Formulae – *Ljud i byggnad och samhälle* (VTAF01)

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I) Some logarithmic rules:

$$y = \log x \quad \Leftrightarrow \quad 10^y = x$$
$$\log(10^x) = x$$
$$10^{\log x} = x$$
$$10^{x+y} = 10^x \cdot 10^y$$
$$\frac{10^x}{10^y} = 10^{x-y}$$

$$\log(x \cdot y) = \log x + \log y$$
$$\log\left(\frac{x}{y}\right) = \log x - \log y$$
$$\log(x^n) = n \cdot \log(x)$$
$$10^0 = 1$$
$$\log 1 = 0$$

II) Fundamental acoustic definitions

Sound pressure

- **One-dimensional plane sound field:** $p(x,t) = \hat{p}\cos\left(\omega t - \frac{\omega}{c}x + \varphi\right) = \hat{p}e^{i(\omega t - kx + \varphi)}$

- Effective value (**RMS**, Root Mean Square) for **sound pressure** in a point: $\tilde{p} = \sqrt{\frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} p^2(x,t)dt}$

NOTE: For a harmonic wave: $\tilde{p} = \frac{\hat{p}}{\sqrt{2}}$

Sound pressure level SPL

- **Sound pressure level:** $L_p = 10\log\left(\frac{\tilde{p}^2}{p_{ref}^2}\right)$, where $p_{ref} = 2 \cdot 10^{-5}$ Pa

(p_{ref}: approximately the quietest sound a young undamaged human hearing can detect at 1000 Hz)

- **Equivalent sound level:** $L_{eq,T} = 10\log\left(\frac{1}{T}\int_0^T \frac{p^2(t)}{p_{ref}^2}dt\right) = 10\log\left(\frac{1}{T}\int_0^T 10^{L_p(t)/10}dt\right)$

- **Summing** of **two** sound **sources** (if uncorrelated the last term vanishes): $\tilde{p}_{tot}^2 = \tilde{p}_1^2 + \tilde{p}_2^2 + \frac{2}{\Delta t}\int_{t_0}^{t_0+\Delta t} p_1(t)p_2(t)dt$

SPL difference (dB) between 2 sources	0	1	2	3	4	5	6	7	8	9	10
Added dB to the highest SPL	3	2,54	2,12	1,76	1,46	1,19	0,97	0,79	0,64	0,51	0,41

- **Summing** of *N* **uncorrelated sources:** $L_{p,tot} = 10\log\left(\sum_{n=1}^{N} 10^{L_{p,n}/10}\right)$

- **Phon curves:**

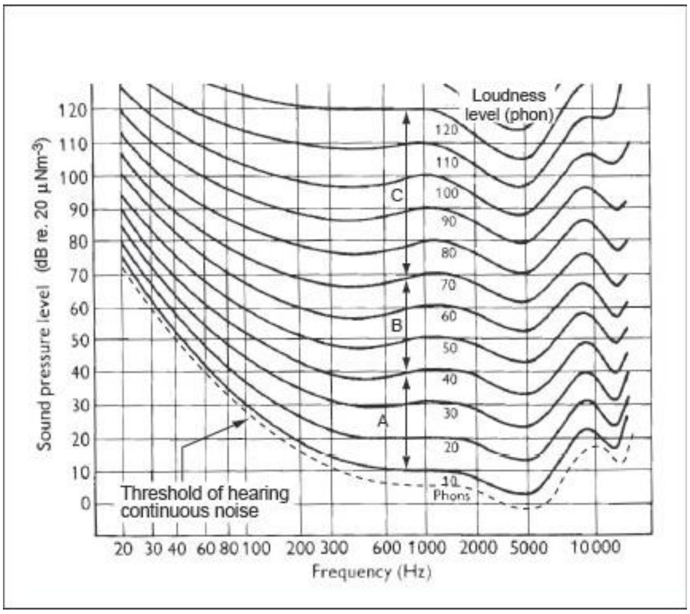


Figure 1 – Phon curves

Sound intensity

- The **sound energy** Π and **sound intensity** *I* is $\left.\begin{array}{l}\Pi(t) = F(t)\cdot v(t)\\ I(t) = \frac{\Pi(t)}{S}\end{array}\right\} \Rightarrow I(t) = p(t)\cdot v(t)$

- In analogy to the sound pressure level, the **sound power level** and **sound intensity level** are calculated (in **decibels**) from the respective time mean value according to

$L_\Pi = 10\log\left(\frac{\bar{\Pi}}{\Pi_{ref}}\right)$ and $L_I = 10\log\left(\frac{\bar{I}}{I_{ref}}\right)$, where $\Pi_{ref} = 10^{-12}$ W and $I_{ref} = 10^{-12}$ W/m^2.

The time mean values are: $\bar{\Pi} = \frac{S}{T}\int_0^T p(t)\cdot v(t)dt$ and $\bar{I} = \frac{1}{T}\int_0^T p(t)\cdot v(t)dt$

- For a wave (free-field) propagating in the positive *x*-direction, the previous integral yields: $\bar{I} = \tilde{p}^2 / \rho c$, with *c* being the speed of sound in m/s and $Z = \rho c$ the specific acoustic impedance.

The speed of sound in air varies with temperature and density according to:

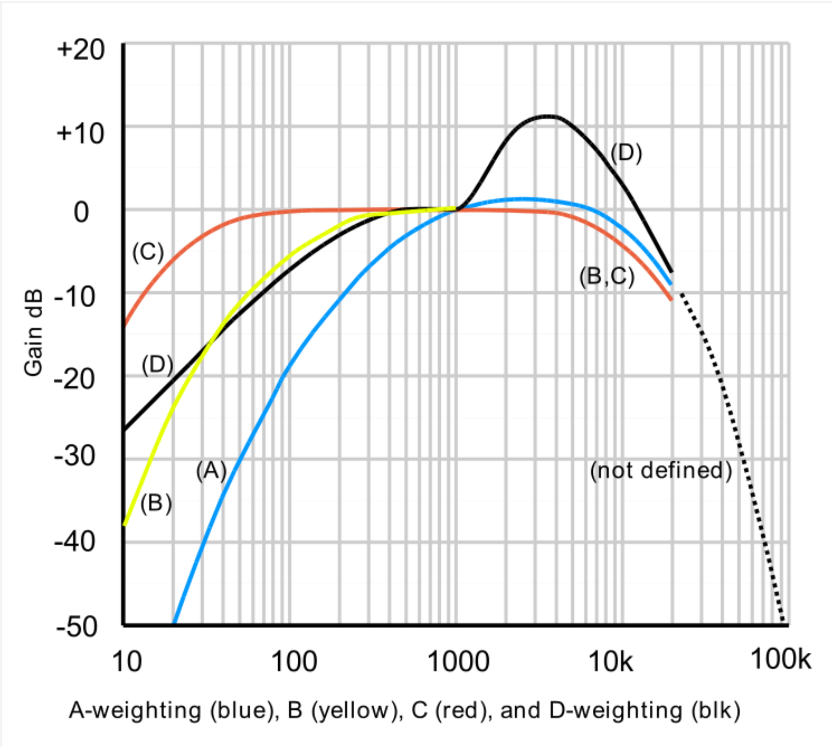
$$c_{air} = \sqrt{\frac{\gamma P_0}{\rho_{air}(T = 0^\circ)}} \left(1 + \frac{T_{air}[^\circ C]}{2 \cdot 273}\right) = 331.4 \cdot \left(1 + \frac{T_{air}[^\circ C]}{2 \cdot 273}\right)$$

II.1) Frequency bandwidths, weighted sound pressure level

II.1.a) Octave bands and third octave bands: $L_{weighed} = 10\log\left(\sum 10^{(L_n + weighing)/10}\right)$

Mittfrekvens f_m (Hz)	Tersfilter $f_u - f_ö$ (Hz)	Oktavfilter $f_u - f_ö$ (Hz)	Mittfrekvens f_m (Hz)	Tersfilter $f_u - f_ö$ (Hz)	Oktavfilter $f_u - f_ö$ (Hz)
50	44,7 – 56,2		800	708 – 891	
63	56,2 – 70,8	44,7 – 89,1	**1000**	891 – 1120	708 – 1410
80	70,8 – 89,1		1250	1120 – 1410	
100	89,1 – 112		1600	1410 – 1780	
125	112 – 141	89,1 – 178	**2000**	1780 – 2240	1410 – 2820
160	141 – 178		2500	2240 – 2820	
200	178 – 224		3150	2820 – 3550	
250	224 – 282	178 – 355	**4000**	3550 – 4470	2820 – 5620
315	282 – 355		5000	4470 – 5620	
400	355 – 447		6300	5620 – 7080	
500	447 – 562	355 – 708	**8000**	7080 – 8910	5620 – 11200
630	562 – 708		10000	8910 – 11200	

II.1.b) Weighed sound level:



Frekvens [Hz]	A-filter [dB]	B-filter [dB]	C-filter [dB]
10	-70.4	-38.2	-14.3
12.5	-63.4	-33.2	-11.2
16	-56.7	-28.5	-8.5
20	-50.5	-24.2	-6.2
25	-44.7	-20.4	-4.4
31.5	-39.4	-17.1	-3.0
40	-34.6	-14.2	-2.0
50	-30.2	-11.6	-1.3
63	-26.2	-9.3	-0.8
80	-22.5	-7.4	-0.5
100	-19.1	-5.6	-0.3
125	-16.1	-4.2	-0.2
160	-13.4	-3.0	-0.1
200	-10.9	-2.0	0
250	-8.6	-1.3	0
315	-6.6	-0.8	0
400	-4.8	-0.5	0
500	-3.2	-0.3	0
630	-1.9	-0.1	0
800	-0.8	0	0
1000	0	0	0
1250	0.6	0	0
1600	1.0	0	-0.1
2000	1.2	-0.1	-0.2
2500	1.3	-0.2	-0.3
3150	1.2	-0.4	-0.5
4000	1.0	-0.7	-0.8
5000	0.5	-1.2	-1.3
6300	-0.1	-1.9	-2.0
8000	-1.1	-2.9	-3.0
10000	-2.5	-4.3	-4.4
12500	-4.3	-6.1	-6.2
16000	-6.6	-8.4	-8.5
20000	-9.3	-11.1	-11.2

Figure 2 - A-, B-, C- and D-weightings (10 Hz – 20 kHz).

II.2) One dimensional wave propagation

The equation of motion in fluids and solid media is

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial v}{\partial t}$$; p: pressure, ρ: specific mass, ν: fluid velocity

For fluids, the relation between pressure and particle velocity is

$$\frac{\partial p}{\partial t} = -\gamma P_0 \frac{\partial v}{\partial x}$$; γ = adiabatic index (γ_{air} =1.4)

For solid media we have the corresponding relation between force and displacement

$$F = -ES\frac{\partial u}{\partial x}$$; E: modulus of elasticity, S: surface

II.2.a) Waves in fluid media

- For a **longitudinal wave propagating in air** (one dimensional propagation in positive *x*-direction) expressed with sound pressure and particle velocity, respectively, as the field variables are:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \cdot \frac{\partial^2 p}{\partial t^2} = 0 \qquad \frac{\partial^2 v}{\partial t^2} = c^2 \cdot \frac{\partial^2 v}{\partial x^2}$$

where $c = \sqrt{\gamma P_0 / \rho}$ is the propagation speed for the pressure wave in the air, with P_0 being the atmospheric pressure. The general solution to the wave equation is

$$p(x,t) = p_+(t - x/c) + p_-(t + x/c)$$

The harmonic solution to the wave equation in complex form is

$$p(x,t) = \hat{p}_+ e^{i(\omega t - kx)} + \hat{p}_- e^{i(\omega t + kx)}$$

For physical interpretation, take the real part of the result, where $\hat{p}_+$ and $\hat{p}_-$ are the pressure amplitudes for the waves propagating in positive and in negative direction, respectively. ω is the angular frequency and $k = 2\pi/\lambda = \omega/c$ is the wave number. This yields the equality $c = f\lambda$

- **Specific acoustic impedanc**e is defined as $Z \equiv \frac{p}{v}$

- For a **wave propagating in air** (1-D propagation in positive *x*-direction) the acoustic impedance is

$$Z = \frac{p_+}{v_+} = \rho c$$

II.2.b) *Waves in solid media*

-For **longitudinal and waves** (infinite medium) and **quasi-longitudinal waves** (finite medium), their wave equations read, respectively:

$$E\frac{\partial^2 v_x}{\partial x^2} - \rho \cdot \frac{\partial^2 v_x}{\partial t^2} = 0 \qquad E'\frac{\partial^2 v_x}{\partial x^2} - \rho \cdot \frac{\partial^2 v_x}{\partial t^2} = 0$$

The **propagation speed for longitudinal waves** is $c_L = \sqrt{E/\rho}$, whereas for **quasi-longitudinal waves** it is, $c_{ql} = \sqrt{\frac{E'}{\rho}} = \sqrt{\frac{E}{\rho(1-\nu^2)}}$, ν being the Poisson's ratio of the material where the wave is propagating through. Particle displacement, particle velocity, strain or force can be used instead of pressure as a field variable in the wave equation.

- For **shear waves** the wave equation can be expressed using the transversal displacement w as:

$$\frac{\partial^2 w}{\partial x^2} - \frac{\rho}{G} \cdot \frac{\partial^2 w}{\partial t^2} = 0 \quad \text{where} \quad c_{sh} = \sqrt{\frac{G}{\rho}} \quad , \; G = \frac{E}{2(1+\upsilon)}$$ being the shear modulus of the material.

- For **bending waves** in beams and plates the wave equation in one dimension is

$$B\frac{\partial^4 w}{\partial x^4} + \rho S \frac{\partial^2 w}{\partial t^2} = 0 \quad \text{where} \quad B = E\frac{bh^3}{12}$$ for a rectangular cross section.

The propagation speed depends on the frequency (dispersive waves), i.e. $c_f(\omega) = \frac{\omega}{k} = \sqrt{\omega} \cdot \sqrt[4]{\frac{B}{\rho S}}$

III) Room acoustics

III.1) Inside a room: reflection and transmission

III.1.a) *Standing waves, eigenfrequencies*

At normal incidence on a hard surface the pressure function is

$$p(x,t) = 2\hat{p}_+ \cos(kx) \cdot e^{i\omega t}$$

and the particle velocity function

$$v(x,t) = 2\hat{v}_+ \sin(kx) \cdot e^{i(\omega t - \pi/2)}$$

The Helmholtz equation: $\frac{\partial^2 p(x)}{\partial x^2} + k^2 p(x) = 0$ has, in the one dimensional case with two hard boundary surfaces at x = 0 and x = L the solution

$$p(x,t) = B\cos\left(\frac{n\pi}{L}x\right) \cdot e^{i2\pi f_n t}$$

where f_n are the resonance frequencies $f_n = \frac{c}{\lambda_n} = \frac{c}{2}\frac{n}{L}$

For the three dimensional case with six hard boundary surfaces the eigenfrequencies are

$$f_{n_x,n_y,n_z} = \frac{c}{2}\sqrt{\left(\frac{n_x}{L}\right)^2 + \left(\frac{n_y}{B}\right)^2 + \left(\frac{n_z}{H}\right)^2}$$

n_x, n_y and n_z being indexes indicating the order mode in each direction of the room (*L*, *B* and *H*).

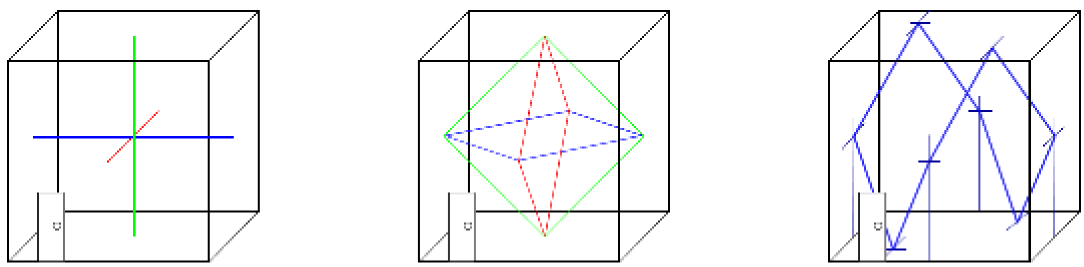
In transmission from one medium with wave impedance $Z_1 = \rho_1 c_1$ to another medium with wave impedance $Z_2 = \rho_2 c_2$ the transmission factor *t* and reflection factor *r* are given, respectively, as:

$$t = \frac{\hat{p}_t}{\hat{p}_i} = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1} = \frac{2Z_2}{Z_2 + Z_1} \qquad r = \frac{\hat{p}_r}{\hat{p}_i} = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

while the transmission coefficient τ and reflection coefficient ρ is

$$\tau = \frac{I_t}{I_i} = \frac{4\rho_2 c_2 \cdot \rho_1 c_1}{(\rho_2 c_2 + \rho_1 c_1)^2} = \frac{4Z_2 Z_1}{(Z_2 + Z_1)^2} \qquad \rho = \frac{I_r}{I_i} = \frac{|\rho_2 c_2 - \rho_1 c_1|^2}{(\rho_2 c_2 + \rho_1 c_1)^2} = \frac{|Z_2 - Z_1|^2}{(Z_2 + Z_1)^2}$$

III.1.b) Eigenfrequencies types:



Axial modes 1D   Tangential modes 2D   Oblique modes 3D

Figure 3 – Related geometry of eigenfrequencies
Source: Sengpielaudio

III.1.c) Reverberation time

Reverberation time can be estimated by using Sabine's formula. Sabine based his empirical formula on the sound decay of 60 dB (1/1000 SPL) after the abruptly end of a test tone (shot, shooting, etc.).

$$T(f) = 0{,}16.\frac{V}{A(f)}$$

with: *V*: volume of the room in square meters
A: effective absorption area of the room in square meter: $A(f) = \sum_{i=1}^{n} \propto_{i(f)}.S_i$

with S_i : surface of every absorption element
$\alpha_{i(f)}$: absorption coefficient of each element

III.2) Between two rooms: sound isolation and absorption

III.2.a) Airborne sound insulation

- **Sound reduction index R**: $R = 10\log\left(\frac{\Pi_i}{\Pi_t}\right) = 10\log\left(\frac{1}{\tau}\right)$

- **Measurement** of sound reduction index ***R***: $R(f) = L_s(f) - L_r(f) + 10\log\left(\frac{S_{wall}}{A(f)}\right)$

- **Combined reduction index**: $R = -10\log\left(\frac{1}{S_{total}}\left(S_1 10^{-R_1/10} + S_2 10^{-R_2/10} + \ldots\right)\right)$

- Slot **leakage**: $R = -10 \cdot \log\left(10^{-R/10} + \frac{S_s}{S_{total}}\right)$

- The **mass law** for a single leaf wall: $R_0(f) \approx 20\log\frac{\pi f m''}{2\rho c}$

- The **mass law** for a double leaf wall: $R_0(f) \approx 40\log\frac{\pi f m''}{\rho c}$

- **Coincidence frequency** (critical frequency): $f_c = \frac{c_0^2}{2\pi}\sqrt{\frac{m''}{BI}} = \frac{K}{t} = \frac{\sqrt{3}c_{air}^2}{\pi c_{LongWave} t}$

-1st **resonance frequency** (eigenfrequency) of a wall (dimensions *a* x *b*): $f_1 = \frac{\pi}{4\sqrt{3}} c_{LongWave} t\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$

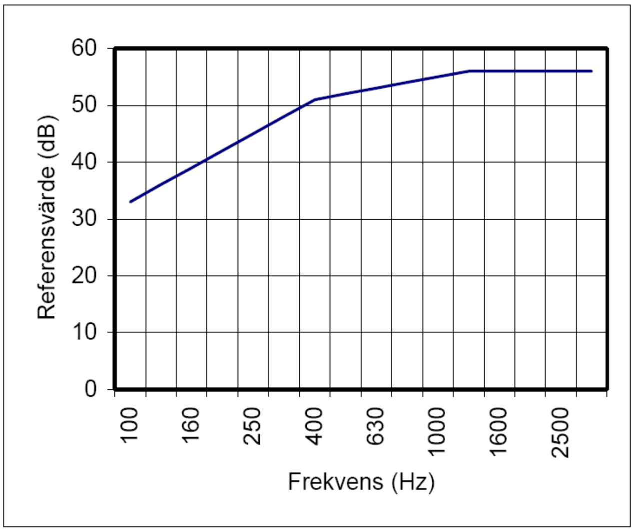
III.2.b) Impact sound insulation

- **Measurement** of **step sound level L**: $L_n(f) = L_r(f) + 10\log\left(\frac{A}{10}\right)$

III.2.c) ISO-REFERENCE CURVES

Airborne sound isolation

Measurement of sound reduction index: $R(f) = L_s(f) - L_r(f) + 10\log\left(\frac{S}{A(f)}\right)$

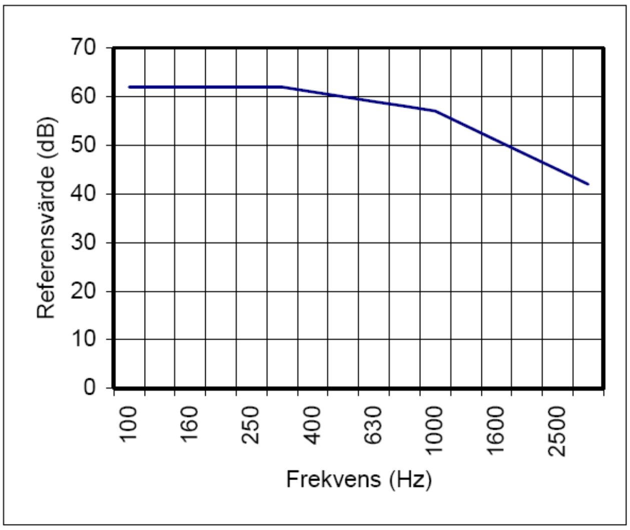


Frekvens, Hz	Ref.värde, dB
100	33
125	36
160	39
200	42
250	45
315	48
400	51
500	52
630	53
800	54
1000	55
1250	56
1600	56
2000	56
2500	56
3150	56

Figure 4 – Reference curve for air borne sound insulation (ISO 717-1)

Impact sound isolation

Measurement of **step sound level L:** $L_n(f) = L_r(f) + 10\log\left(\frac{A}{10}\right)$



Frekvens, Hz	Ref.värde, dB
100	62
125	62
160	62
200	62
250	62
315	62
400	61
500	60
630	59
800	58
1000	57
1250	54
1600	51
2000	48
2500	45
3150	42

Figure 5 – Reference curve for step sound insulation (ISO 717-2)