. (4.166); (••••••), Calculated according Eq. (4.159)-Eq. (4.163); (— — —), Calculated according to Eq.(4.30) and Eq.(4.34) for infinite plates.

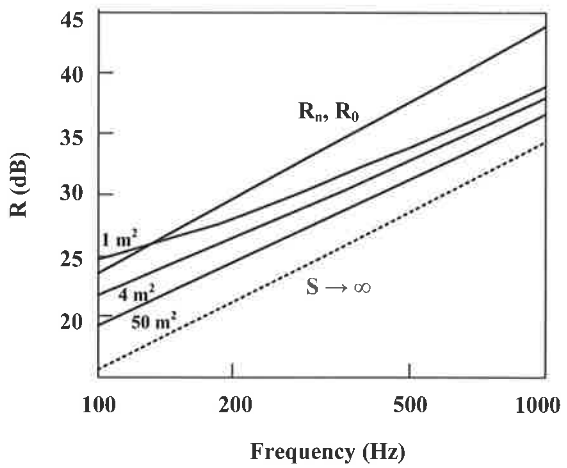


Fig. 4.30 Random-incidence sound transmission loss of 25 kg/m^2 panels as a function of frequency with panel surface area S as a parameter, mass-controlled low-frequency region ($f << f_c$); computed according to Eq. (4.150) and Eq. (4.144). R_0 is the approximation of R_n, the sound reduction index for normal incidence.

Further, the loss factor also influences the transmission loss of finite plate; for the forced part of transmission; an increase of the loss factor will increase the transmission loss at and above the critical frequency. However, for the resonant part of the transmission, an increase of the loss factor will increase the sound reduction index at all frequencies. It should be pointed out that the sound transmission loss formulas of finite plates as derived above are only valid in the frequency region well above the first bending-wave resonance of the panel (f_{11}).

4.9 Transmission of Sound through Infinite Plate

In this section, the case of oblique incidence and diffuse sound field through the plate are discussed, based on the transmission coefficient. For the theory, it is assumed that the panels are of constant thickness and the panel material is homogeneous and isotropic. Furthermore, the panels are assumed to be thin i.e., the bending wavelength of the forced vibration is at least six times the panel thickness and of infinite extent; i.e., the lateral dimensions must be much greater than the bending wavelength of the forced vibration

4.9.1 Transmission loss of oblique incidence sound waves

Solids can transmit shear forces so that in addition to longitudinal compression waves, shear waves and bending waves can be transmitted. When a plane sound waves is incident at oblique angle to the normal of the plate, φ, (see e.g., Fig. 2.45) on the plate that is laterally infinite, the solution may be formulated by considering that sound excite both longitudinal (P-waves) and shear waves (S-waves) in the plate material, similar to the case of a plates or half space excited by an indenter or disk. The two wave type travel between the two faces of the plate in a similar manner as the normally incident wave, except that the compressional and shear waves run at oblique incidence. Subsequently, it is expected that the transmission loss must be a function of φ. The oblique transmission loss (sound reduction index) for thin infinite plates can be written as (Cremer, 1971):

$$R(\varphi) = 10\log\frac{1}{\tau(\varphi)} \approx 10\log\left[1+\left|\frac{\pi f M_s \cos\varphi}{\rho_0 c}\left[1-\left(\frac{f}{f_c}\right)^2 \sin^4\varphi\right]\right|^2\right] \quad (4.167)$$

where τ is the sound (power) transmission coefficient, as discussed later on. Eq. (4.167) is valid for homogeneous and inhomogeneous, isotropic plates, but plates must be thin compared with shear wavelength. It indicates that when φ = 0 (normal incidence), then Eq. (4.167) is reduced to Eq. (4.27), the classical mass law. Moreover, when $f = f_c/\sin^2\varphi$ (the coincidence frequency), trace coincidence between incident sound waves and free bending waves in the plate occurs and would result in complete transmission if the plate has no internal damping. Consequently, one may introduce internal damping, η, of the plate by making the Young modulus as complex, according to Eq. (2.194). Subsequently, the complex critical frequency, f_c', may now be expressed as

$$f_c' = f_c\left(1 - j\frac{\eta}{2}\right) \tag{4.168}$$

Accordingly, Eq. (4.167) may be rewritten as

$$R(\varphi) \approx 10\log\left[1 + \left|\frac{\pi f M_s \cos\varphi}{\rho_0 c}\left[1 - \left(\frac{f}{f_c(1 - j\eta/2)}\right)^2 \sin^4\varphi\right]\right|^2\right] \tag{4.169}$$

Eq. (4.169) indicates that near $f = f_c/\sin^2\varphi$, the curve of sound transmission loss will be minimum (the trace-matching dip), which is controlled by the damping; see Fig. 4.31. Eq. (4.167) and Eq. (4.169) indicate that zero transmission loss can be obtained at grazing incidence ($\varphi = 90°$) and when $f = 0$. At coincidence, when trace matching occurs, i.e., $f = f_c/\sin^2\varphi$, then Eq. (4.167) becomes zero, while with damping in plate, the transmission loss, Eq. (4.169) will exhibit a minimum value.

For infinite thick plate, the sound transmission loss of plate may be written as (Cremer, 1971):

$$R(\varphi) = 10\log\frac{1}{\tau(\varphi)} = 10\log\left|1 + \frac{Z_s\cos\varphi}{2\rho_0 c}\right|^2 \tag{4.170}$$

where Z_s is the separation impedance, defined as: $(p_s - p_{rec})/v_n$, where p_s is the complex amplitude of the sound pressure at the source-side face of the plate (the sum of incident and reflected pressure), p_{rec} is the complex amplitude of sound pressure on the receiver-side face of the plate, and v_n is the complex amplitude of normal vibration velocity of the receiver side face of the plate. For single panels, it is assumed that both faces of the plate vibrate in phase with the same velocity. The separation impedance of thick isotropic plate (same B in all directions) may be expressed as (Sharp, 1973):

$$Z_s = \frac{j\omega\left[\rho h + \left(\frac{\rho h^3}{12} + \frac{\rho B}{G}\right)\frac{\omega^2}{c^2}\sin^2\varphi\right] - j\omega^3\left[\frac{B}{c^4}\sin^4\varphi + \frac{\rho^2 h^3}{12G}\right]}{\left[1 + \frac{B\omega^2\sin^2\varphi}{Gc^2 h} - \frac{\rho h^2\omega^2}{12G}\right]} \tag{4.171}$$

where ρ is the panel density, G the shear modulus, B is the bending stiffness, h is the thickness of panel, and c is the speed of sound in air. Eq. (4.170) indicates complete transmission occurs at the trace coincidence between incident sound wave and the free shear waves in the plate, and a plate can either be predominately controlled by shear or by bending. Note that Eq. (4.170) is general in the sense that it comprises thick or thin plates. In thin plates, the resistance to bending in much lower than the resistance to shear, and that $k_s/h << 1$, where $k_s = \omega/c_s$ is the shear wavenumber, where c_s is obtained from Eq. (4.131). This means that, in this case, the condition of thin plates is that the plate is thin compared to shear wavelength. Consequently, for thin plates, Eq. (4.170) in combination with Eq. (4.171) reduces to Eq. (4.167).

4.9.2 Transmission of random-incidence (diffuse) sound through infinite plate

A plane wave incident on the plate at one angle is not a practical problem. The sound field in the room is better modeled as diffuse sound field in which plane sound waves have same average intensity and travel with equal probability in all directions; these waves are uncorrelated and have same intensity. A region of unit area on the plate will be exposed at any instance to incident waves from all areas on a hemisphere (half of a sphere), whose centre is the area of the plate. The sound reduction index (or sound transmission loss) of the plate is defined in terms of the sound (power) transmission coefficient, τ, as

$$R = -10\log(\tau) \tag{4.172}$$

In general, the sound transmission coefficient is a function of both the angle of incidence of the sound field and the frequency. The angle of incidence, φ, is measured from the normal to the plane of the panel as illustrated in Fig. 2.45, which indicates that the incident sound field is composed of waves strikes the plate at different angles and frequencies. To simplify the situation, it is necessary to average both the incident and the transmitted intensities over both frequency and angle of incidence. At a fixed frequency, the incident intensity, $I_{in}(\varphi)$, establishes both the average incident intensity, I_n (subscript n denotes the normal to the panel), and the average transmitted intensity, I_t, (subscript t denotes transmitted). Subsequently, an average transmission coefficient, $\bar{\tau}$, is defined as

$$\bar{\tau} = I_t / I_n \tag{4.173}$$

The incident sound intensity on the unit area of the plate from any particular angle equals to the intensity of the plane wave at angle φ times the cosine of the angle of incidence. The random-incidence sound reduction index (R_{random}) is defined as

$$R_{random} = -10\log(\bar{\tau}) \tag{4.174}$$

The average incident intensity (W/m^2) and the average transmitted intensity are defined, respectively, as

$$I_t(\varphi) = \int_\Omega \tau(\varphi) I_{in}(\varphi)\cos\varphi\, d\Omega \quad \text{and} \quad I_n(\varphi) = \int_\Omega I_{in}(\varphi)\cos\varphi\, d\Omega \tag{4.175a, b}$$

The integration is taken over a hemisphere of solid angle, Ω, where $d\Omega = \sin\varphi\, d\varphi\, d\theta$, where $0 \le \varphi \le 2\pi$ and $0 \le \varphi \le 2\pi$.

For a diffuse incident sound field, $I(\varphi)$ is independent of direction and is the same for all plane waves ($I(\varphi) = p^2/4\rho_0 c$) and τ is independent of the polar angle, θ. Subsequently, the expression for the average sound transmission coefficient is obtained by combining Eq. (4.175) and Eq. (4.173):

$$\bar{\tau} = \frac{\int_0^{\varphi_{lim}} \tau(\varphi)\cos\varphi \sin\varphi\, d\varphi}{\int_0^{\varphi_{lim}} \cos\varphi \sin\varphi\, d\varphi} \tag{4.176}$$

The angle φ_{lim} is the limiting angle of incidence of the sound field, which is taken as an empirical limit for the integration. If the sound field is truly diffuse (random incidence), then $\varphi_{lim} = \pi/2$ radians or 90°. The sound transmission coefficient $\tau(\varphi)$ may be obtained from the expression for the thin and thick plates, as discussed above.

At low frequencies ($f << f_c$), the random-incidence sound transmission loss of a thin plate is found by combining Eq. (4.176), Eq. (4.174) and Eq. (4.169) with $\varphi_{im} = 90°$. The result may be approximated as (see Sec. 4.2.1)

$$R_{random} \approx R_n - 10\log(0.2303 R_n) \tag{4.177}$$

where R_n is transmission loss at normal wave incidence as expressed in Eq. (4.27); R_{random} is commonly referred to as the random-incidence mass law. The approximation of Eq. (4.177) may be compared with direct calculation of Eq. (4.176) as shown in Fig. 4.32. As can be seen, Eq. (4.177) yields good approximation for $f \le 0.5 f_c$. Further, it can be shown that this approximation is valid for $R > 15$ dB; for $R < 15$ dB, a difference up to 3 dB should be expected between the approximate expression and the exact one, Eq. (4.176).

In practice and based on laboratory measurements of the sound transmission loss for a wide range of construction, it appears that an empirical value of φ_{lim} is in the range of 78° to 85° (Cremer et al., 1942); however, $\varphi_{lim} = 78°$ is commonly used in practice. Subsequently, Eq. (4.176) must be solved by numerical integration in combination with Eq. (4.169) and Eq. (4.174) to obtain the field-incidence sound transmission for thin isotropic plates, R_{field}. The solution may be simplified at low frequencies ($f << f_c$) and the result becomes:

$$\frac{1}{\bar{\tau}} \approx \frac{1}{\pi}\left[1 + \left(\frac{\pi f M_s}{\rho_0 c}\right)^2\right] \tag{4.178}$$

Subsequently the sound transmission loss is obtained using Eq. (4.172) as

$$R_{field} = 10\log(1/\bar{\tau}) \approx R_n - 5 \tag{4.179}$$

The latter expression is commonly referred to as the field-incidence mass law and the approximation is valid for $R_n \ge 15$ dB. The approximation of Eq. (4.179) is compared with direct calculation of Eq. (4.176) as shown in Fig. 4.32. As can be seen, Eq. (4.179) yields good approximation for $f \le 0.5 f_c$.

4.9.3 Discussion

Fig. 4.31 shows the curve of transmission loss for a steel sheet at different angle of incidence, according to Eq. (4.169); the random field-incidence transmission loss is obtained from Eq. (4.176) and Eq. (4.174), with $\varphi_{lim} = 78°$. As can be seen, the sound transmission loss increases as angle of incidence, φ, decreases (because of the $\cos\varphi$ term) and that at each φ there a coincidence frequency defined by Eq. (4.33). As the incidence angle approaches grazing incidence (i.e., $\varphi = 90°$), the transmission loss approaches zero at f_c. Fig. 4.32 shows the transmission loss calculated using approximate and exact expressions, which indicates that the agreement is good up to $f \le 0.5 f_c$. For example, when, $R_n = 40$ dB, then $R_{random} = 30.4$ dB and $R_{field} = 35$ dB. It is interesting to see that the random-incidence transmission loss becomes equal to the random field-incidence transmission loss for frequencies above the critical frequency.

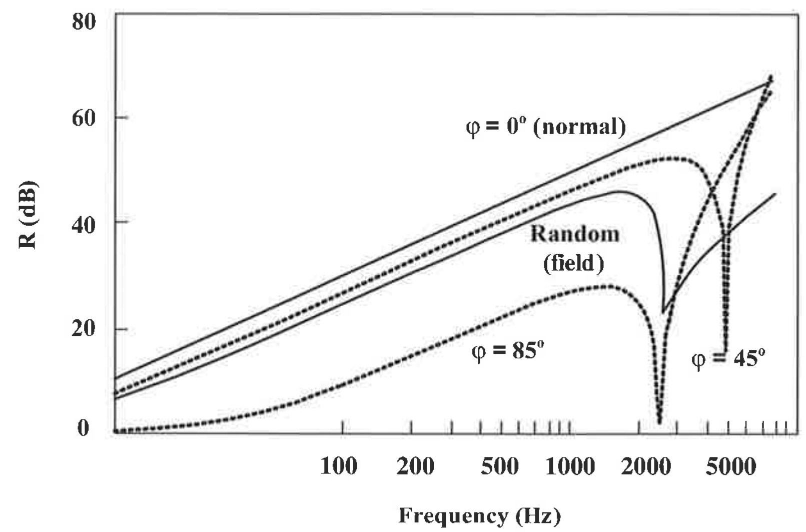


Fig. 4.31 Calculated transmission loss versus frequency of versus frequency of 5 mm (3/16 in) thick infinite steel plate: η = 0.5%, μ = 0.3, E = 200 GPa, ρ = 7700 kg/m^3 for various angles of incidence.

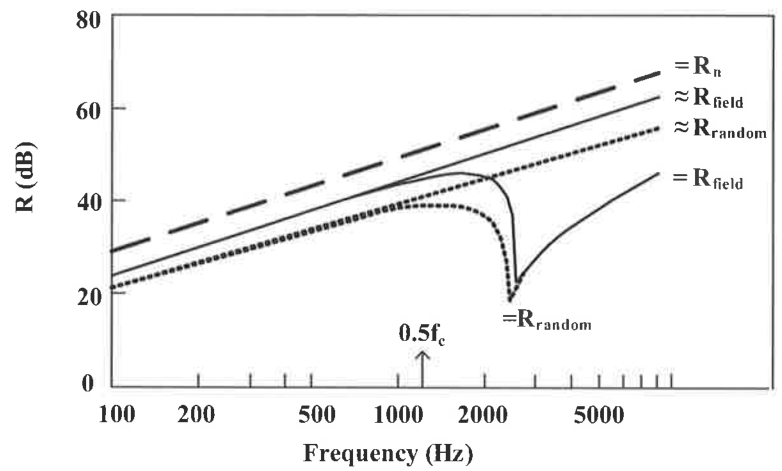


Fig. 4.32 Calculated transmission loss versus frequency of 5 mm (3/16 in) thick infinite steel plate: η = 0.5%, μ = 0.3, E = 200 GPa, ρ =7700 kg/m^3. Field incidence mass law, R_{field}, assumes a sound field that allows all angles of incidence up to 78° from normal and is calculated using Eq. (4.176) and Eq. (4.174), while the approximate R_{field} (denoted by ≈) is calculated from Eq. (4.179).The random-incidence sound transmission loss, R_{random}, is calculated using Eq. (4.176) and Eq. (4.174) with φ_{lim} = 90° while the approximate R_{random} is calculated from Eq. (4.177). The approximations are valid up to $f \leq 0.5f_c$.

4.10 The Loss Factor in Practice and Theory

The calculation procedure of sound insulation of partitions requires knowing the loss or damping factor, η either experimentally or theoretically. To measure η, it is used small mechanical transducers called accelerometers, which are attached to the vibrating structure, as shown in Fig. 4.33. The accelerometer is connected to a preamplifier which can contain networks allowing the measurement of vibration displacement, velocity, and acceleration. The output signal is analyzed by the same type of instrumentation as used for sound measurements e.g., a frequency analysis of the vibration signal. The loss factor, η, is determined from the mechanical reverberation time of a partition which is excited by a shaker driven by white noise in 1/3 octave bands. When the partition has reached a steady level of vibration, the shaker will immediately be stopped. The reverberation time for each 1/3 octave band is determined from the decay curves recorded by an accelerometer, and η is calculated from the general expression, Eq. (4.181).

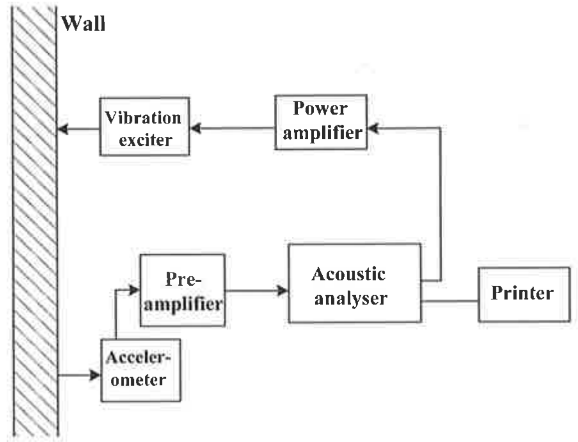


Fig. 4.33 Illustration of measuring the loss factor of a partition.

In theory, the *eigen* vibrations (free modes) that are not fed by energy, will diminish out. At a given point (x, y) of the plate, the damped *eigen* vibration amplitude, ξ, can be written as

$$\xi = \hat{\xi}_0 e^{-\delta t} \cos(2\pi f_{nm} t) \tag{4.180}$$

where $\hat{\xi}_0$ is the amplitude at time $t = 0$, f_{nm} is the resonant frequencies as read from Eq. (4.22), and δ is the damping coefficient, which is equal to logarithmic decrement divided by the period time; see Chapter 11 for more details on damping factors. The total loss factor η is related to the damping coefficient δ, which in turn is related to the reverberation time T as (see also Sec. 2.12.2):

$$\eta = \frac{\delta}{\pi f} = \frac{3\ln 10}{\pi f T} \approx \frac{2.2}{fT} \qquad (4.181)$$

where f is the centre frequency of the 1/3 octave band and T the mechanical reverberation time of the partition. Further, to calculate the energy loss of vibrations in plate, η, the following expression is used:

$$\eta = \frac{W_{tr}}{\omega E_{mech}} = \frac{W_{tr}}{2\pi f M_s S < \tilde{v}^2 >} \qquad (4.182)$$

where W_{tr} is the radiated power (energy per time unit) and E_{mech} is the total energy of vibrations, which can be expressed by the square RMS velocity averaged over plate area times the plate mass, M_sS; refer to Chapter 11.

As discussed before, the damping of structures, in general, is due to either transmission to other parts of the structure or radiation to air, or as conversion to heat (material damping). When measuring isolated samples such as plates in laboratory, the loss factor is only the radiation loss and material (or internal) damping loss, η_i. If the radiation loss to air is small, the only losses are due to material losses, as given Table A (Appendix). However, in buildings the losses due to transmission to other structure parts (coupling losses) are important and the internal loss factor η_i, is not practically enough to describe the total loss factor, η, in buildings since the energy loss at edges (boundary losses) or coupling loss factor, η_b, can also increase the final value of η. There is also the energy loss due to radiation from plate to air, η_r. Consequently, the loss factor that is measured and calculated by Eq. (4.181) is the total loss factor, which is can be written as

$$\eta = \eta_i + \eta_b + \eta_r \qquad (4.183)$$

Additional loss factor, η_p, may also be added to the right-hand side of Eq. (4.183), where η_p is the loss factor due to special case of energy transmission from one floor to another through the structure; if no data are available then $\eta_p \approx 0$. The loss factor of sound radiation to surrounding air η_r is expressed in Eq. (2.247a, b). Loss factor due to internal losses, η_i, can be estimated to 0.4% for most building materials (Cremer et al., 1988). It is suggested that the sum $\eta_i + \eta_p + \eta_r \cong 0.55\%$ for building materials (Ljunggren and Ottosson, 1995); however, the case can differ from one situation to another since in case of lightly damped metal structures can have relatively high η_r. The boundary or edge or so called

coupling loss factor for a plate with area S and critical frequency f_c can be written as (see Eq. (2.254)):

$$\eta_b = \frac{c\sum_i L_i\alpha_i}{\pi^2 S\sqrt{ff_c}} \approx \frac{cU\alpha}{\pi^2 S\sqrt{ff_c}} = \frac{\lambda_B U\alpha}{\pi^2 S} \qquad (4.184)$$

where L_i and α_i are edge lengths and their absorptions coefficients, which can simplified by setting the plate circumference, U, and the average absorption coefficient, α, which can lie in the interval: $0 < \alpha \leq 0.3$; one can also use Fig. 6.9, Chapter 6 to determine α. Eq. (4.184) indicates that the boundary losses decrease as area increases. If there is sufficient data on the surrounding building elements, it is possible to calculate the coupling losses according to the method described in Chapter 6. In most masonry walls and floors, most of the damping is due to the coupling loss at the edges, which is frequency dependent.

An approximate empirical expression for the coupling loss factor for common walls and floors made of concrete, lightweight concrete and masonry is (Craik, 1981):

$$\eta_b \approx 1/\sqrt{f} \qquad (4.185)$$

This means that average absorption coefficient in Eq. (4.181), $\alpha \cong 0.2$. However, the latter expression cannot be made general to most building materials, and as such it may be used as very rough estimate.

A general approximate expression for the total loss factor for most typical building elements is presented as (EN ISO, 1997):

$$\eta \approx \eta_i + \frac{M_s}{485\sqrt{f}} \qquad (4.186)$$

Eq. (4.186) indicates that as frequency increases, the total energy losses decreases and approaches that of material damping.

The energy losses due to radiation of sound into air, η_r, in many building structures are usually small but can be important for lightly damped metal structures. In addition, the material loss factors, η_i, of metals can increase with strain amplitude, but the loss factors of plastics and rubbers can approximately be independent of strain amplitude up to strains of the order of unity (Beranek and Ver, 1992). The loss factors of some flowing or creeping materials tend to vary noticeably with temperature and frequency. Further, η_i is weakly dependent on the frequency and so it can be taken as constant, for practical reasons.

It is noteworthy to indicate that when calculating the transmission loss of infinite plates, the total loss factor is often taken as the material damping only. However, as indicated earlier, this is not totally accurate as there is no infinite plate in reality and, therefore, the total loss factor for a plate in a building should be calculated considering all the energy losses, as discussed earlier.

4.11 Sound Transmission Loss of Holes and Slits

Sound leakage occurs often in typical building systems, for instance, airborne sound leaks through an open air path at the wall connections between two rooms. Ventilation systems and ceilings that are not airtight can be a source of sound leakage. In some wall sections cracks can be seen. These are very undesirable because of sound leakage. All sound leaks are important because sound will travel through any opening with little loss. For example, a very small air hole in a brick wall can easily reduce insulation from 50 dB to as low as 20 dB. Cracks, gap around pipe work through partition, louvred doors, porous construction, etc. are, therefore, to be avoided. For example, the lightweight, porous sound-absorbing tiles or panels are relatively poor isolators. The sound transmission loss of holes and slits (circular and slip-shaped apertures in wall) can be increased substantially by sealing them with either porous sound-absorbing material or an elastomeric material or designing them as silencer joints. Prediction of the sound transmission loss of such acoustically sealed openings is presented below.

4.11.1 Calculation procedure

The sound energy that is transmitted through a typical partition, W_{tr} is very little in relation to the amount of incident energy, W_{in}. For example, one can expect that a concrete wall can have a transmission loss (sound reduction index) of about 50 dB in the mid- frequencies range, i.e.

$$\tau = W_{tr}/W_{in} = 10^{-5} \tag{4.187}$$

where τ is the sound transmission coefficient. Many slits and openings in the ceiling and/or wall can allow sound energy to escape the room. For such slits and openings, the reduction sound index is roughly 0 dB. This implies that even a very narrow slit or opening deteriorates the wall insulation. For instance, a crack of about 0.1 mm along the edge of concrete partition (between flats) can be very deteriorating for the sound insulation.

Circular holes

Approximately, the sound reduction index of a round hole, at quite low frequencies obeys (see Gomperts, and Kihlman, 1967):

$$R_h = 10\log\left(\frac{n^2\sin^2[K_h(L+2e)]+4K_h^4}{16K_h^2}\right) \quad (K_h < 2) \tag{4.188}$$

where $K_h = 2\pi f r_0 / c$, f is the frequency (Hz), r_0 the radius of the hole (m), c the sound speed in the air (m/s), $L = \delta / r_0$, δ is the hole length (m), e is the end correction (non-dimensional); in general, the end correction for a circular hole e is taken as, $e \approx 0.6$. The factor n depends on the location of the hole in the wall: $n = 2$, if the hole is at the middle of the wall; $n = 1$, if the hole is along one of the wall's edge; and $n = 1/2$ if the hole is near to the corner.

In general, a hole in the middle of the partition gives a less deterioration in sound insulation than a hole along the edges and at the corner; the difference can amount to 6 and 12 dB, respectively (for long holes and low frequencies).

At high frequencies ($K_h \geq 2$), one can approximate the hole transmission loss as $R_h \approx 0$; see also Mechel (1986, 1987).

Slits

For oblong slits, the sound reduction index with respect to frequency is calculated according to a similar approximate formula:

$$R_{sl} = 10\log\left(\frac{2n^2\sin^2[K_{sl}(L+2e)]+4K_{sl}^4}{8K_{sl}}\right) \quad (K_{sl} < 1.5) \tag{4.189}$$

where $K_{sl} = 2\pi f \beta / c$, β is the width of the slit (m) (the lowest cross-dimension), $L = \delta / \beta$, δ is the depth of the slit (m); which is often taken as the wall thickness. The coefficient, n, takes the following values: $n = 1$, if the slit is in the middle of the wall; $n = 1/2$, if the slit is along the wall's edge. The end correction, e, in this case, is strongly dependent on the frequency as compared with the circular hole. In calculations, e is obtained using the following expression (only for long narrow slits):

$$e = (1/\pi)\{\ln(8/K_{sl}) - \gamma\} \tag{4.190}$$

where γ is Euler's constant = 0.57722. Thus, the factor e is a function of K_{sl}.

In practice, the sound reduction index of a slit is often negative: –10 dB to –15 dB for fairly broad frequency range and for long narrow slits. This should not be understood as a contradiction with the definition of transmission loss, R; owing to the sound wave character, a greater part of the wave front of the incident sound is transmitted than what is given by the geometric area of the opening. At high frequencies, i.e., $K_{sl} \geq 1.5$, the reduction index is approximated as 0 dB.

Approximate formula for small holes

The sound transmission loss of a small round hole with radius $r_0 << \lambda$ (= c/f) in a thin plate of thickness h is well approximated by (Cremer, 1971):

$$R_h \approx 20\log\left(\frac{h+1.6r_0}{r_0\sqrt{2}}\right) \quad (4.191)$$

Eq. (4.191) indicates that small holes in thin partitions ($h << r$) yield a frequency-independent sound transmission loss of $R_h \approx 1$ dB; however, $R_h \approx 0$ dB is frequently used in practice. Note, however, that Eq. (4.191) is only valid for small round holes. Long narrow slits can have a negative sound transmission loss, as discussed above.

The net (composite) sound transmission loss for a plate with an opening can be determined using Eq. (4.116) in which sound transmission loss of the opening is obtained using the relevant expressions of the opening, as presented in this section; refer also to Example 4.8 and Example 4.11.

Discussion

The sound leakage via small holes and long slits has quite different effects on the measured sound reduction index curve, as shown in Fig. 4.34. The leakage from slits results in a wide dip in the transmission loss curve, while leakage over a smaller surface gives a characteristically spiny curve.

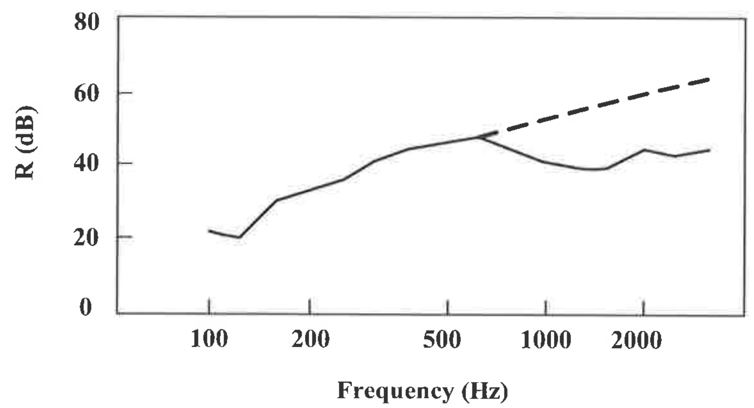


Fig. 4.34 Sound reduction index for a wall with a narrow opening (slit) along the adjoining wall. (——), measured R with sound leakage; (— —), estimated R after soundproofing (sealing or caulking) the opening.

Usually, slits are essentially more dangerous to sound insulation than round holes; the conventional keyhole in a door is thus totally safe compared to the slits or opening between the door leaf and frame. This concerns also the situations whereby the door is over-folded since the type of slits has inconsiderable effect in this case. A wall with a potential R_w of 60 dB, which has a hole of only 0.001% of the total wall area (10 × 10 mm hole in a 2.4 × 4.2 m wall), can be reduced to an effective R_w of 50 dB.

As the hole area increases, eventually the rating is determined entirely by the hole area. The higher the acoustical isolation that is sought, the more important it is to eliminate all sound leaks. Sealing the outlets (such as electrical) properly increases the R_w. Even if the holes are rather large, the presence of glass fibre in the cavity helps to reduce transmission through the leak. Because of the losses in the sound absorber, the transmission loss of the hole is no longer zero. If electrical outlets must be installed in a party wall, they should be well sealed and be offset from each other along the wall. Further details are presented in the following section.

Example 4.11: A wall has a transmission loss of 30 dB with no opening in the wall. If an opening having an area equal to 10% of the total wall area is added in the wall, determine the overall transmission loss (sound reduction index) for the wall with the opening included. If the transmission loss of the wall is increased to 50 dB, would this improve the overall transmission loss? Discuss the practical consequences.

Solution

The transmission loss for the opening is $R \approx 0$. From Eq. (4.116), the total transmission loss for the wall with the opening included is:

$$R_{tot} = 10\log\left(\frac{S}{0.1S + 10^{-30/10}(0.9S)}\right) \approx 10 \text{ dB}$$

As can be observed, an opening of only 10% of the total wall area reduces the transmission loss from 30 dB to a value of 10 dB. After the wall's transmission loss increases to 50 dB, R_{tot} = 10 dB, which means that there is no advantage of increasing the transmission loss of the wall, itself. If the noise reduction for a wall is to be effective, any openings must be as small as possible or completely removed, if practical.

4.11.2 Acoustical treatment

Sound leaks can significantly reduce the effectiveness of a system. They can easily occur at the perimeter of walls and floors where caulking is absent or improperly installed, or where a penetration is made to add some service such as electricity or plumbing. The higher the acoustical isolation that is sought, the more important it is to eliminate all sound leaks. In practice, slits and openings cannot always be avoided and so a remedy plan must in this case be considered. The principle for soundproofing (sealing or caulking) to reduce sound leaks is depicted in Fig. 4.35. All penetrations and fissures in a wall or floor must be thoroughly caulked, all windows and doors must be tightly weather stripped and holes for services properly sealed. Joints between the top of walls and roof or floor assemblies should be sealed with elastomeric joint sealants. The joint space behind the sealant backing can be filled with mortar, grout, foam, cellulose fiber, glass fiber or mineral wool.

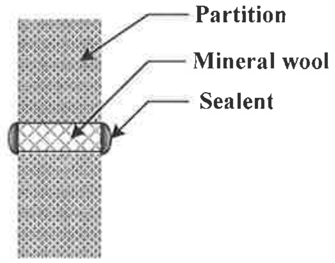


Fig. 4.35 The principle of soundproofing of an opening or hole in a wall/floor.

In order to increase the sound insulation of ceilings, a layer of fibreglass batt insulation is installed on top of the ceiling tiles. This increases the thickness and effectiveness of the ceiling assembly, helps to seal some cracks, and also helps to absorb sound in the plenum. Wherever electrical boxes or other recessed fixtures are installed in gypsum board partitions, the space between the gypsum board opening and the box should be caulked.

The windows and glass doors must be installed and sealed into the building façade so that it would not introduce significant extra air infiltration. Higher air infiltration will give lower sound insulation. The detrimental effect of air infiltration increases as the sound insulation increases. Doors and windows are a particular problem for sound transmission. With doors, the problem is the cracks

around the edges, particularly the large gap at the bottom. With windows, the problem is the glass. Suggestions for treatments are:

a. If acoustical privacy or noise is a particular problem, automatic door bottoms to seal the gap at the bottom of the door should be installed. They can easily be installed at any time in any wood door.
b. If acoustical privacy or noise is a particular problem and windows are needed, double glazing should be used and operable windows gasketed.
c. Wherever possible, hollow metal frames that include gasketing for all doors may be installed. This permits sealing of any door in the future simply by installing the automatic door bottom.

Appendix

Table A. Mechanical properties of solids under standard room conditions. c_L is the longitudinal speed of sound; ρ is the material density; η is the material damping coefficient (loss factor); E is Young's modulus; and is μ Poisson's ratio [a].

Material	ρ (kg/m^3)	η [b]	E (GPa) ($\times 10^9$ N/m^2)	μ
Aluminum [c]	2700-2800	0.0001-0.001	65.2-72	0.33-0.34
Asbestos concrete	2000	0.007-0.02	28	0.1-0.05
Asphalt	1800-2300	0.3-0.4	7.7	0.35-0.38
Brass [d]	8500-8710	0.002	89-95	0.33-0.37
Brick	1800-2300	0.01-0.02	16.7-24	0.20
Chipboard	625-750	0.020	0.34-1.5	0.08
Concrete (dense) [e]	2300-2400	0.005-0.02	20-36	0.1-0.30
Concrete (light)	1300-1500	0.005-0.02	3.8	0.13
Concrete (porous)	500-600	0.01	1.5-2	0.13
Copper	8900	0.002	46-125	0.35
Cork	120-125	0.13-0.17	0.025	0.0001
Glass	2500-2600	0.0006-0.0013	60-70	0.2-0.21
Granite	2690	0.001	44.5	0.28
Gypsum board	650-1200	0.006-0.03	2.3-29	0.13-0.3
Lead	11000-11300	0.015-0.02	11.6-13.8	0.40-0.43
Lexan [f]	1200	0.015	1.80	0.38-0.40
Marble	2800	0.001	51.5	0.25-0.26
Masonry block	1100	0.006-0.007	10.5	0.09-0.10
(0.15 m = 6 in)				
Plaster, solid	1700	0.005-0.10	29.2	0.30
Plexiglas [g]	1150	0.020-0.04	3.4-5.6	0.40
Plywood	600	0.01-0.040	5-12	0.40
Polyethylene [h]	935	0.010	0.42	0.34-0.35
Pyrex [i]	2280-2300	0.004	58.4	0.23-0.24
Rubber (hard) [j]	950-1250	0.080	1.7-1.9	0.40-0.50
Sand, dry [k]	1500	0.06-0.12	0.03	0.20-0.45
Steel [l]	7700-7850	0.0001-0.01	186-210	0.27-0.31
Tin	7280	0.002	4.4	0.39
Wood (oak, along	700-1000	0.008-0.01	10.9-11.2	0.15
grain)	640	0.020	9-13.4	0.15
Wood (pine) [m]	7130-7140	0.0002-0.0003	13.1	0.33
Zinc				

Table notes

[a] The critical frequency f_c of the plate can be obtained from the following expression: $f_c = M_s f_c / M_s$, where the frequency product: $M_s f_c = \sqrt{3}\, c^2 \rho / \pi c_L$ (unit: Hz-kg/m^2) and $M_s = \rho h$ (unit kg/m^2). The sound speed in plate is found from: $c_L = \{E/\rho(1-\mu^2)\}^{1/2}$.

[b] The material damping factor of metals can increase with strain amplitude, but the damping factors of plastics and rubbers can approximately be independent of strain amplitude up to strains of unity. The damping factors of some flowing or creeping materials can also vary with temperature and frequency.

[c] The second value concerns Aluminum type 2014. The Poisson's ratio of aluminum can also be 0.345.

[d] The second value concerns brass (red). Some kind of brasses have E = 125 GPa.

[e] For *in-situ* normal concrete used in composite structures, dynamic E = 30-38 GPa. For lightweight concrete (density of 1800 kg/m^3), dynamic E =22 GPa. For reinforced concrete, it is recommended that ρ = 2400 kg/m^3, while for concrete without reinforcement ρ = 2300 kg/m^3. Concrete type K 250, E =26 GPa; concrete type K 350, E =31.5 GPa. Poisson's ratio, μ = 0.3, seems to be reasonable for many types of concrete.

[f] Trade name for polycarbonate (plastic) sheet.

[g] Trade name for plastic sheets of various thickness used in residential and commercial glazing. Some types of Plexiglas or Lucite has η = 0.002.

[h] The most common form of plastic (e.g., in plastic bags).

[i] A brand name for heat-resistant glass.

[j] For rubber (small strain), E = 0.01-0.1 GPa. The dynamic E of rubber differs from its static E due to elasticity. However for a very hard rubber that behaves similar to a rigid material, the two values are approximately equal.

[k] In sand as well as plaster, the parameters in table are very rough as they depends on grading, stress history, etc. The same thing concerns concrete and similar structures.

[l] The results applies approximately to iron. The loss factor of structures of these materials are sensitive to construction techniques and edge conditions.

[m] E = 9 GPa is for pine (along grain). For wood waste material bonded with plastic of density 23 kg/m^3, ρ = 750 kg/m^3, η = 0.005-0.01.