

## Laboratory exercise instruction 2 – Acoustics VTAF05

The laboratory exercise is carried out by 2-3 students in the measurement room in the acoustic hall.

### Part 1. Impedance measurement in Kundt-tube

#### Theory

When a sound effect  $\Pi_i$  with frequency  $f$  incidents normal to a surface a certain amount of it is absorbed  $\Pi_a$  and the rest is reflected  $\Pi_r$ . The fraction that is absorbed is given by  $\alpha = \Pi_a / \Pi_i$  and the reflected is  $\tau = \Pi_r / \Pi_i$ .  $\alpha$  is used as a measure of the absorption efficiency of absorbing materials, where  $0 \leq \alpha \leq 1$ . The sound effect is proportional to the pressure squared,  $\Pi \propto \tilde{p}^2$

A Kundt-tube, or impedance tube, is used for measuring materials absorption. It sends a plane wave with a certain frequency from one end of the tube to the other, where we have a sample of the material of interest. The sample has a certain absorption coefficient  $\alpha$  and part of the sound pressure will be reflected. A microphone probe that can be moved in the length direction measures the sound pressure in the tube, which is the sum of the incident and reflected wave. The sum of them creates a standing wave with a maximum and a minimum. The sound pressure is given as a value of the voltage from the microphone.

By moving the microphone back and forth it can be identified where the incident and reflected wave interferes constructive (maximum pressure) and destructive (minimum pressure). If nothing gets reflected, i.e.  $\alpha = 1$ , we will see no difference between maximum and minimum, and if all is reflected, i.e.  $\alpha = 0$ , we will see a total annihilation at destructive interference. From the relation between maximum and minimum we can express the absorption coefficient.

The reflection at the hard surface produces a standing wave pattern, where the sound pressure can be expressed, with the amplitude of the incident wave  $\hat{p}_+$ , as

$$p(x,t) = 2\hat{p}_+ \cos(kx) \cdot e^{i\omega t},$$

where the wave number can be expressed with wavelength or frequency according to  $k = 2\pi / \lambda = 2\pi f / c$ .

At constructive interference  $\hat{p}_{tot} = \hat{p}_i + \hat{p}_r$  which means that  $\tilde{p}_{tot} = \tilde{p}_i + \tilde{p}_r$  and at destructive interference  $\hat{p}_{tot} = \hat{p}_i - \hat{p}_r$  which means that  $\tilde{p}_{tot} = \tilde{p}_i - \tilde{p}_r$ .

#### Material

- Kundt-tube.
- One absorption material sample
- Function generator
- Microphone on a cart with a length scale
- Voltmeter/frequency meter / oscilloscope

#### Execution

For the material sample, evaluate  $\alpha$  as a function of the following frequencies:  $f = 125, 250, 500$ ,

1k, 2k och 4kHz (octave bands). Start with setting the frequency on the function generator at the appropriate level. The frequency can be set exactly, and the distance between two nodes or two anti-nodes can be measured as well. What distance, expressed in wavelengths, does this correspond to? Use this relation to give an approximate speed of sound in air. Do this before inserting absorbents in the tube, start by measuring the standing wave without absorbent.

The absorption coefficient is calculated by means of the sound pressure in a node and the pressure in the anti-node. The microphone voltage amplitude is proportional to the amplitude of the sound pressure  $\hat{p}_{tot}$ . The following expressions are used:

$$F = \frac{\hat{p}_i - \hat{p}_r}{\hat{p}_i + \hat{p}_r} \text{ and } \alpha = \frac{2F}{(1 + F)^2}$$

where  $F$  is the ratio between maximum and minimum of the amplitude function of the standing wave in the tube,  $\hat{p}_i$  and  $\hat{p}_r$  are the amplitudes of the incoming and reflected wave, respectively. Give the absorption coefficient in a diagram as a function of frequency (logarithmic scale) for the absorbent.

## Part 2. Resonance frequency in plate

### Theory

For simple geometries and boundary conditions it is possible to find analytical eigenfrequencies and eigenmodes (or mode shapes) to beams and plates. In these two cases we will consider a homogeneous plate that is either simply supported or free-free. We will approximate the 2-dimensional plate with a 1-dimensional (wide) beam. Contemplate what kind of information that gets lost in this idealization.

For a transversal bending wave propagating in a beam the wave equation can be expressed as

$$B \frac{\partial^4 w}{\partial x^4} + \rho S \frac{\partial^2 w}{\partial t^2} = 0$$

where  $B$  is the bending stiffness of the beam, which for a rectangular cross section is

$$B = EI = E \frac{bh^3}{12}$$

$S = b \cdot h$  is the cross sectional area of the beam,  $E$  is Young's modulus or stiffness modulus in the longitudinal direction and  $\rho$  is the material's density.  $h$  is the height of the beam, i.e. the dimension of the beam in the direction of the motion. The propagation speed of a bending wave is dependent on frequency according to

$$c_f = \frac{\omega}{k} = \sqrt{\omega} \cdot \sqrt[4]{\frac{B}{\rho S}} = \sqrt{\omega} \cdot \sqrt[4]{\frac{Eh^2}{12\rho}}$$

Eigenfrequencies occurs in the beam if the wavelength of the wave, the mode shape, corresponds to the given boundary conditions.

For a **simply supported beam** (supports at both ends, not clamped) the first (lowest) eigenfrequency occurs when the wave length is half of the length of the beam,  $L = \lambda_1/2$ .

The eigenmodes of the beam in that case is a simple sine-function.

$$\Psi_n(x) = \sin(k_n x)$$

From this condition, the expression above and the simple relation between frequency and wave length

$$k_n = \frac{2\pi f_n}{c} = \frac{2\pi}{\lambda_n}$$

the first (lowest) eigenfrequency can be derived

$$f_1 = \frac{1}{2\pi} \cdot \left(\frac{\pi}{L}\right)^2 \cdot \sqrt{\frac{Eh^2}{12\rho}}$$

The following eigenfrequencies can be calculated using the expression

$$f_n = f_1 \cdot n^2$$

For a beam with a total length  $L$  **clamped in the middle** (at  $x = 0$ ) the eigenfrequency can be shown to be

$$f_1 = \frac{1}{2\pi} \cdot \left(\frac{0.597\pi}{L/2}\right)^2 \cdot \sqrt{\frac{Eh^2}{12\rho}}$$

And for higher frequencies,  $n = 2, 3, 4, \dots$ , the eigenfrequencies can be approximated with

$$f_n = \frac{1}{2\pi} \cdot \left(\frac{(n-1/2)\pi}{L/2}\right)^2 \cdot \sqrt{\frac{Eh^2}{12\rho}}$$

The eigenmodes are given by (with  $x = 0$  in the clamped end and with a beam length of  $L/2$ ):

$$\Psi_n(x) = (\sin(k_n L/2) - \sinh(k_n L/2))(\sin(k_n x) - \sinh(k_n x)) + (\cos(k_n L/2) - \cosh(k_n L/2))(\cos(k_n x) - \cosh(k_n x))$$

For a **free-free** beam (no supports, floating in space) the expression becomes slightly more complicated

$$f_1 = \frac{1}{2\pi} \cdot \left(\frac{1.506\pi}{L}\right)^2 \cdot \sqrt{\frac{Eh^2}{12\rho}}, n = 2, 3, 4, \dots$$

and for higher frequencies

$$f_n = \frac{1}{2\pi} \cdot \left(\frac{(n+1/2)\pi}{L}\right)^2 \cdot \sqrt{\frac{Eh^2}{12\rho}}$$

The eigenmodes becomes

$$\Psi_n(x) = \cosh(k_n x) + \cos(k_n x) - \gamma_n (\sinh(k_n x) + \sin(k_n x))$$

where  $\gamma_1 = 0.983$

which is enough to give us the first eigenmode. Further eigenmodes are given by the expression

$$\Psi_n(x) = \cosh(k_n x) + \cos(k_n x) - \gamma_n (\sinh(k_n x) + \sin(k_n x))$$

where a general expression for  $\gamma_n$  is given by

$$\gamma_n = \frac{\cos(\alpha_n) - \cosh(\alpha_n)}{\sin(\alpha_n) - \sinh(\alpha_n)}$$

and where  $\alpha_1 = 1.506\pi$ ,  $\alpha_2 = 2.5\pi$ ,  $\alpha_3 = 3.5\pi, \dots$

## Material

- Function generator
- Amplifier
- Shaker
- Plywood plate

## Execution

The plate can be considered free-free, resting on the single support of the shaker. Get acquainted with the equipment, the function generator, the amplifier and the shaker so you know their purposes. The function generator should, however, be operated by the lab supervisor since the equipment is sensitive.

The function generator can be started and one can attempt to perform a frequency sweep from 5 Hz and increase it slowly. Then set the frequency on the ones that were calculated in the preparation task.

When you find the lowest eigenfrequency, try to feel with your fingertips, or by straying dirt, where on the plate the vibrations are at its maximum and try to find places on the plate that don't move as much.

Proceed to the following eigenfrequencies and try to use your touch to identify corresponding eigenmodes. If you find "too many" eigenfrequencies, try to explain why they appear and where they come from.

Continue the frequency sweep into the audible range so that you can hear a clear tone. Explain how the sound that you hear is created.

## Laboratory report

Generally, a laboratory report should describe

- The theory behind the laboratory exercise,
- A description of what has been done and how it has been done
- The results that you have come up with
- An explanation, discussion or reflection of the results,
- A discussion of how accurate and reliable the measurements are, and of possible source of errors

A thought target group of the report is another student that shall be able to take the report as an instruction, go down in the laboratory, repeat the measurement and get the same results.

Try to discuss and explain the appearance of all plots you present. Illustrative pictures of the measurement are always in place!

The laboratory report shall be written on computer, printed and stapled. It shall be handed in Kristian's tray at the fourth floor next to the lunch room, at latest one week after the laboratory exercise have been carried out. The report is either passed or returned for completion.