Våg 1 - lösningar

1. a) $\omega = 2\pi f = 3142$ rad/s

b) $T = \frac{1}{f} = 2$ ms

c) $\lambda = \frac{c}{f} = \frac{340}{500} = 0{,}68$ m

d) $k = \frac{2\pi}{\lambda} = 9{,}24$ rad/m

2. a) $c = 331{,}4\left(1+\frac{T}{2\cdot 273}\right) = 344$ m/s $\qquad Z = \rho c = 405$ Pa·s/m

b) $c = 331{,}4\,(1+0) = 331{,}4$ m/s $\qquad Z = \rho(T{=}0)c = 428$ Pa·s/m

$$\rho(T{=}0) = \frac{\gamma P_0}{331{,}4^2} = 1{,}29 \text{ kg/m}^3$$

c) $c = \sqrt{\frac{D}{\rho}} = 1480$ m/s $\qquad Z = \rho c = \sqrt{\rho D} = 1{,}48\cdot 10^6$ Pa·s/m

d) $c = \sqrt{\frac{E}{\rho}} = 5060$ m/s $\qquad Z = \rho c S = 77{,}6\cdot 10^3$ Pa·s·m

e) $c = \sqrt{\frac{E(1-\nu^2)}{\rho}} = 4830$ m/s $\qquad Z = \rho c S = 74{,}0\cdot 10^3$ Pa·s·m

f) $c = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{2(1+\nu)\rho}} = 3140$ m/s $\qquad Z = \rho c S = 48{,}1\cdot 10^3$ Pa·s·m

g) $c = \sqrt{\frac{E}{\rho}} = 4470$ m/s $\qquad Z = \rho c S = 34{,}9\cdot 10^3$ Pa·s·m

h) $c = \sqrt{\frac{E}{\rho}} = 3360$ m/s $\qquad Z = \rho c S = 1{,}24\cdot 10^6$ Pa·s·m

3.
$$\frac{\partial p}{\partial x} = -\hat{p}ike^{i(\omega t - kx)} \qquad \frac{\partial p}{\partial t} = \hat{p}i\omega e^{i(\omega t - kx)}$$

$$\frac{\partial^2 p}{\partial x^2} = +\hat{p}(ik)^2 e^{i(\omega t - kx)} = -\hat{p}k^2 e^{i(\omega t - kx)} \qquad \frac{\partial^2 p}{\partial t^2} = -\hat{p}\omega^2 e^{i(\omega t - kx)}$$

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\hat{p}k^2 e^{i(\omega t - kx)} - \frac{1}{c^2}\left(-\hat{p}\omega^2 e^{i(\omega t - kx)}\right) =$$

$$= \left(-k^2 + \frac{\omega^2}{c^2}\right)\hat{p}e^{i(\omega t - kx)} = \left(-k^2 + k^2\right)\hat{p}e^{i(\omega t - kx)} = 0 \qquad \text{v.s.v.}$$

5. $p(x,t) = \hat{p} e^{i(\omega t + kx)}$

$$\frac{\partial p}{\partial x} = -\varrho \frac{\partial v}{\partial t} \Rightarrow v = -\frac{1}{\varrho}\int \frac{\partial p}{\partial x} dt = -\frac{1}{\varrho}\int ik\hat{p} e^{i(\omega t + kx)} dt =$$

$$= -\frac{ik\hat{p}}{\varrho} \cdot \frac{1}{i\omega} e^{i(\omega t + kx)} = -\frac{1}{\varrho c}\hat{p} e^{i(\omega t + kx)}$$

$$Z_- = \frac{p_-}{v_-} = \frac{\hat{p} e^{i(\omega t + kx)}}{-\frac{1}{\varrho c}\hat{p} e^{i(\omega t + kx)}} = -\varrho c$$

6. $I(x,t) = p(x,t) \cdot v(x,t) = \frac{1}{\varrho c} p(x,t)^2$

$$\bar{I} = \frac{1}{T}\int_0^T \frac{1}{\varrho c} p(t)^2 dt = \frac{1}{\varrho c}\tilde{p}^2$$

$$L_I = 10 \log\left(\frac{\bar{I}}{I_{ref}}\right) = 10 \log\left(\frac{\tilde{p}^2}{\varrho c I_{ref}}\right) = 10 \log\left(\frac{\tilde{p}^2}{p_{ref}^2} \cdot \frac{p_{ref}^2}{\varrho c I_{ref}}\right) =$$

$$= L_p + 10 \log\left(\frac{p_{ref}^2}{\varrho c I_{ref}}\right) = L_p + 10 \log\left(\frac{(2 \cdot 10^{-5})^2}{405 \cdot 1 \cdot 10^{-12}}\right) = L_p - 0{,}05$$

≈ Ingen skillnad!

7. $u(x,t) = \hat{u} e^{i(\omega t - kx)}$

a) $v(x,t) = \frac{\partial u}{\partial t} = i\omega\hat{u} e^{i(\omega t - kx)} = \omega\hat{u} e^{i(\omega t - kx + \frac{\pi}{2})} = \hat{v} e^{i(\omega t - kx + \frac{\pi}{2})}$

$\hat{v} = 0{,}096\,m/s$

b) $p(x,t) = Z \cdot v(x,t) = \varrho c \hat{v} e^{i(\omega t - kx + \frac{\pi}{2})} = \hat{p} e^{i(\omega t - kx + \frac{\pi}{2})}$ $k = 2{,}35$ rad/m

$\hat{p} = 38{,}9$ Pa

c) $L_p = 10 \log\left(\frac{(\hat{p}/\sqrt{2})^2}{p_{ref}^2}\right) = 75{,}8$ dB

d) $\bar{I} = \frac{\tilde{p}^2}{\varrho c} = \frac{\hat{p}^2}{2\varrho c} = 1{,}87$ W/m²

e) $W = \int I dS = I \cdot S = 18{,}7$ mW

8. a) $\sigma(x,t) = -\frac{1}{S} F(x,t) = -\frac{F_{driv}}{S} e^{i(\omega t - kx)}$

b) $\varepsilon(x,t) = \frac{\sigma(x,t)}{E} = -\frac{F_{driv}}{SE} e^{i(\omega t - kx)}$

c) $u(x,t) = \int \varepsilon(x,t)\,dx = -\frac{1}{ik} \cdot \frac{-F_{driv}}{SE} e^{i(\omega t - kx)} = -\frac{F_{driv}\,c}{\omega SE} e^{i(\omega t - kx + \frac{\pi}{2})} =$

$= -\frac{F_{driv}}{\omega S\sqrt{E\rho}} e^{i(\omega t - kx + \frac{\pi}{2})} = \frac{F_{driv}}{\omega S\sqrt{E\rho}} e^{i(\omega t - kx - \frac{\pi}{2})}$ $\quad \hat{u} = 6{,}45 \cdot 10^{-4}\,m$

d) $v(x,t) = \frac{\partial u(x,t)}{\partial t} = \frac{-i\omega F_{driv}}{\omega S\sqrt{E\rho}} e^{i(\omega t - kx + \frac{\pi}{2})} = \frac{F_{driv}}{S\sqrt{E\rho}} e^{i(\omega t - kx)}$

$= 12{,}9\, e^{\cdots}$ mm

e) $a(x,t) = \frac{\partial v(x,t)}{\partial t} = -\frac{i\omega F_{driv}}{S\sqrt{E\rho}} e^{i(\omega t - kx)}$ $\quad \hat{a} = \frac{\omega F_{driv}}{S\sqrt{E\rho}} = 5{,}1\ m/s^2$

f) $Z = \frac{F}{V} = \frac{F_{driv}\, e^{i(\omega t - kx)}}{-\frac{F_{driv}}{S\sqrt{E\rho}} e^{i(\omega t - kx)}} = S\sqrt{E\rho} = 77{,}6 \cdot 10^3$ Pa·s·m

9. a) $c_f = \frac{\omega}{k} = \sqrt{\omega}\sqrt[4]{\frac{D_0}{\rho S}}$ $\quad f_c = \frac{\omega_c}{2\pi} = \frac{c_f^2}{2\pi\sqrt{\frac{Eh^2}{12\rho}}} = 6300$ Hz

b) $\lambda_c = \frac{2\pi}{k} = \frac{2\pi c_f}{\omega_c} = \frac{c_f}{f_c} = 54$ mm

c) $c_l = c_f = 6300$ Hz

d) $\lambda_l = \lambda_c = 54$ mm

e) Parallellt med ytan (strykande)

7e skall vara $18{,}7$ mW

8c) $u = 0{,}645 \cdot e^{\cdots}$

e $\quad \hat{a} = 0{,}258\ m/s^2$