Modal analysis for floors in lightweight buildings

Lars-Göran Sjökvist *
Engineering Acoustics
Lund University
Box 118
SE-221 00 Lund
Sweden

Jonas Brunskog †
Acoustic Technology
Technical University of Denmark
Ørsteds Plads
DK-2800 Kgs. Lyngby
Denmark

ABSTRACT

High rise buildings built with wooden framing systems is of major interest for several reasons. It is said to be environmental friendly material, the production cost might be lower and the weight of the house is very low. The development of wooden housing is faster than the development of acoustical prediction methods for those houses. The calculation standard EN 12354 is under evaluation since it cannot include most of the wooden houses that are built. It is important during such a work to have a great understanding of the acoustical behaviour for the wooden houses. The floors in lightweight constructions usually consist of plates that are stiffened by beams and by the dividing walls. In this study the wave equation for a plate is expanded by Fourier series and an analytical solution in terms of the eigenmodes of the entire system is presented. The studied system consists of one lightweigt floor that is simply supported at its boundaries. Preliminary results show that the vibration level is directed by the presence of the beams.

1 INTRODUCTION

The modern human have high demands on residential qualities. You shall at one time watch a movie and feel like you are at the cinema while next evening you like to quietly read a book. In multi-family houses those demands give rise to high requirements for sound insulation. No house will today be considered comfortable if the noise from neighbours is disturbing.

Sound insulation between apartments often have to be estimated before the actual building is built. For houses built with lightweight technique this is today mostly done by method of best practise. That method works rather well but have discrepancies when new circumstances appear. Details and sizes of buildings change during development of the buildings and this yield a bit of uncertainty for the sound insulation. A reliable tool for predicting the sound insulation would be valuable. When designing lightweight structures, as wood constructions with boards, such a tool cannot be found. The prediction models of lightweight structures are still under development.

To improve calculation methods, more knowledge about the sound propagation in the lightweight systems is needed. The present study aims at gaining knowledge of rib stiffened plate by using Fourier sinus series. From the model the vibration level of the floor is examined and evaluated by means of attenuation rate, vibration distribution and expected uncertainties.

Studies on wooden floor with use of Fourier series have been made by Chung and Fox [**?**]. They model a structure with two plates, beams and a cavity in their studies and the model show good agrement with experiments made by Emms et. al. [1]. Also Nightingale and Bosmans used Fourier series expansion for a ribbed plate [2]. They used the expansion on smaller plates that was put together in order to have a system that simulates the beam enforced plate. The

*Email address: lars-goran.sjokvist@acoustics.lth.se
†Email address: jbr@oersted.dtu.dk

calculated input mobility showed that the distance between a beam and the driving point was what mostly effected the mobility.

The present model is built from the bending wave equation for a plate that is simply supported. Beams influence the vibrations on the plate by means of reaction forces and moments. Those forces is a result from the bending and torsion equations for the beams. The plate is also excited by a harmonic point force. The pressure from the surrounding air is not included which probably lead to less attenuation than if it was included. To give more clarity, the harmonic component, $e^{i\omega t}$, is omitted in all equations.

2 THEORY

A plate that is stiffened by beams can be described by the inhomogeneous bending wave equation as

$$\mathcal{S}w = p_e - p_M(x,y) - p_F(x,y) \tag{1}$$

where $\mathcal{S}$ is the bending wave operator for the plate, w is the displacement and p_e is the excitation pressure. The remaining parameters are reaction pressures from the beams, p_M of moments from the beams, p_F of forces from the beams. The bending wave operator for a plate is from e.g. Cremer et. al.[3, p.286]

$$\mathcal{S} = B\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 - \rho h \omega^2 \tag{2}$$

where B is the bending stiffness for a plate, ρ is the plate density, h is the thickness of the plate and ω is the radian frequency. The bending stiffness for a plate is

$$B = \frac{h^3 E}{12(1-\mu^2)} \tag{3}$$

where E is the modulus of elasticity and μ is the Poisson ratio.

The whole system is assumed to have Dirichlet boundary conditions and one can thereby assume that the displacement have the form

$$w(x,y) = \sum_{n=1}^{\infty} c_{m,n}\Psi_{m,n}(x,y) \tag{4}$$

where the modal chapes, Ψ , is sinusoidal and the constants, c, remains to establish.

First, merge equation 4 into the left hand side of equation 1. Then change the order between summation and derivation. And finally perform the derivations. The result will be

$$\sum_{m,n=1}^{\infty} c_{m,n}\left[B\left(\Omega_m^2 + \Omega_n^2\right)^2 - \rho h \omega^2\right]\Psi_{m,n}(x,y) = p_e - p_M(x,y) - p_F(x,y) \tag{5}$$

The following notation is used in this proceeding

$$\Omega_m = m\frac{\pi}{L_x}\ , \Omega_n = n\frac{\pi}{L_y}\ , \Omega_p = p\frac{\pi}{L_x}\ , \Omega_q = q\frac{\pi}{L_y}, \tag{6}$$

where L_y and L_x are the length of the plate in x and y direction respectively and m, n, p and q is integers. Note that Ω_m and Ω_p are related to L_x while Ω_n and Ω_q are related to L_y.

2.1 The forces that acts on the plate

In general the forces on the plate will be developed into sine series in the form

$$p(x,y) = \sum_{m,n}^{\infty} d_{m,n} \Psi_{m,n}(x,y). \tag{7}$$

The Fourier coefficients, $d_{m,n}$, is then known as

$$d_{m,n} = \frac{4}{L_x L_y} \iint_{0}^{L_x,L_y} p(x,y) \Psi_{m,n}(x,y) \mathrm{d}x \mathrm{d}y \tag{8}$$

In the following this will be employed for each type of the pressures that appear on the right hand side of equation 1.

2.2 The impact force

The model will use a time harmonic point force with the amplitude Q as the impact on the system. The point of impact is denoted (x_0, y_0). This leads to a pressure that is described as

$$p_e = Q\delta(x - x_0)\delta(y - y_0) \tag{9}$$

The Fourier coefficients that is related to this function is

$$e_{m,n} = \frac{4Q}{L_x L_y} \Psi_{m,n}(x_0, y_0) \tag{10}$$

2.3 Vertical line reaction forces

The line reaction force at beam r have the position $x = x_r$. It is expanded into the sinus series as

$$p_{F,r}(x,y) = \sum_{p,q=1}^{\infty} f_{p,q} \Psi_{p,q}(x,y) \tag{11}$$

$f_{p,q}$ is then known from equation 8. We know that the vertical reaction pressure is a function from the vertical reaction force at the line $x = x_r$,

$$p_{F,r}(x,y) = F_r(y)\delta(x - x_r) \tag{12}$$

The vertical line reaction force, F_r is a function of the the displacement in the plate and the bending wave equation for the beam that is attached to the plate as

$$F_r = B_f \frac{\partial^4 w}{\partial y^4} - m' \omega^2 w \tag{13}$$

where B_f is the Bending stiffness of the beam and m' is the mass per meter for the beam. Using the fourier expanded displacement together with the two equations above will yield the following expression for the pressure from one vertical line reaction force;

$$p_{F,r} = \sum_{m,n=1}^{\infty} c_{m,n} \left(B_f \Omega_n^4 - m' \omega^2 \right) \Psi_{m,n} \delta(x - x_r) \tag{14}$$

Inserting this into the integral for the Fourier coefficients will yield many orthogonal functions in the y direction and they are all zero. For $q = n$ will though the integration in the y-direction result in the constant times $L_y/2$, as usual for orthogonal sinus functions. Therefor is

$$f_{p,q} = \frac{4}{L_x L_y} \frac{L_y}{2} \int_0^{L_x} \sum_{m=1}^{\infty} c_{m,q} \left(B_f \Omega_q^4 - m' \omega^2 \right) \Phi_{m,p}(x) \delta(x - x_r) \mathrm{d}x \tag{15}$$

where $\Phi_{m,p}(x) = \sin(\Omega_m x) \sin(\Omega_p x)$ shows that the whole sine series across the beams have to be considered when calculating one Fourier coefficient for the reaction force. Performing the integration means only to evaluate that $0 < x_r < L_x$. This is always true since the beam should be connected to the plate. Thereby is the value of the integral the same as the value of the integrand, except the functional, at $x = x_r$. The fourier coefficient is now

$$f_{p,q} = \frac{2}{L_x} \sum_{m=1}^{\infty} c_{m,q} \left(B_f \Omega_q^4 - m' \omega^2 \right) \Phi_{m,p}(x_r) \tag{16}$$

2.4 The moment reaction

The fourier coefficients will here be derived for the sine series that belongs to the r line moment reaction force. The reaction force is expanded into a sine series

$$p_{M,r} = \sum_{p,q=1}^{\infty} d_{p,q} \Psi_{p,q}(x, y) \tag{17}$$

We want to know the fourier coefficients $d_{m,n}$. Those are known for a sinus expansion from equation 8. The moment reaction pressure is modelled as

$$p_{M,r} = -M_r \delta'(x - x_r) \tag{18}$$

and the rotational stiffness of the beam is

$$M_r = T \frac{\partial^3 w}{\partial x \partial^2 y} + \Theta \omega^2 \frac{\partial w}{\partial x} \tag{19}$$

The two equations above will be used together with the expanded displacement to yield

$$p_{M,r} = -\sum_{m,n=1}^{\infty} c_{m,n} \left(\Theta \omega^2 \Omega_m - T \Omega_m \Omega_n^2 \right) \cos(\Omega_m x) \sin(\Omega_n y) \delta'(x - x_r) \tag{20}$$

When inserting this into equation 8 the integration in y direction will yield the same constant as for the vertical line force above. The integration in x will have a value only when the dirac distribution is zero since $\int f(x)\delta'(x-a)dx = -f'(a)$. The result yield that

$$d_{p,q} = -\frac{2}{L_x}\sum_{m=1}^{\infty} c_{m,q}\left(\Theta\omega^2\Omega_m - T\Omega_m\Omega_q^2\right)\left(-\Omega_m\Phi_{m,p}(x_r) + \Omega_p\Upsilon_{m,p}(x_r)\right) \tag{21}$$

where $\Upsilon_{m,p}(x) = \cos(\Omega_m x)\cos(\Omega_p x)$.

2.5 Solving the Fourier coefficients

The fourier coefficients of the displacement, $c_{m,n}$ is the unknown in the equations above that is needed. To have the value for these we first consider the wave equation in the form with all parameters expanded into fourier series. Then using the uniqueness of the Fourier coefficients will yield

$$c_{p,q}\left[B\left(\Omega_p^2+\Omega_q^2\right)^2 - \rho h\omega^2\right] = e_{p,q} + \sum_r \left(d_{p,q} + f_{p,q}\right) \tag{22}$$

Now the Fourier coefficients for the reaction forces includes expressions that involves $c_{m,n}$. Since one Fourier coefficient for a reaction force contains a sum over several $c_{m,n}$:s there will arise an equation system with the same number of equations and unknown coefficients. The sum over r above expresses beams that are added to the plate.

One can here use the formulations that is derived for the fourier coefficients on the right hand side of equation 22 and also $g_{p,q} = \left[B\left(\Omega_p^2+\Omega_q^2\right)^2 - \rho h\omega^2\right]$. Then the equation system to solve is

$$\begin{aligned} c_{p,q}g_{p,q} = {} & \frac{4Q}{L_xL_y}\Psi_{p,q}(x_0,y_0) - \\ & \sum_r \frac{2}{L_x}\sum_{m=1}^{\infty} c_{m,q}\left(\Theta\omega^2\Omega_m - T\Omega_m\Omega_q^2\right)\left(-\Omega_m\Phi_{m,p}(x_r) + \Omega_p\Upsilon_{m,p}(x_r)\right) \\ & + \sum_r \frac{2}{L_x}\sum_{m=1}^{\infty} c_{m,q}\left(B_f\Omega_q^4 - m'\omega^2\right)\Phi_{m,p}(x_r) \end{aligned} \tag{23}$$

This can be a bit rewritten to see the structure of the equations more easily as

$$\begin{aligned} c_{p,q}g_{p,q} - e_{p,q} = {} & \\ & \frac{2}{L_x}\sum_{m=1}^{\infty} c_{m,q}\sum_r\Big[-\left(\Theta\omega^2\Omega_m - T\Omega_m\Omega_q^2\right)\left(-\Omega_m\Phi_{m,p}(x_r) + \Omega_p\Upsilon_{m,p}(x_r)\right) \\ & + \left(B_f\Omega_q^4 - m'\omega^2\right)\Phi_{m,p}(x_r)\Big] \end{aligned} \tag{24}$$

For one q it is possible to put up a system of equations as

$$\mathbf{c} = \mathbf{e} + \mathbf{A}\mathbf{c} \tag{25}$$

where vectors and matrix are of the form

$$\mathbf{c} = \begin{bmatrix} c_{1,q} \\ c_{2,q} \\ etc. \end{bmatrix} \quad \mathbf{e} = \begin{bmatrix} e_{1,q}/g_{1,q} \\ e_{2,q}/g_{2,q} \\ etc. \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} a_{1,1}/g_{1,q} & a_{1,2}/g_{1,q} & etc. \\ a_{2,1}/g_{2,q} & a_{2,2}/g_{2,q} & etc. \\ etc. & etc. & etc. \end{bmatrix} \tag{26}$$

and

$$a_{p,m} = \sum_r \left[- \left(\Theta\omega^2\Omega_m - T\Omega_m\Omega_q^2\right)\left(-\Omega_m\Phi_{m,p}(x_r) + \Omega_p\Upsilon_{m,p}(x_r)\right) + \left(B_f\Omega_q^4 - m'\omega^2\right)\Phi_{m,p}(x_r)\right] \tag{27}$$

The system above have the solution

$$\mathbf{c} = (\mathbf{I} - \mathbf{A})^{-\mathbf{1}}\,\mathbf{e} \tag{28}$$

where $\mathbf{I}$ is an identity matrix. Hereby is the fourier coefficients for the displacement solved.

2.6 Calculation of the vibration level

The displacements over the plate is known from equation 4 when the fourier coefficients have been calculated. The velocity level can then be calculated from the displacements as

$$L_v = 10\log\frac{|\mathrm{i}\omega w/\sqrt{2}|^2}{v_{ref}^2} \tag{29}$$

where $v_{ref} = 5 \cdot 10^{-8}$ is the reference velocity.

3 PRELIMINARY RESULTS

The vibrations for a plate that measures 4 times 4 meter and have beams with 0.5 m distance was calculated and some results from those calculations will here be shown. The first beam was placed at $x_1 = 0.25$ m. The summations was truncated after 50 terms. Damping is introduced by means of complex bending stiffness. The system also have the following properties: $\omega = 500 \cdot 2\pi$ rad, $(x_0, y_0) = (1, 1)$ m, $(L_x, L_y) = (4, 4)$ m, $Q = 10$ N, $\rho h = 10.8$ kg/m^2, $\mu = 0.3$, $B = 2800(1+\mathrm{i}\eta)$ $\eta = 0.02$, $B_f = 2.87\mathrm{e}6(1+\mathrm{i}\eta)$, $\Theta = 0.073$, $T = 3.72e5$.

The vibration pattern is shown in figure 1. One might here notice that the vibrations are fairly much affected by the attached beams. The vibration level within a bay is not varied as much as it is in the direction across the beams. The attenuation seems to be very directional: In a homogenous plate one would expect too se a radial attenuation from the point of excitation and at some distance would the vibration level not attenuate any more due to reflections from the plates edges. Here the attenuation is weak within each bay but strong across the beams, nearly leading to a one dimensional attenuation behaviour. This is in accordance with measures presented earlier [4].

inspired by figure 1 the behaviour in the direction across the beams is examined a bit closer in figure 2. Here the mean vibration levels at each x-position as well as for each bay is plotted. It is shown for this example case that the attenuation in this direction is strong. Pherhaps the attenuation is strongest near the source and somewhat weaker a few bays away. Nightingale and Bosmans have already concluded that the attenuation is strongest across the first beam [2]. They though suggest that one needs to account for longitudinal waves in the model to discover

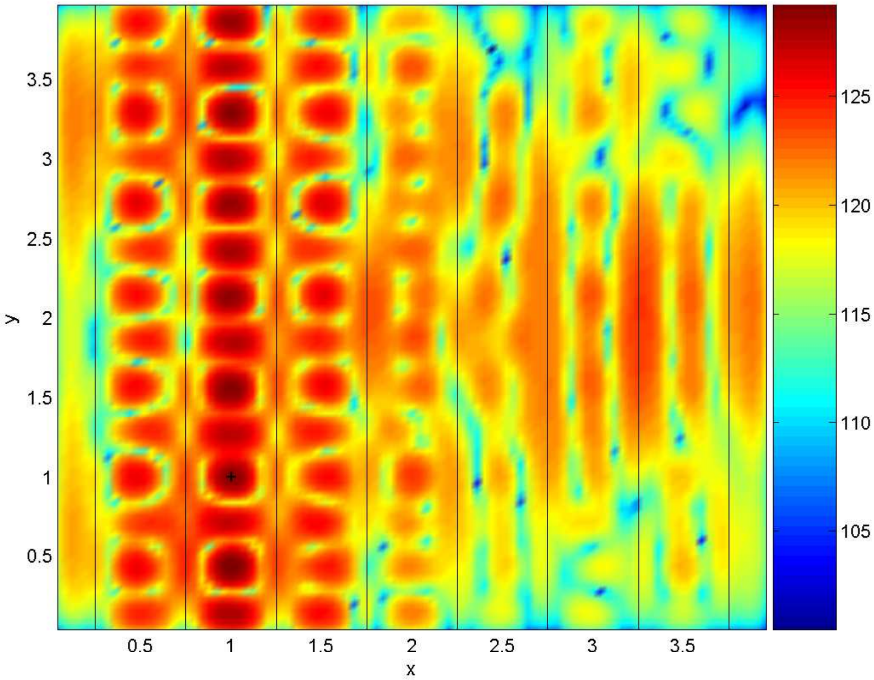

Figure 1: The plate seen from above. The position of the beams is marked with black lines and the excitation point is marked with a '+' sign. The vibration level is displayed by colour, 'colder colour is less vibration', see the colour bar at the right hand side of the figure.

such attenuation behaviour.

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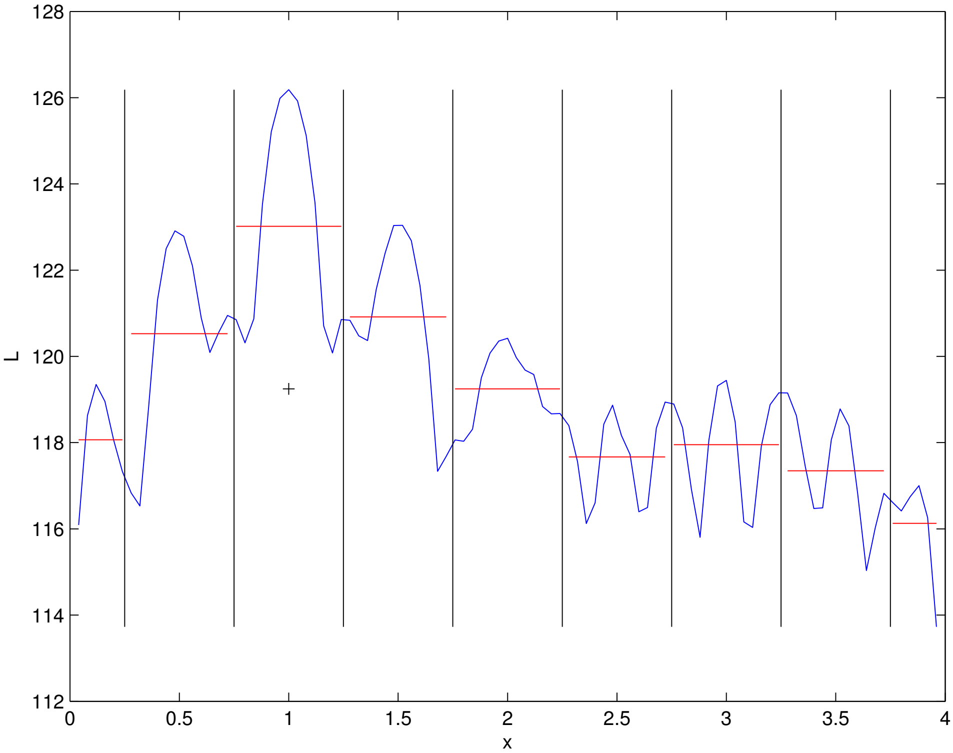

Figure 2: Mean vibrations of the plate in the direction across the beams. The blue line is the mean at each x-position and the red lines is the mean in each bay. The position of the beams is marked with vertical black lines and the excitation point is marked with a '+' sign.