Formulas – Acoustics VTAF05

Some logarithmic laws:

$y = \log x \quad \Leftrightarrow \quad 10^y = x$

$\log(10^x) = x$

$10^{\log x} = x$

$10^{x+y} = 10^x \cdot 10^y$

$\frac{10^x}{10^y} = 10^{x-y}$

$\log(x \cdot y) = \log x + \log y$

$\log\left(\frac{x}{y}\right) = \log x - \log y$

$\log(x^n) = n \cdot \log(x)$

$10^0 = 1$

$\log 1 = 0$

Fundamental acoustic definitions

1-dimensional plane sound field:

$$p(x,t) = A\cos\left(\omega t - \frac{\omega}{c}x + \varphi\right)$$

Effective value (rms) for sound pressure in a point:

$$\tilde{p} = \sqrt{\frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} p^2(t)dt}$$

Sound pressure level (sound level):

$$L_p = 10\log\left(\frac{\tilde{p}^2}{p_{ref}^2}\right)$$

, where $p_{ref} = 2 \cdot 10^{-5}$ Pa

Equivalent sound level:

$$L_{eq,T} = 10\log\left(\frac{1}{T}\int_0^T \frac{p^2(t)}{p_{ref}^2}dt\right) =$$

$$= 10\log\left(\frac{1}{T}\int_0^T 10^{L_p(t)/10}dt\right)$$

Weighed sound level:

$$L_{weighed} = 10\log\left(\Sigma 10^{(L_n + weighing)/10}\right)$$

Summing of two sound sources (if uncorrelated the last term vanishes):

$$\tilde{p}_{tot}^2 = \tilde{p}_1^2 + \tilde{p}_2^2 + \frac{2}{\Delta t}\int_{t_0}^{t_0+\Delta t} p_1(t)\, p_2(t)\, dt$$

Summing of *N* uncorrelated sources:

$$L_{p,tot} = 10\log\left(\sum_{n=1}^{N} 10^{L_{p,n}/10}\right)$$

Frekvens [Hz]	A-filter [dB]	B-filter [dB]	C-filter [dB]
10	-70.4	-38.2	-14.3
12.5	-63.4	-33.2	-11.2
16	-56.7	-28.5	-8.5
20	-50.5	-24.2	-6.2
25	-44.7	-20.4	-4.4
31.5	-39.4	-17.1	-3.0
40	-34.6	-14.2	-2.0
50	-30.2	-11.6	-1.3
63	-26.2	-9.3	-0.8
80	-22.5	-7.4	-0.5
100	-19.1	-5.6	-0.3
125	-16.1	-4.2	-0.2
160	-13.4	-3.0	-0.1
200	-10.9	-2.0	0
250	-8.6	-1.3	0
315	-6.6	-0.8	0
400	-4.8	-0.5	0
500	-3.2	-0.3	0
630	-1.9	-0.1	0
800	-0.8	0	0
1000	0	0	0
1250	0.6	0	0
1600	1.0	0	-0.1
2000	1.2	-0.1	-0.2
2500	1.3	-0.2	-0.3
3150	1.2	-0.4	-0.5
4000	1.0	-0.7	-0.8
5000	0.5	-1.2	-1.3
6300	-0.1	-1.9	-2.0
8000	-1.1	-2.9	-3.0
10000	-2.5	-4.3	-4.4
12500	-4.3	-6.1	-6.2
16000	-6.6	-8.4	-8.5
20000	-9.3	-11.1	-11.2

One dimensional wave propagation

The equation of motion in fluids and solid media is

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial v}{\partial t}$$

For fluids the relation between pressure and particle velocity is

$$\frac{\partial p}{\partial t} = -\gamma P_0 \frac{\partial v}{\partial x}$$

For solid media we have the corresponding relation between force and displacement

$$F = -ES \frac{\partial u}{\partial x}$$

The wave equation for longitudinal waves in one dimension expressed in sound pressure is

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \cdot \frac{\partial^2 p}{\partial t^2} = 0$$

where $c = \sqrt{\gamma P_0 / \rho}$ is the propagation speed for the pressure wave in air, $\gamma = 1.4$ and P_0 is the atmospheric pressure. In solid media the propagation speed for longitudinal waves is $c = \sqrt{E/\rho}$. Particle displacement, particle velocity, strain or force can be used instead of pressure as a field variable in the wave equation.

The general solution to the wave equation is

$$p(x,t) = p_+(t - x/c) + p_-(t + x/c)$$

The harmonic solution to the wave equation on complex form (for physical interpretation, take the real part of the result) is

$$p(x,t) = \hat{p}_+ \, e^{i(\omega t - kx)} + \hat{p}_- \, e^{i(\omega t + kx)}$$

where $\hat{p}_+$ and $\hat{p}_-$ are the pressure amplitudes for the waves propagating in positive and in negative direction, respectively. ω is the angular frequency and $k = 2\pi/\lambda = \omega/c$ is the wave number. This gives

$$c = f\lambda$$

Specific acoustic impedance is defined as

$$Z \equiv \frac{p}{v}$$

For a wave propagating in air (one dimensional propagation in positive *x*-direction) the acoustic impedance is

$$Z = \frac{p_+}{v_+} = \rho c$$

For shear waves the wave equation can be expressed using the transversal displacement *w*

$$\frac{\partial^2 w}{\partial x^2} - \frac{\rho}{G} \cdot \frac{\partial^2 w}{\partial t^2} = 0 \text{ where } c = \sqrt{\frac{G}{\rho}}$$

For bending waves in beams and plates the wave equation in one dimension is

$$B \frac{\partial^4 w}{\partial x^4} + \rho S \frac{\partial^2 w}{\partial t^2} = 0$$

where $B = E \frac{bh^3}{12}$ for a rectangular cross section and the propagation speed (of the phase)

$$c_f = \frac{\omega}{k} = \sqrt{\omega} \cdot \sqrt[4]{\frac{B}{\rho S}}$$

The sound energy Π and sound intensity I is

$$\left.\begin{aligned} \Pi(t) &= F(t) \cdot v(t) \\ I(t) &= \frac{\Pi(t)}{S} \end{aligned}\right\} \quad \Rightarrow \quad I(t) = p(t) \cdot v(t)$$

From the sound pressure, the sound pressure level is defined as

$$L_p = 10 \log \left(\frac{\tilde{p}^2}{p_{ref}^2} \right) \qquad \text{where } p_{ref} = 2 \cdot 10^{-5} \text{ Pa}$$

Sound effect level and sound intensity level are calculated from the respective time mean value according to

$$L_\Pi = 10 \log \left(\frac{\overline{\Pi}}{\Pi_{ref}} \right) \text{ and } L_I = 10 \log \left(\frac{\bar{I}}{I_{ref}} \right)$$

where $\Pi_{ref} = 10^{-12}$ W and $I_{ref} = 10^{-12}$ W/m^2. The time mean values are

$$\overline{\Pi} = \frac{S}{T} \int_0^T p(t) \cdot v(t) dt \text{ and } \bar{I} = \frac{1}{T} \int_0^T p(t) \cdot v(t) dt$$

For a wave propagating in the positive *x*-direction $\bar{I} = \tilde{p}^2 / \rho c$.

Reflection and transmission

At normal incidence on a hard surface the pressure function is

$$p(x,t) = 2\hat{p}_+ \cos(kx) \cdot e^{i\omega t}$$

and the particle velocity function

$$v(x,t) = 2\hat{v}_+ \sin\left(kx\right) \cdot e^{i(\omega t - \pi/2)}$$

Helmholtz equation

$$\frac{\partial^2 p(x)}{\partial x^2} + k^2 p(x) = 0$$

has in the one dimensional case with two hard boundary surfaces at $x = 0$ and $x = L$ the solution

$$p(x,t) = B \cos\left(\frac{n\pi}{L} x\right) \cdot e^{i 2\pi f_n t}$$

where f_n are the resonance frequencies

$$f_n = \frac{c}{\lambda_n} = \frac{c}{2} \frac{n}{L}$$

For the three dimensional case with six hard boundary surfaces the eigen frequencies is

$$f_{n_x, n_y, n_z} = \frac{c}{2} \sqrt{\left(\frac{n_x}{L}\right)^2 + \left(\frac{n_y}{B}\right)^2 + \left(\frac{n_z}{H}\right)^2}$$

In transmission from one medium with wave impedance $Z_1 = \rho_1 c_1$ to another medium with wave impedance $Z_2 = \rho_2 c_2$ the transmission factor t and reflection factor r

$$t = \frac{\hat{p}_t}{\hat{p}_i} = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1} = \frac{2Z_2}{Z_2 + Z_1}$$

$$r = \frac{\hat{p}_r}{\hat{p}_i} = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

while the transmission coefficient τ and reflection coefficient ρ is

$$\tau = \frac{I_t}{I_i} = \frac{4\rho_2 c_2 \cdot \rho_1 c_1}{\left(\rho_2 c_2 + \rho_1 c_1\right)^2} = \frac{4 Z_2 Z_1}{\left(Z_2 + Z_1\right)^2}$$

$$\rho = \frac{I_r}{I_i} = \frac{\left|\rho_2 c_2 - \rho_1 c_1\right|^2}{\left(\rho_2 c_2 + \rho_1 c_1\right)^2} = \frac{\left|Z_2 - Z_1\right|^2}{\left(Z_2 + Z_1\right)^2}$$

Sound isolation and absorption

Sound reduction index:

$$R \equiv 10\log\left(\frac{\Pi_i}{\Pi_t}\right) = 10\log\left(\frac{1}{\tau}\right)$$

Measurement of sound reduction index *R*:

$$R = L_s - L_m + 10\log\left(\frac{S}{A}\right)$$

Measurement of step sound level:

$$L_n = L_m + 10\log\left(\frac{A}{10}\right)$$

Combined reduction index:

$$R = -10\log\left(\frac{1}{S}(S_1 10^{-R_1/10} + S_2 10^{-R_2/10} + \ldots)\right)$$

Slot leakage:

$$R = -10\cdot\log\left(10^{-R/10} + \frac{S_s}{S}\right)$$

Sabine's formula:

$$T_{60} = 0.16\cdot\frac{V}{A}$$

The mass law for a single leaf wall:

$$R_0 = 20\log\frac{\pi f m''}{2\rho c}$$

Coincidence frequency (critical frequency)

$$f_c = \frac{c_0^2}{2\pi}\sqrt{\frac{m''}{B}} = K/h$$

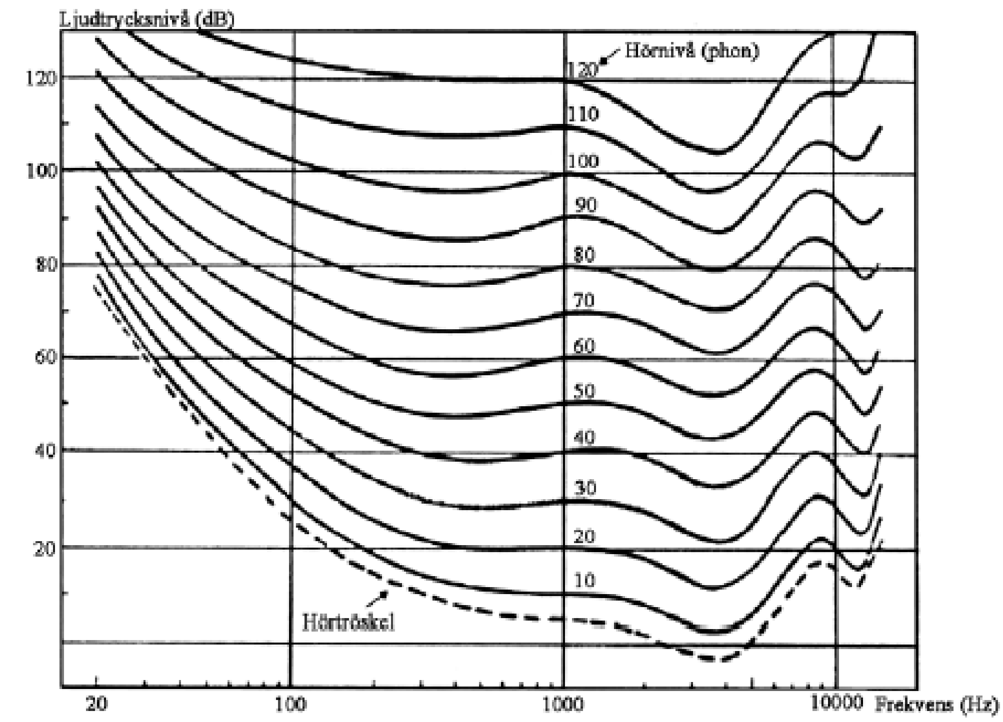
where *B* is bending stiffness per unit width, $B = E\frac{h^3}{12}$

Tables

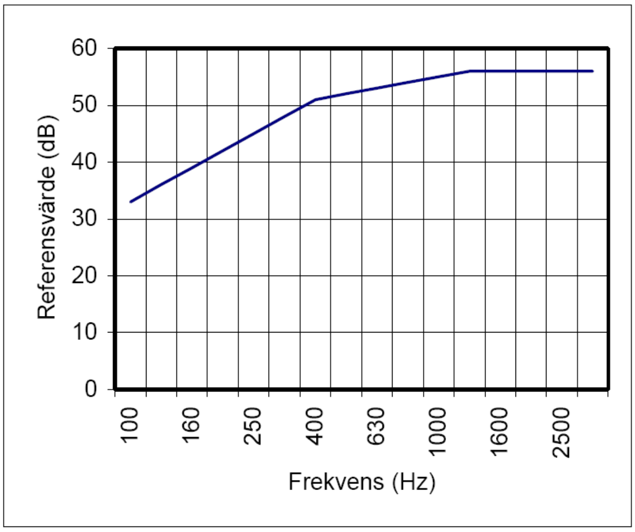
Octave bands and third octave bands:

Mittfrekvens f_m (Hz)	Tersfilter $f_u - f_ö$ (Hz)	Oktavfilter $f_u - f_ö$ (Hz)	Mittfrekvens f_m (Hz)	Tersfilter $f_u - f_ö$ (Hz)	Oktavfilter $f_u - f_ö$ (Hz)
50	44,7 – 56,2		800	708 – 891	
63	56,2 – 70,8	44,7 – 89,1	**1000**	891 – 1120	708 – 1410
80	70,8 – 89,1		1250	1120 – 1410	
100	89,1 – 112		1600	1410 – 1780	
125	112 – 141	89,1 – 178	**2000**	1780 – 2240	1410 – 2820
160	141 – 178		2500	2240 – 2820	
200	178 – 224		3150	2820 – 3550	
250	224 – 282	178 – 355	**4000**	3550 – 4470	2820 – 5620
315	282 – 355		5000	4470 – 5620	
400	355 – 447		6300	5620 – 7080	
500	447 – 562	355 – 708	**8000**	7080 – 8910	5620 – 11200
630	562 – 708		10000	8910 – 11200	

Phon curves:



Reference curve for air bourn sound isolation:

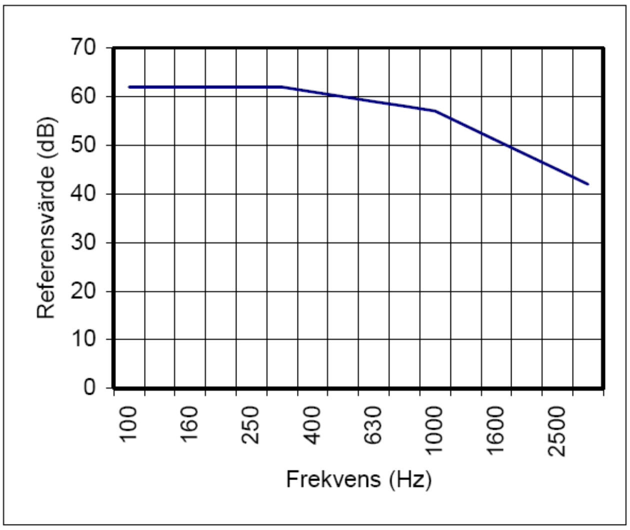


Frekvens, Hz	Ref.värde, dB
100	33
125	36
160	39
200	42
250	45
315	48
400	51
500	52
630	53
800	54
1000	55
1250	56
1600	56
2000	56
2500	56
3150	56

In calculating the result the reference curve is moved in steps of 1 dB towards the measured curved until the non-beneficial deviation is as large as possible, but not larger than 32 dB. A non-beneficial displacement at a certain frequency occurs when the results of the measurements are *lower* than the reference value. Only non-beneficial displacements are considered.

The value in dB of the reference curve at 500 Hz, after being moved according to the above, is R'_w.

Reference curve for step sound level:



Frekvens, Hz	Ref.värde, dB
100	62
125	62
160	62
200	62
250	62
315	62
400	61
500	60
630	59
800	58
1000	57
1250	54
1600	51
2000	48
2500	45
3150	42

In order to calculate the result, the reference curve is moved in steps of 1 dB towards the measured curve until the non-beneficial displacement is as large as possible, but not larger than 32 dB. A non-beneficial displacement at a certain frequency occurs when the result *exceeds* the reference value. Only non-beneficial displacements are considered.

The value in dB of the reference curve at 500 Hz, after being moved according to the above, is $L'_{n,w}$.