Ljud i byggnad och samhälle (VTAF01) – Summary – F1 to F6

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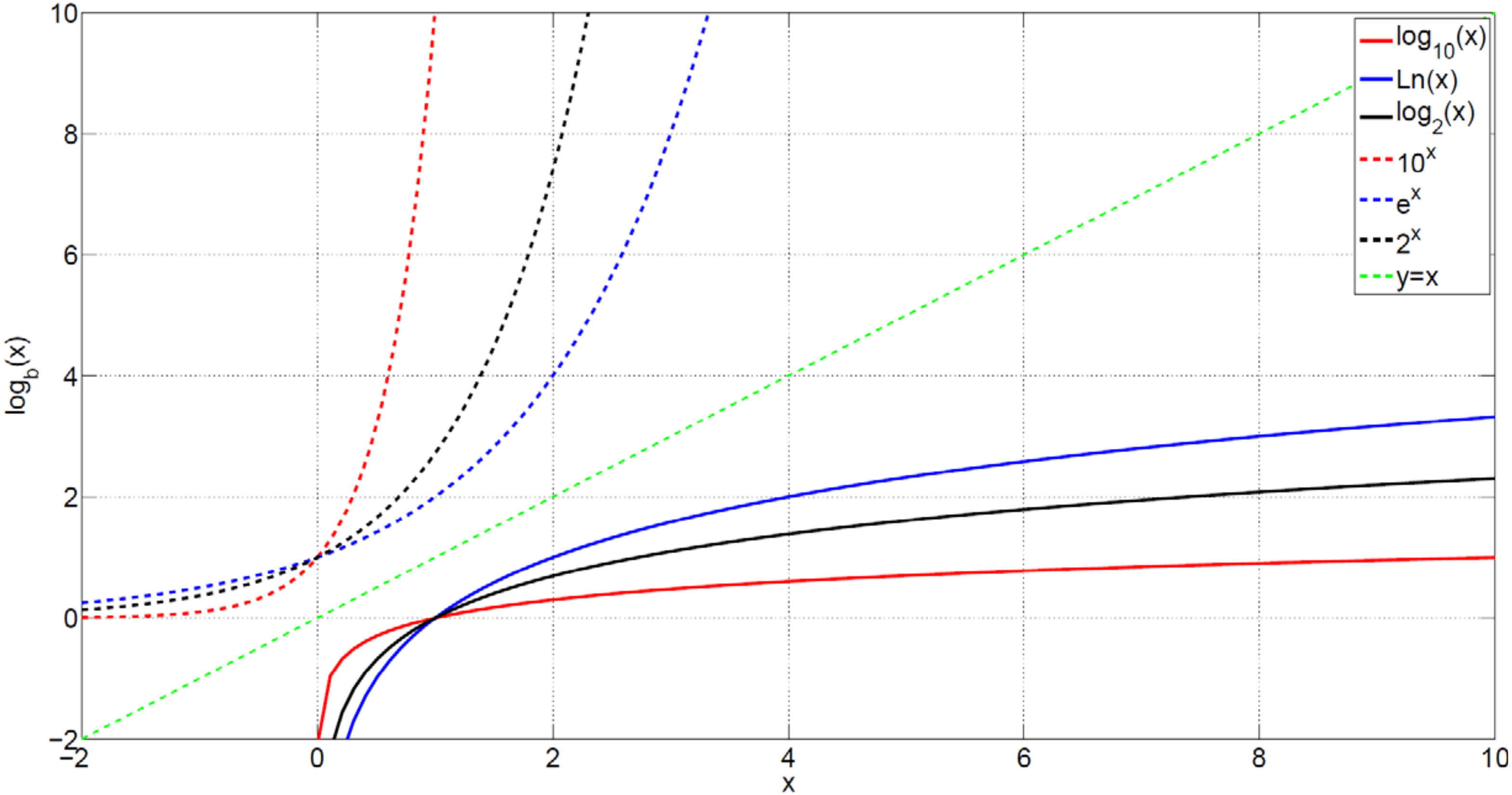
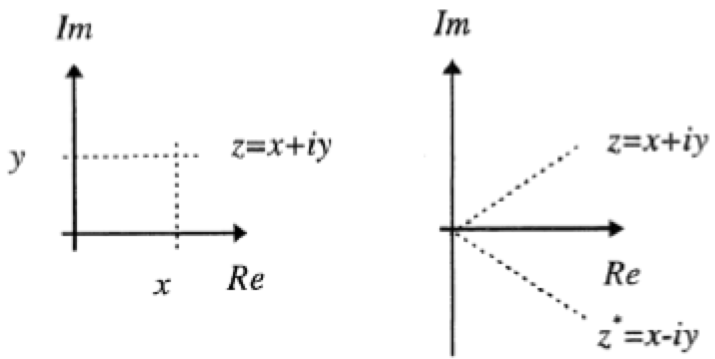
Intro maths – Logarithms



Intro math – Complex numbers

- Rectangular form



Figur 8 Komplexa tal

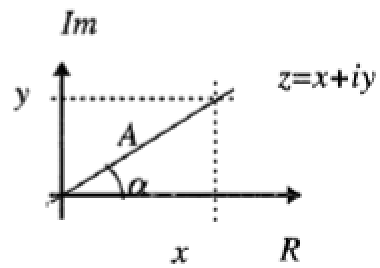
- Imaginära enheten:

$$i=\sqrt{-1},\quad i^2=-1,\quad i^3=-i,\ldots\quad i^{-1}=-i$$

- Komplexa tal:

$$z=x+iy$$

- Polar form



Figur 9 Amplitud och fas

- Amplitud (eller absolutbelopp):

$$A=|z|=\sqrt{x^2+y^2}$$

- Fas (alltid i radianer!):

$$\alpha=\arctan\left(\frac{y}{x}\right)$$

$$x=A\cos(\alpha),\quad y=A\sin(\alpha)$$

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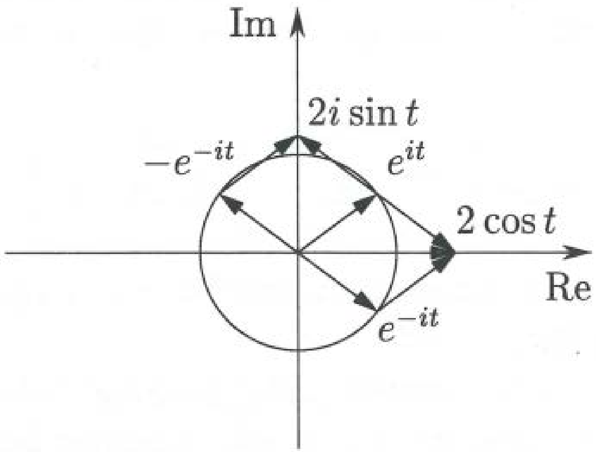
Intro math – Complex numbers

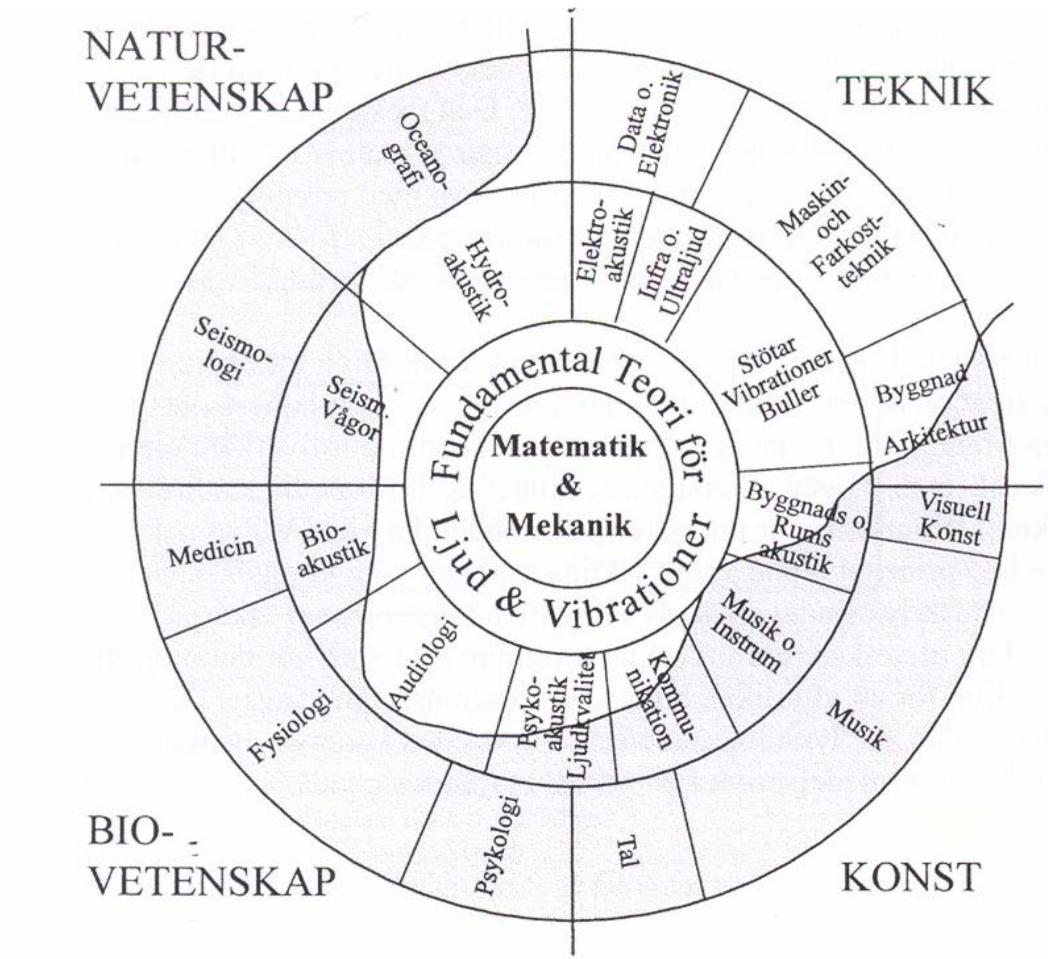
- Euler's identity

$$e^{it} = \cos t + i \sin t \iff \begin{cases} \cos t = \frac{1}{2}\left(e^{it} + e^{-it}\right) = \operatorname{Re} e^{it} \\ \sin t = \frac{1}{2i}\left(e^{it} - e^{-it}\right) = \operatorname{Im} e^{it} \end{cases}$$



- Practical to describe rotating motion with complex numbers. Euler's identity is the background.

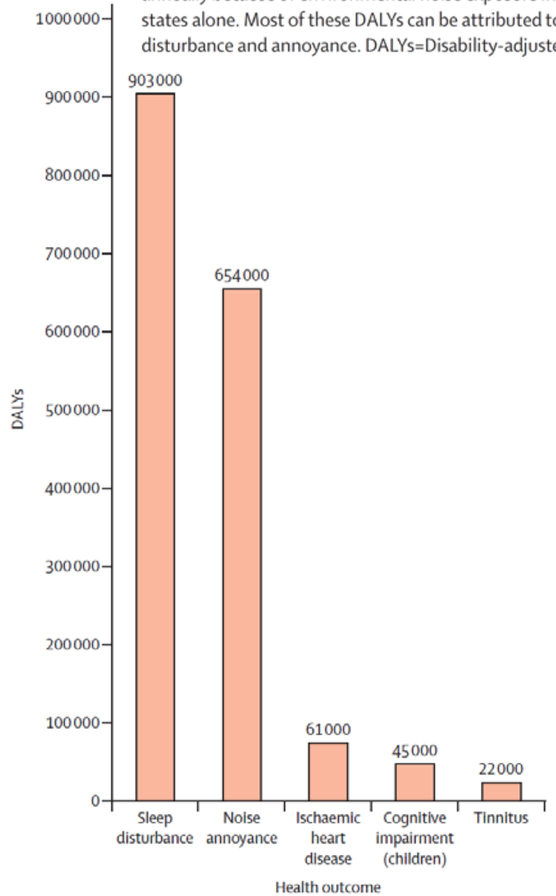
What is acoustics? (II)



Why address sound issues?

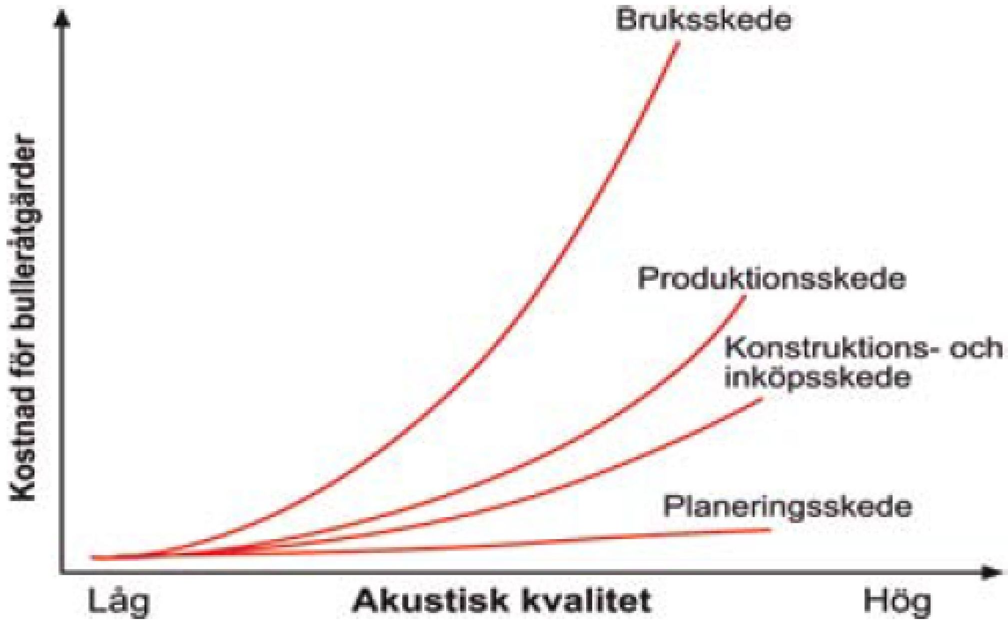
- Noise affects people physiologically and psychologically
  - At least 25 % of EU citizens are exposed to noise in such extent that it affects health and quality of life
  - Approximately 2 million people in Sweden are exposed to a noise level that exceeds the regulations set up by the Swedish parliament

DALYs attributed to environmental noise exposure in Europe. According to WHO, more than 1 million healthy life years (DALYs) are lost annually because of environmental noise exposure in European A-member states alone. Most of these DALYs can be attributed to noise-induced sleep disturbance and annoyance. DALYs=Disability-adjusted life years.



Why address sound issues?

Figur 1.1
Samband mellan akustisk kvalitet och kostnad för bulleråtgärder.
Källa: SOU 1993:65

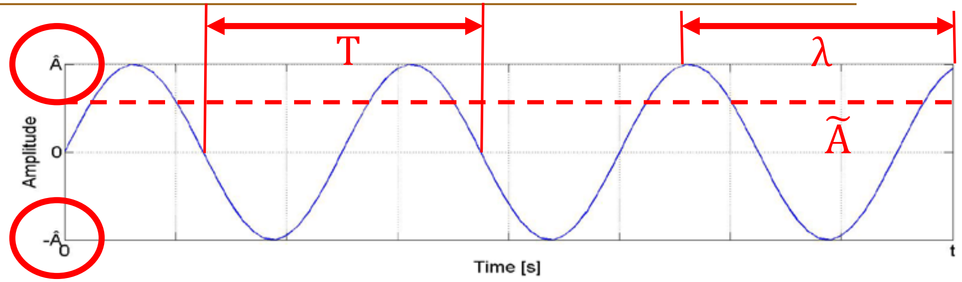


Basic concepts

- Time domain
  - Harmonic signals
- Frequency domain
- The Decibel

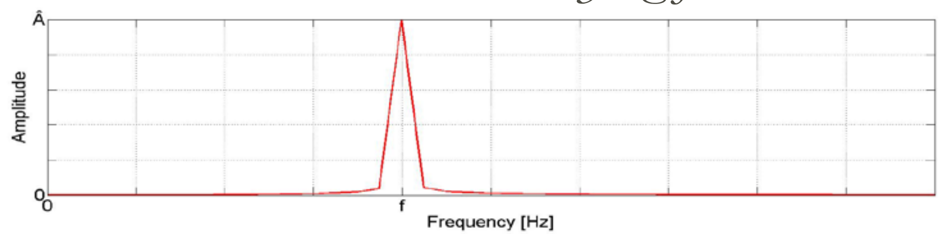
$$L_p = 10\log\left(\frac{\tilde{p}^2}{p_{ref}^2}\right) = 20\log\left(\frac{\tilde{p}}{p_{ref}}\right)$$

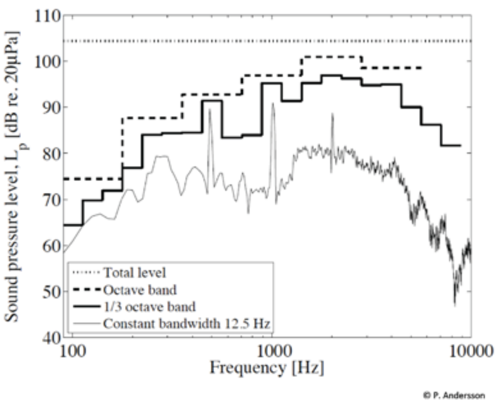
- Frequency weightings
- Frequency bands

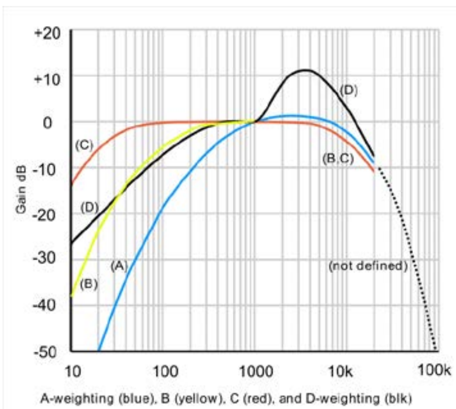


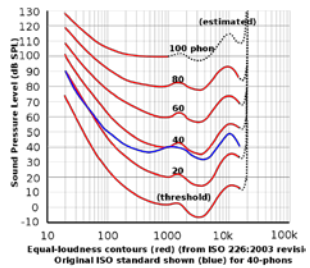
FFT

How much a time-signal resembles sine waves with varying f



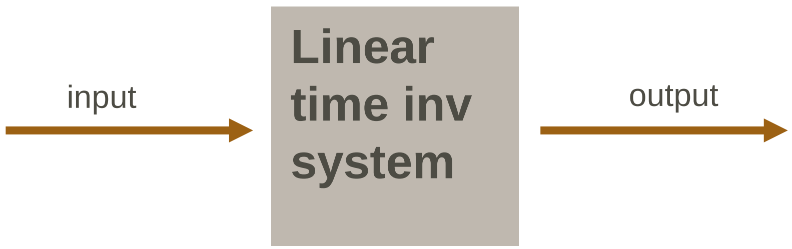






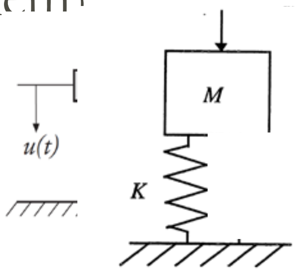
Linear system

- What effect does an input signal has on an ouput signal?
  - What effect does a force on a body has on its velocity?
- A way to answer it using theory of linear time-invariant systems.



Free vibrations - SDOF

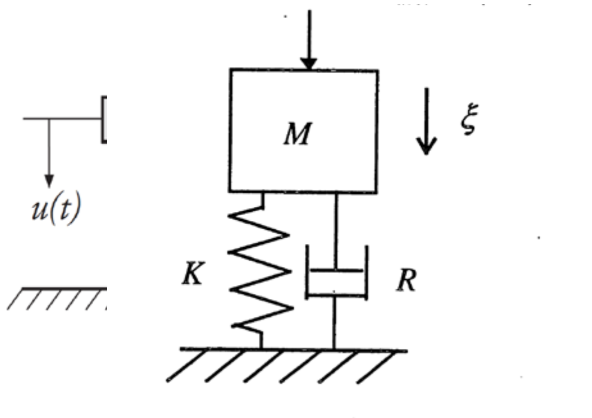
- EoM: $M\ddot{x} + Kx = 0$
- $M\lambda^2 ae^{\lambda t}$+K $ae^{\lambda t}$=0 (mathematically eigenvalue problem)
  - $\lambda^2 + \frac{K}{M} = 0$
    - » $\lambda_1 = \mathrm{i}\sqrt{\frac{K}{M}} = \mathrm{i}\omega_0$; $\lambda_2 = -\mathrm{i}\sqrt{\frac{K}{M}} = -\mathrm{i}\omega_0$.



- What are those solutions λ? (mathematically eigenvalues)
  - K has dimensions N/m = [kg m/s^2]/m = kg/s^2
  - M has dimensions of kg.
  - Then λ is 1/s = Hz. Frequency!
- $x(t) = ae^{\mathrm{i}\omega_0 t} + be^{-\mathrm{i}\omega_0 t}$=$A\sin(\omega_0 t) + B\cos(\omega_0 t)$.
- Oscillatory motion!

Analysis of dimensions is very helpful to check for mistakes and understand what is going on!

Free vibrations with damping



- $\lambda_1 = -\mathrm{R} + \mathrm{i}\omega_R = \mathrm{i}\omega_0$; $\lambda_2 = -\mathrm{R} - \mathrm{i}\omega_R$; $\omega_R = \sqrt{\omega_0^2 - R^2}$.
- $x(t) = ae^{-\mathrm{R}t+\mathrm{i}\omega_R t} + be^{-\mathrm{R}t-\mathrm{i}\omega_R t} =$

$$= [\mathrm{A}\sin(\omega_R t) + \mathrm{B}\cos(\omega_R t)]\, e^{-\mathrm{R}t}$$

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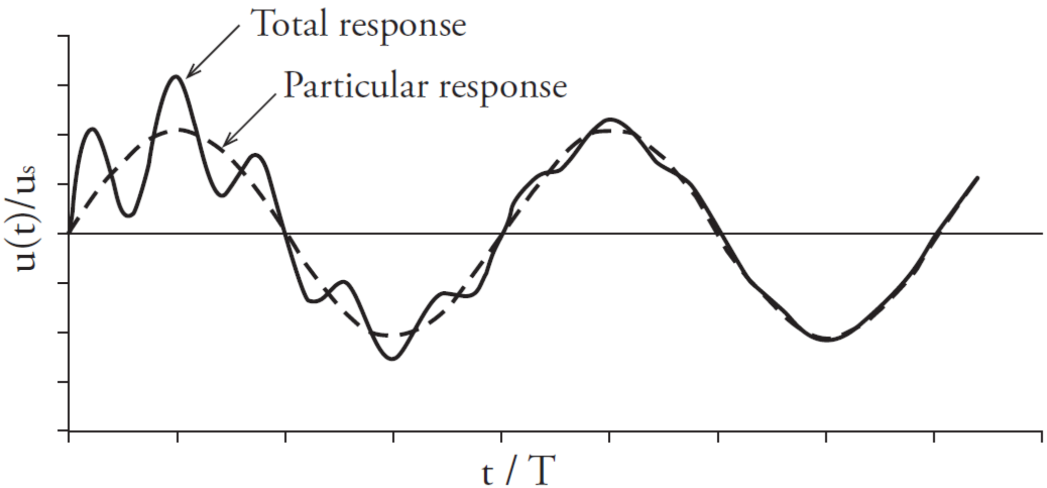
Damped SDOF – Total solution

- Total solution = homogeneous + particular
  - The homogeneous solution vanishes with increasing time. After some time: $u(t) \approx u_p(t)$



Total response of a damped system subjected to a harmonic force,

$$u_{total}(t) = e^{-\frac{\eta}{2}\omega_0 t}\left(B_1 \sin(\omega_d t) + B_2 \cos(\omega_d t)\right) + D_1 \sin(\omega t) + D_2 \cos(\omega t)$$

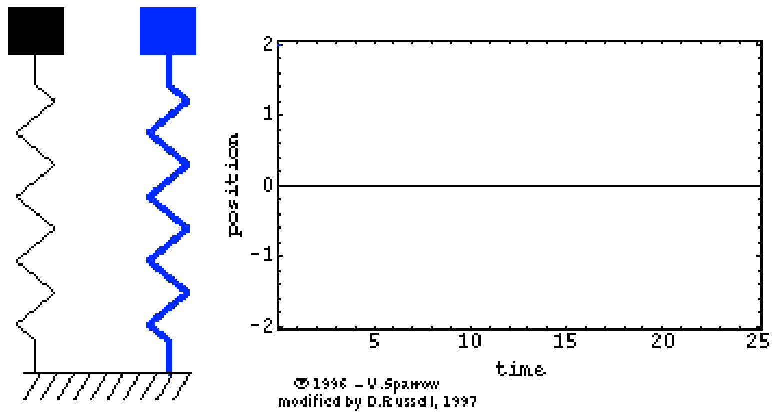
Homogeneous

Particular

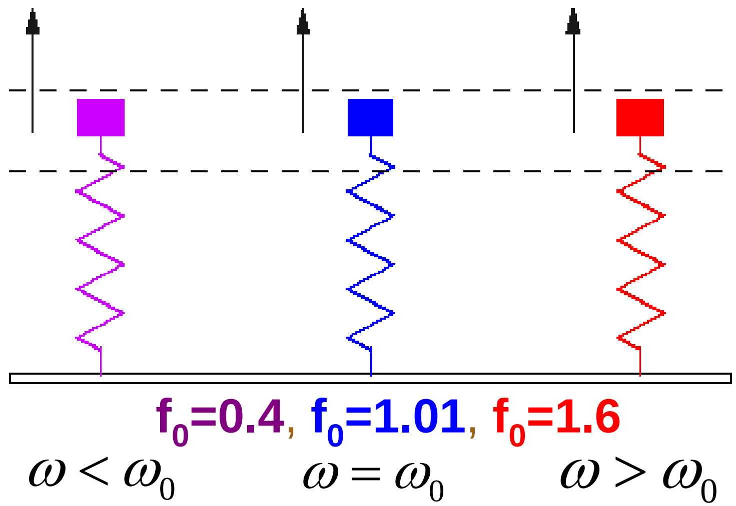
$$\omega_d = \omega_0 \sqrt{1-\zeta^2}$$

SDOF – Driving frequencies

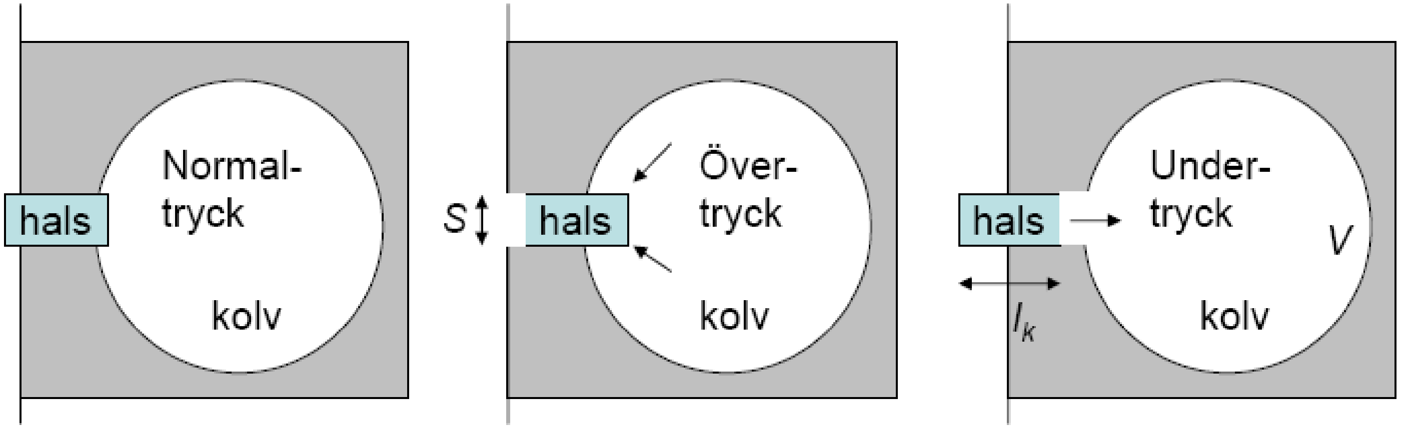
- Ex:
  - Without damping
  - With damping

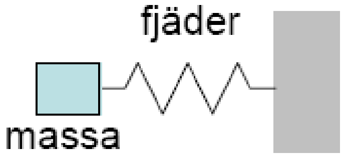


- Same natural frequency f=1
- Different driving frequency



Helmholtz resonator

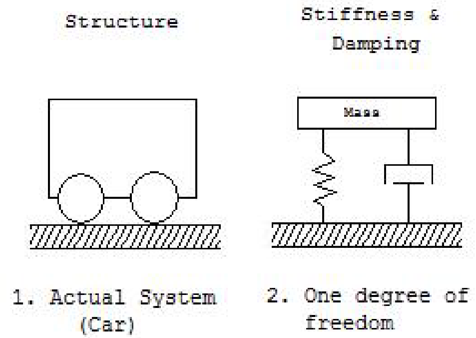




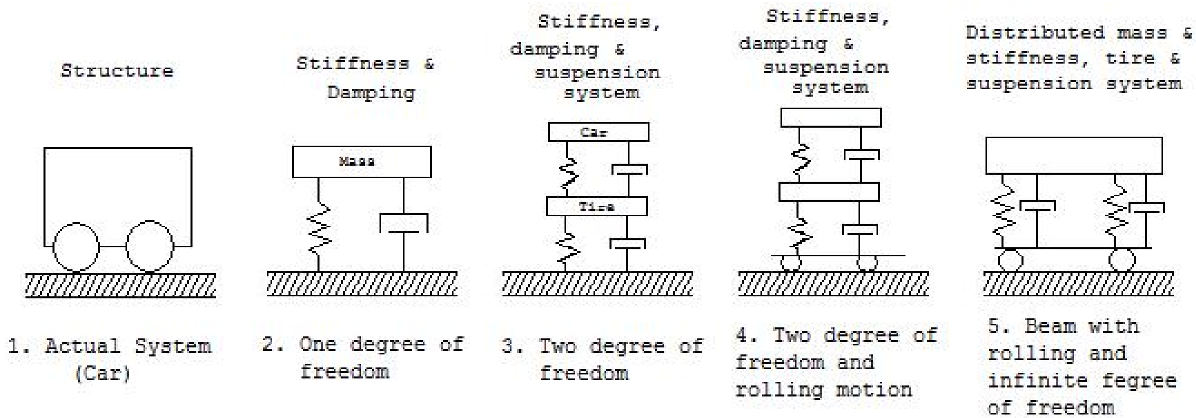
$$f_r = \frac{c}{2\pi}\sqrt{\frac{S}{l_k V}}$$

Single-degree-of-freedom

- One can do a lot with SDOF



- And one can use many SDOF to have a multiple degree-of-freedom system (MDOF).



https://commons.wikimedia.org/wiki/File:Structureandideal2.jpg

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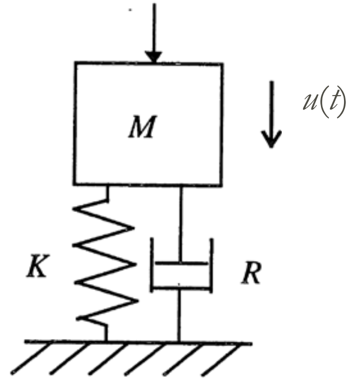
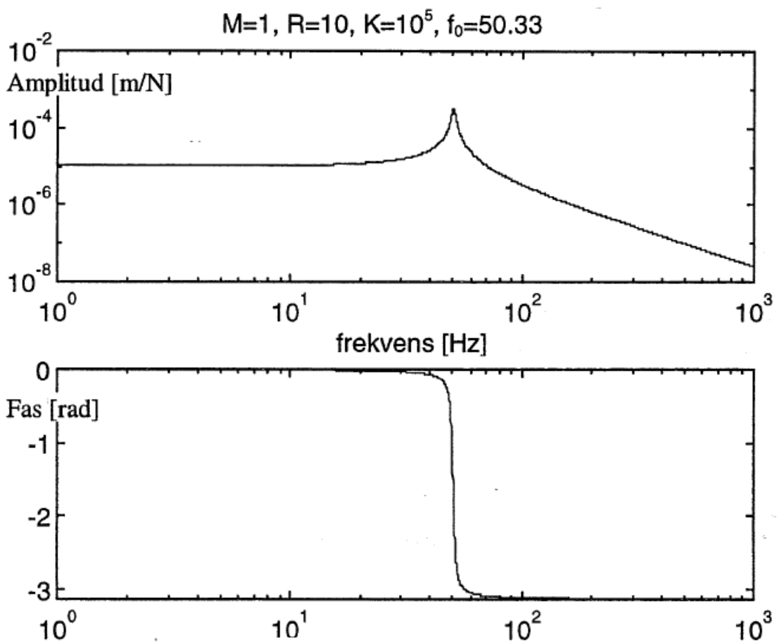
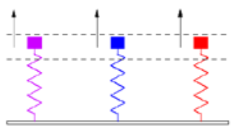
SDOF – Frequency response function

- Output / Input $\frac{\tilde{u}(\omega)}{F_{driv}} = \frac{1}{(K - M\omega^2) + Ri\omega}$
- Kvoten är ett komplext tal och heter överföringsfunktion

$F(t) = F_{driv} \cdot \cos(\omega t)$







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SDOF – Frequency response functions (FRF)

- In general, FRF = transfer function, i.e.:
  - Contains system information
  - Independent of outer conditions
  - **Frequency domain relationship between input and output of a linear time-invariant system**

$$H_{ij}(\omega) = \frac{\tilde{s}_i(\omega)}{\tilde{s}_j(\omega)} = \frac{\text{output}}{\text{input}}$$

- Different FRFs can be obtained depending on the measured quantity

Measured quantity	FRF	
Acceleration (a)	Accelerance = $N_{dyn}(\omega)$ = a/F	Dynamic Mass = $M_{dyn}(\omega)$ = F/a
Velocity (v)	Mobility/admitance = $Y(\omega)$ = v/F	Impedance = $Z(\omega)$ = F/v
Displacement (u)	Receptance/compliance = $C_{dyn}(\omega)$= u/F	Dynamic stiffness = $K_{dyn}(\omega)$ = F/u

$$C_{dyn}(\omega) = \frac{\tilde{u}(\omega)}{F_{driv}(\omega)} = \frac{1}{(K - M\omega^2) + Ri\omega}$$

$$K_{dyn}(\omega) = C_{dyn}(\omega)^{-1} = -M\omega^2 + Ri\omega + K$$

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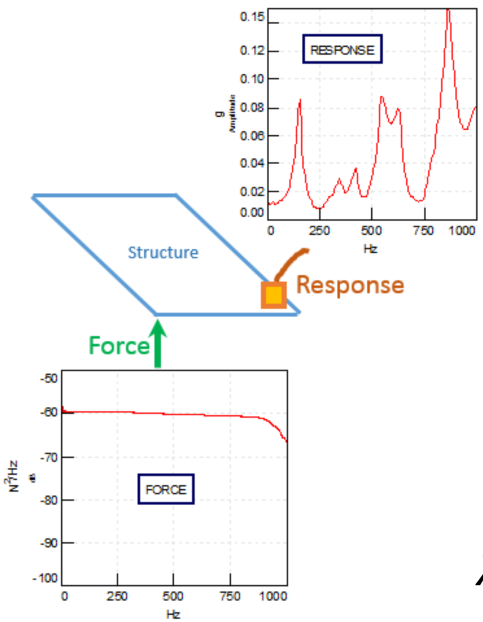
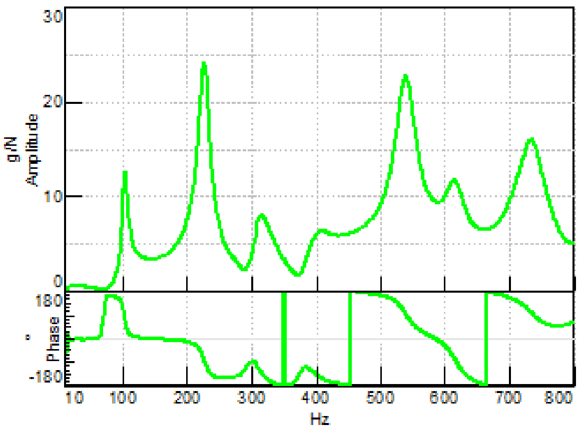
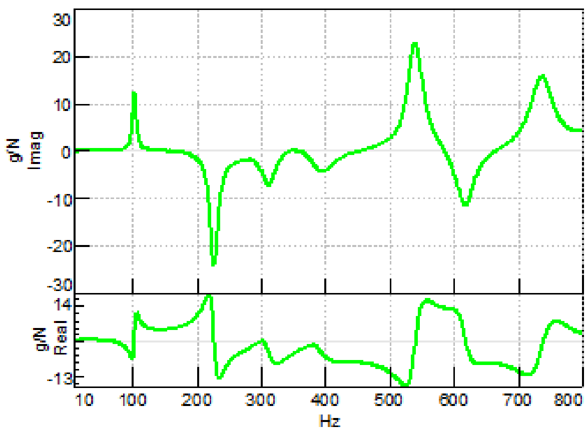
FRF of a complex system

- FRFs are complex
  - Amplitude/Phase
  - Real / imaginary part

$$H_{ij}(\omega) = \frac{\widetilde{s}_i(\omega)}{\widetilde{s}_j(\omega)} = \frac{\text{output}}{\text{input}}$$







$$\lambda_1 = -\mathrm{R} + \mathrm{i}\omega_R = \mathrm{i}\omega_0 \;;\; \lambda_2 = -\mathrm{R} - \mathrm{i}\omega_R \;;\; \omega_R = \sqrt{\omega_0^2 - R^2}.$$

Source: https://community.sw.siemens.com/s/article/what-is-a-frequency-response-function-frf

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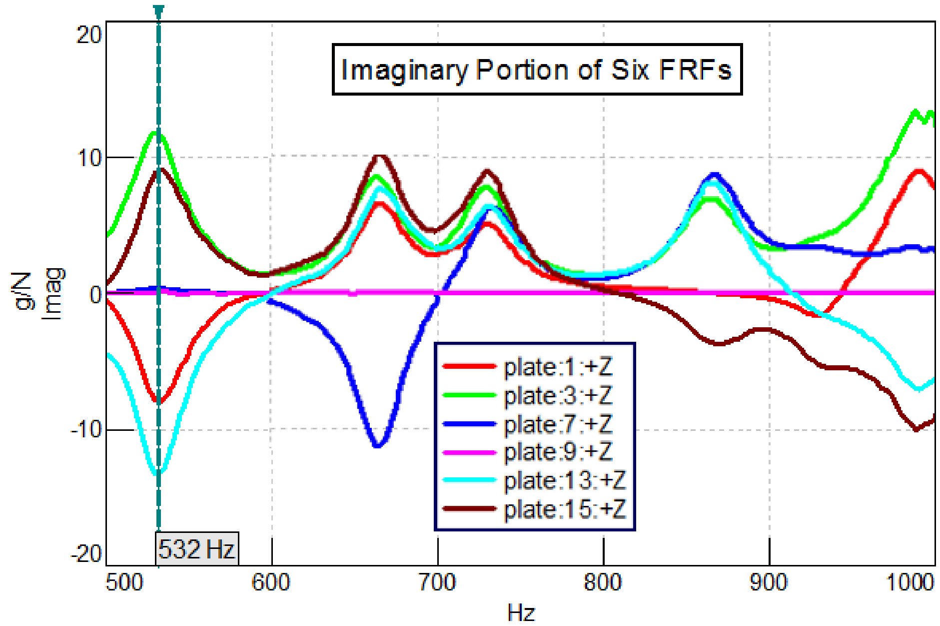
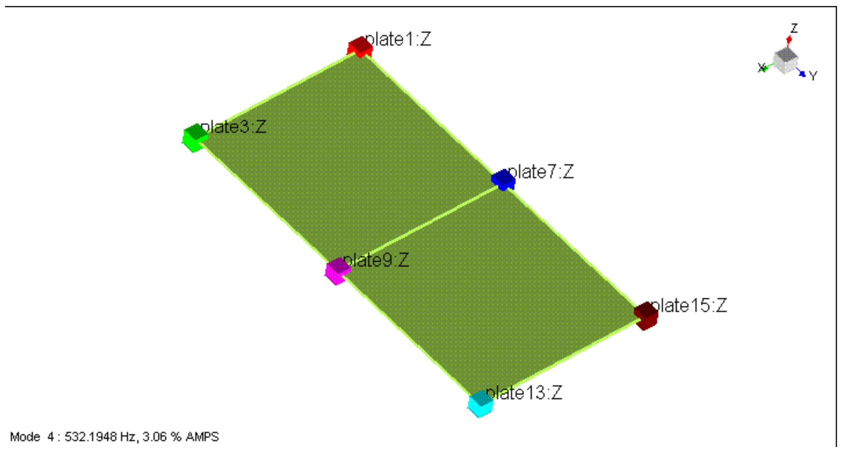
FRF of a complex system

- Real and imaginary parts – the imaginary part has interesting information





Source: https://community.sw.siemens.com/s/article/what-is-a-frequency-response-function-frf

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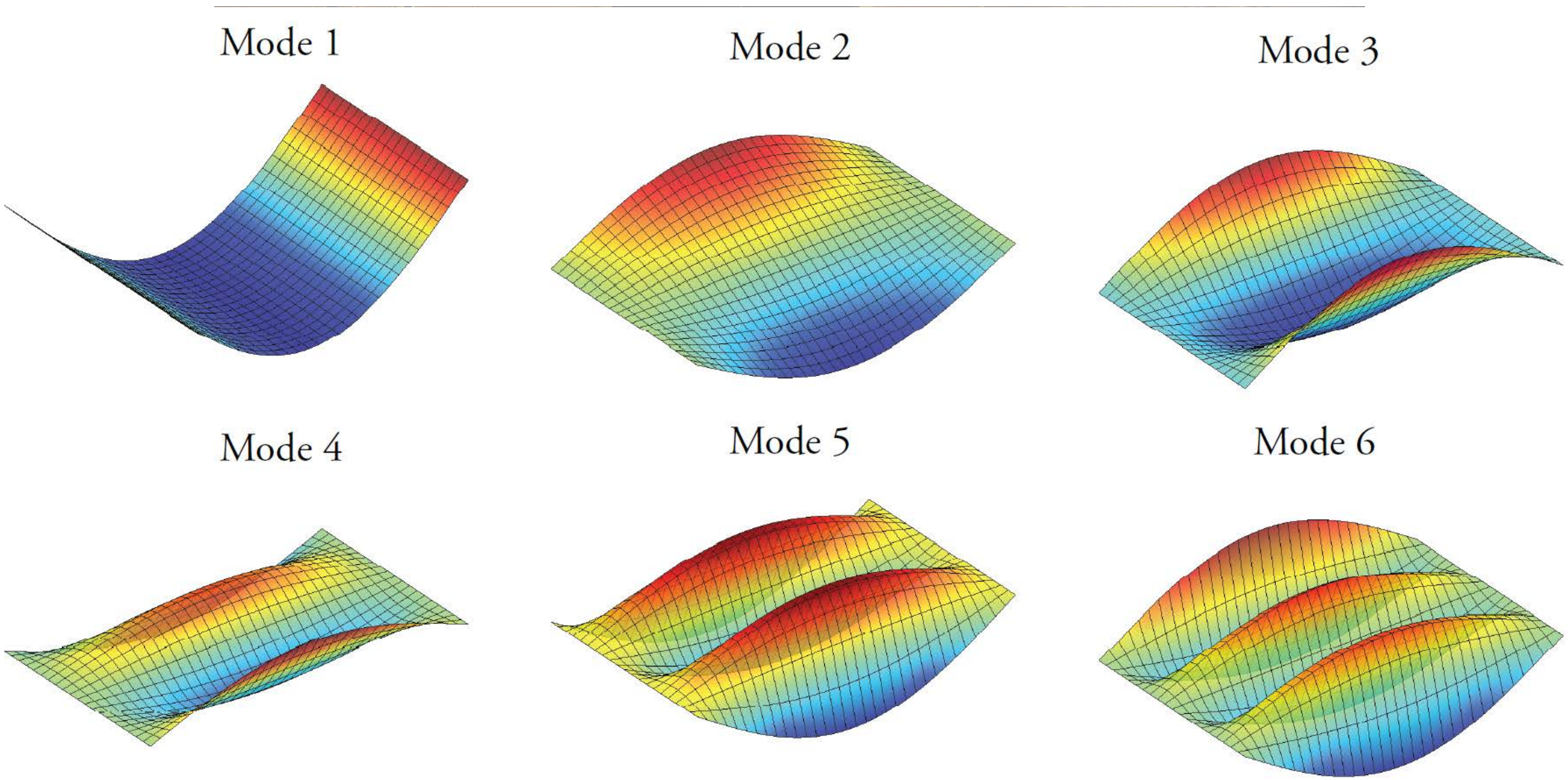
Mode shapes – Example floor



NOTE: In floor vibrations, modes are superimposed on one another to give the overall response of the system. Fortunately it is generally sufficient to consider only the first 3 or 4 modes, since the higher modes are quickly extinguished by damping.

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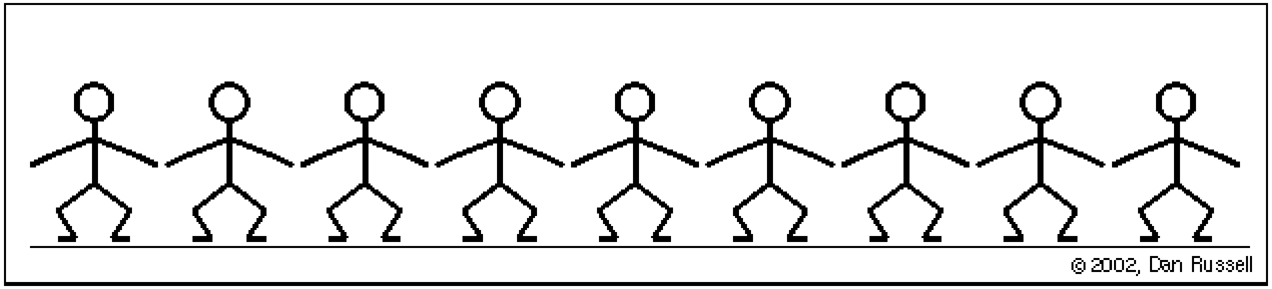
What is a wave?



- A disturbance or deviation from a pre-existing condition. Its motion constitutes a transfer of information from one point in space to another.
- Time plays a key role – static displacement of a rubber band is a disturbance but not a wave.
  - Wave travels at finite speed (hitting a perfectly rigid rod making the rod moving as a unit is no wave, just rigid body motion)
  - The rod is elastic, the impulse travels from one end to the other.
- All mechanical waves travel in a material medium (unlike e.g. electromagnetic waves)
- Many waves satisfy $c^2\nabla^2 u - \ddot{u}=0$ – but not all!

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Recap - Types of waves in solid media

- Longitudinal waves (∞ medium ≈ beams)
  - Quasi-longitudunal waves (finite ≈ plates)

$$\frac{\partial^2 u_x}{\partial x^2} - \frac{\rho}{E'}\frac{\partial^2 u_x}{\partial t^2} = 0$$

$$c_L = \sqrt{\frac{E}{\rho}}$$

$$c_{qL} = \sqrt{\frac{E'}{\rho}} = \sqrt{\frac{E}{\rho(1-\upsilon^2)}}$$

- Shear waves

$$\frac{\partial^2 u_y}{\partial x^2} - \frac{\rho}{G}\frac{\partial^2 u_y}{\partial t^2} = 0$$

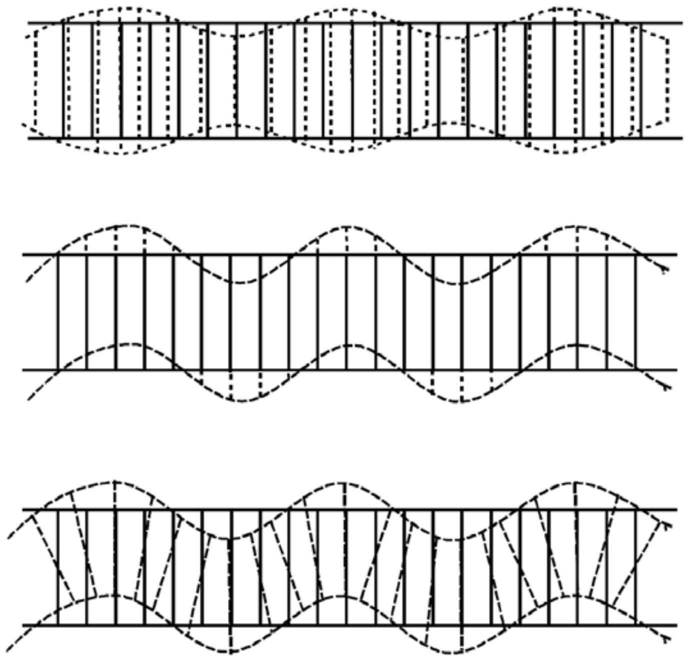
$$c_{sh} = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{2(1+\upsilon)\rho}}$$

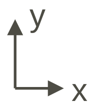
- Bending waves (dispersive)

$$B\frac{\partial^4 u_y}{\partial x^4} + m\frac{\partial^2 u_y}{\partial t^2} = 0$$

$$c_{B(\omega)} = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$$

Plate: E, G, ρ, υ, h





$$m = \rho h \qquad B_{beam} = E\frac{bh^3}{12}$$

$$B_{plate} = \frac{Eh^3}{12(1-\upsilon^2)}$$

NOTE: torsional waves (beams and columns) are not addressed here

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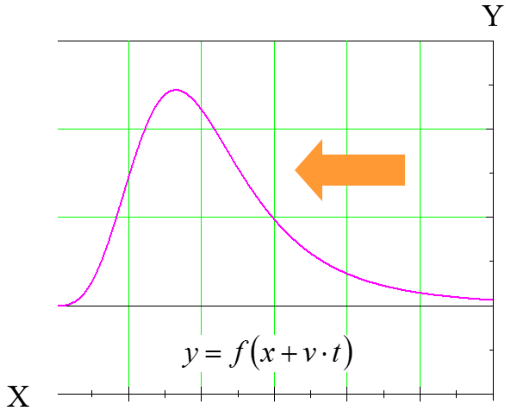
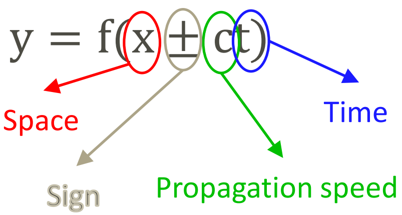
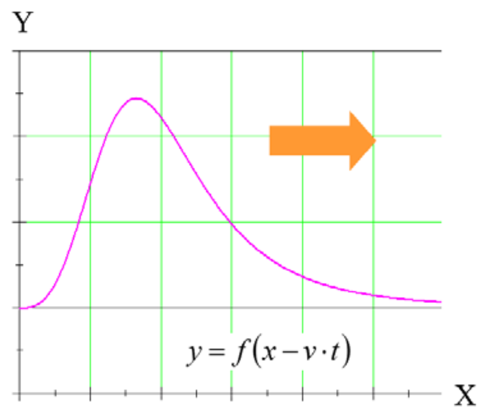
Wave equation solution

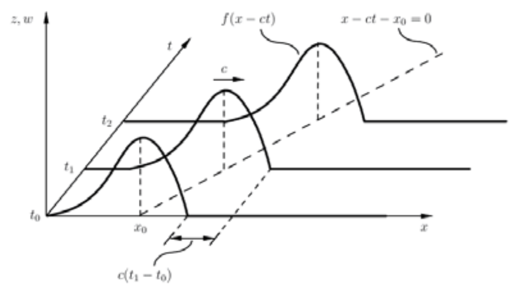
- Most general solution is forward and backward travelling wave.
    - d'Alambert's solution







$$y = f\left(x \pm \frac{\omega}{k} t\right) = f\left(\frac{kx \pm \omega t}{k}\right) = f(kx \pm \omega t)$$



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Waves in fluid media

- Sound waves: longitudinal waves
  - Pressure as field variable

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \longrightarrow p(x,t) = \widehat{p_\pm}\cos(\omega t \pm kx) = \widehat{p_\pm} e^{-i(\omega t \pm kx)}$$

  - Velocity as field variable

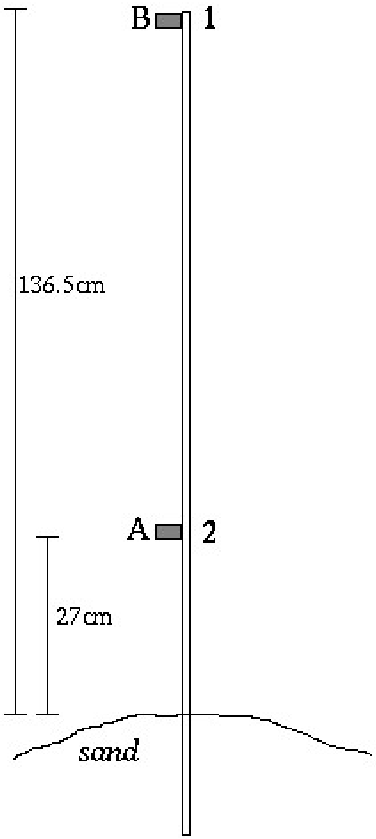
$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2} \longrightarrow v(x,t) = \frac{1}{\rho c}\widehat{p_\pm}\, e^{-i(\omega t \pm kx)}$$

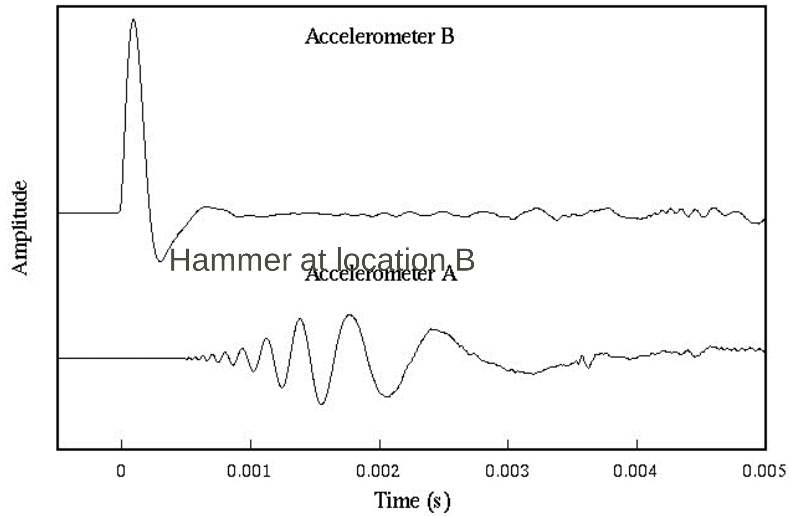
Comparing both equations: $Z \equiv \frac{p_\pm}{v_\pm} = \pm\rho c$ (acoustic impedance)

Bending waves

$$B\frac{\partial^4 u_y}{\partial x^4} + m\frac{\partial^2 u_y}{\partial t^2} = 0$$

$$c_{B(\omega)} = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$$





- Force pulse is very clean at location B
- Pulse disperses by the time it reaches location A --- higher frequency waves travel faster and arrive first --- lower frequency waves travel slower and arrive later

https://www.acs.psu.edu/drussell/Demos/Dispersion/Flexural.html

Bending waves - solution

$$B\frac{\partial^4 u_y}{\partial x^4} + m\frac{\partial^2 u_y}{\partial t^2} = 0$$

- Due to the different form of equations with four-times spatial derivatives, the solution is more complex solutions including *near-field* terms

$$\zeta(x,t) = \hat{\zeta} e^{i(\omega t - k_B x)}. \qquad \zeta(x,t) = (\mathbf{A}e^{ik_B x} + \mathbf{B}e^{-ik_B x} + \mathbf{C}e^{k_B x} + \mathbf{D}e^{-k_B x})e^{i\omega t},$$

- Why near field!?
  - Because they decay rather quickly away from the *boundary* or *load point!*

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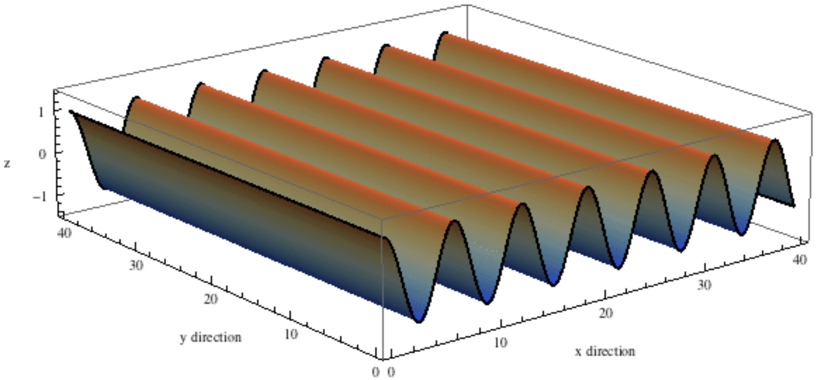
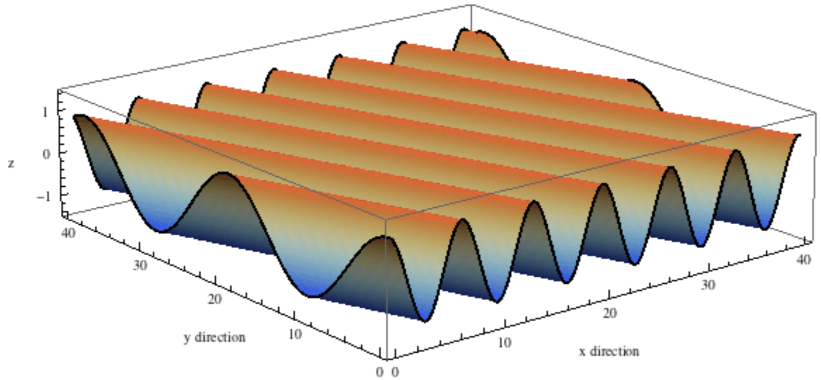
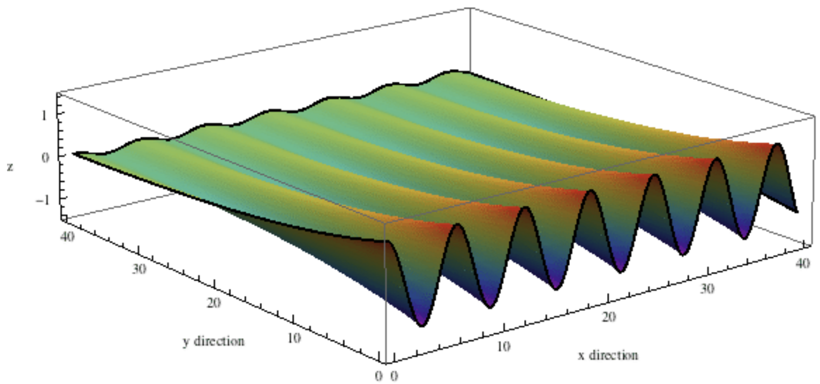
Propagating waves VS evanescent waves

- Waves may thus propagate or not propagate!





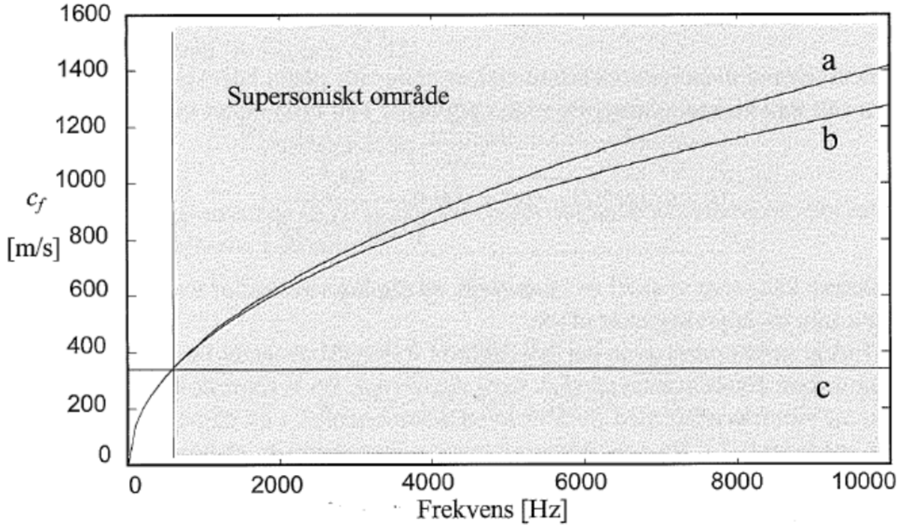


https://www.acs.psu.edu/drussell/Demos/EvanescentWaves/EvanescentWaves.html

Coincidence

- Dispersion curves

$$c_{B(\omega)} = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$$

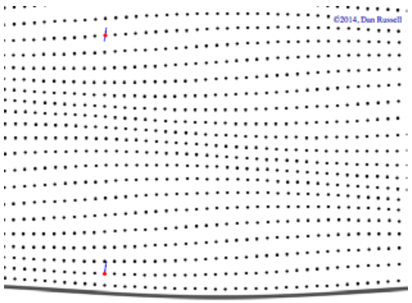


Figur 6-28 Fashastigheten för en cirkulärcylindrisk stålbalk med diametern 5 cm. a) Enligt Bernoulli-Eulerteori, b) enligt Timoshenkoteori. c) Fashastigheten för en kompressionsvåg i luft. Den frekvens där böjvågens fashastighet är lika med ljudhastigheten i det omgivande mediet kallas koincidensfrekvens.

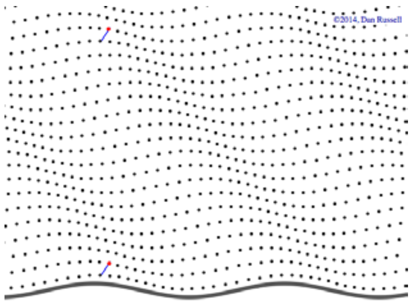
Coincidence

- Animations – look at the red particle and its trajectory!
- Bending wave in the plate which create wave in the air next to it.

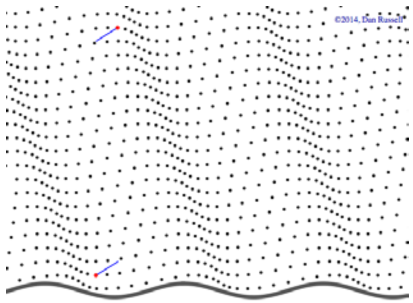
$c_{plate} = 5\ c_{fluid}$ (plane wave)



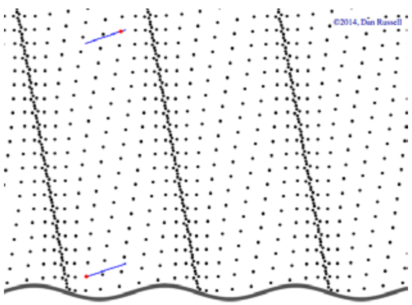
$c_{plate} = 1.5\ c_{fluid}$ (plane wave)



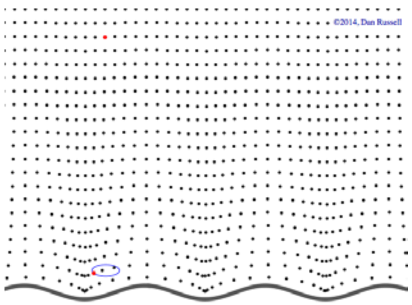
$c_{plate} = 1.1\ c_{fluid}$ (plane wave)



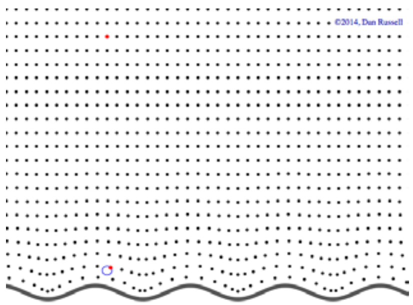
$c_{plate} = 1.03\ c_{fluid}$ (plane wave)



$c_{plate} = 0.95\ c_{fluid}$ (plane wave)



$c_{plate} = 0.75\ c_{fluid}$ (plane wave)



https://www.acs.psu.edu/drussell/Demos/EvanescentWaves/EvanescentWaves.html

Coincidence

- Speed in structure < speed in air

$$\lambda = \frac{c}{f}$$

- wavelength in structure < wavelength in air

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

- Wavenumber in structure > wavenumber in air

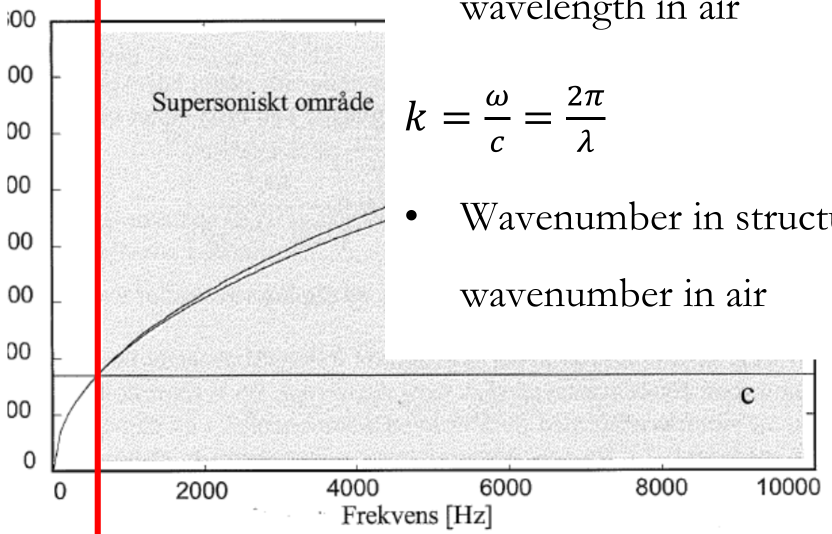
- Speed in structure > speed in air

$$\lambda = \frac{c}{f}$$

- wavelength in structure > wavelength in air

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

- Wavenumber in structure < wavenumber in air

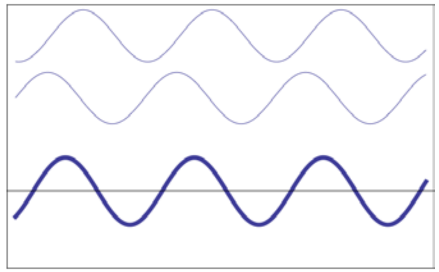


Figur 6-28 Fashastigheten för en cirkulärcylindrisk stålbalk med diametern 5 cm. a) Enligt Bernoulli-Eulerteori, b) enligt Timoshenkoteori. c) Fashastigheten för en kompressionsvåg i luft. Den frekvens där böjvågens fashastighet är lika med ljudhastigheten i det omgivande mediet kallas koincidensfrekvens.

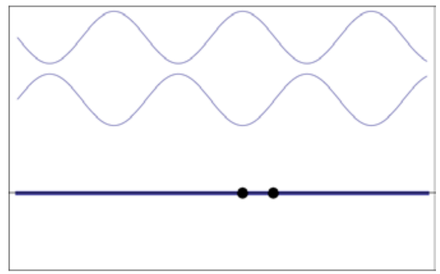
Some more wave phenomena…

- Superposition of two waves

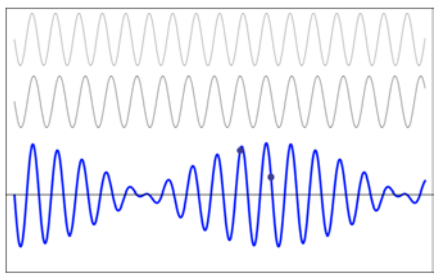
- Constructive and destructive interference (phase shift)



- Standing waves



- Beats



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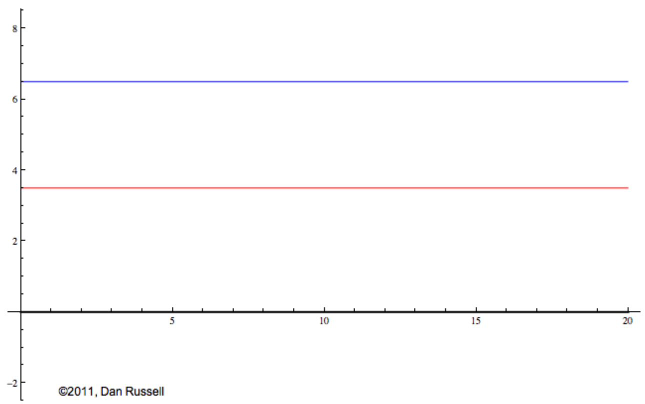
Wave phenomena

- What happens when waves are travelling in a structure and interact with each other?
  - A sound exciting waves in a wall
  - An hammer exciting waves in a plate
- The waves in the structure will travel, bounce on boundaries, travel back, interact with each and create phenomena of constructive and destructive interference.



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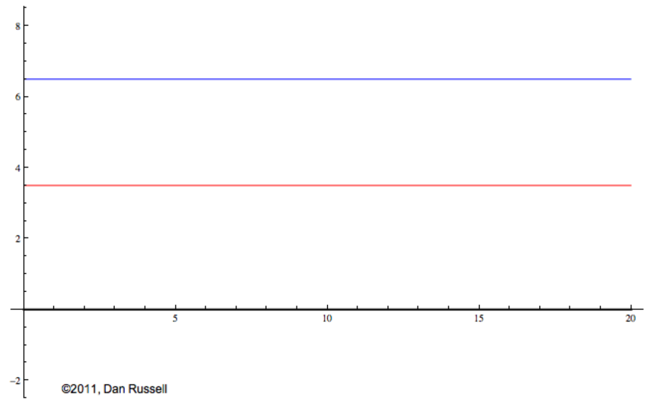
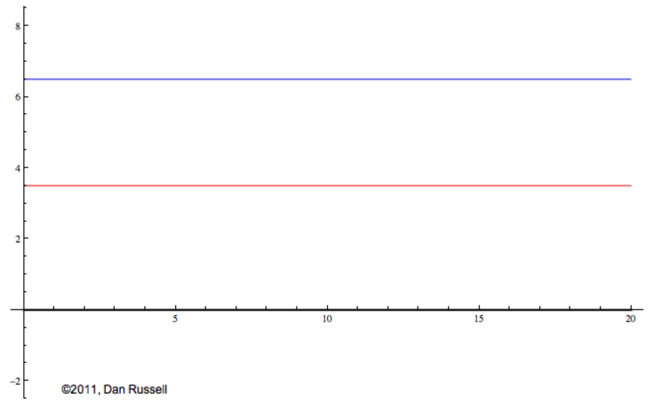
Wave phenomena



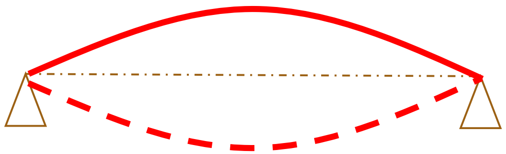
- A pattern in space is created from these reflections resulting in waves interfering constructively and destructively with each other.
- This pattern is called mode (eigenmode)
- The frequency at which this pattern occurs is natural frequency (eigenfrequency)

Wave phenomena



- Just as for the SDOF mass-spring system, natural frequencies and the corresponding spatial pattern of natural modes are intrinsic characteristics of any structure – structure *like* to move at those frequencies.

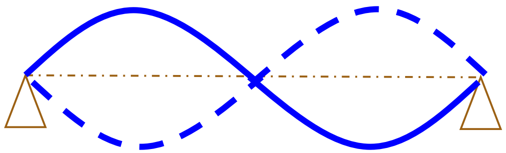
Standing waves in a string



$\lambda=2L$

$f_1=c/2L$ (natural frequency)

Fundamental eigenfrequency / 1st harmonic



$\lambda=L$

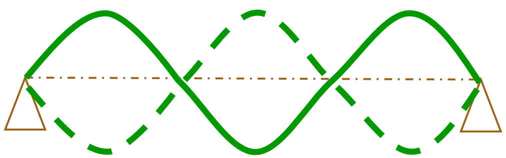
$f_2=2f_1$ (natural frequency)

Second eigenfrequency / 2nd harmonic

In general:

$\lambda=2L/n$

$f_n=n \cdot c/2L$



$\lambda=(2/3)L$

$f_3=3f_1$ (natural frequency)

Third eigenfrequency / 3rd harmonic

Eigenmode: different ways a string (structure in general) can vibrate generating standing waves

Examples: 1 / 2 / 3 / 4.

Modes in structures - plates

- Standing waves in a plate

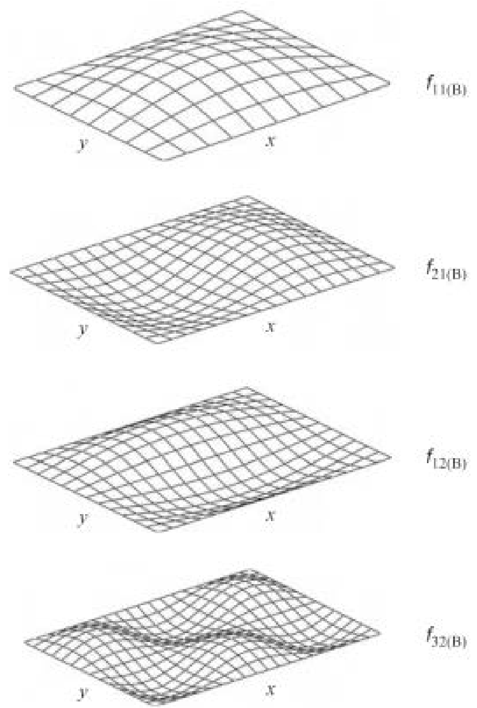


Figure 2.32

Mode shapes for bending modes on a plate with simply supported boundaries.

Source: Carl Hopkins, *Sound Insulation*

Eigenmode: different ways a plate (structure in general) can vibrate generating standing waves

Musical instruments

- Systems' eigenfrequencies are derived without external forces – the homogeneous solution.
- Interesting phenomena happens on the other hand when external forces with their own driving frequencies interact with systems' eigenfrequencies – i.e. resonance phenomena happen – the particular solution.
- Systems eigenfrequencies, and accordingly systems' response to sound and vibrations, will therefore sustain, maintain and add character to external driving frequencies. In musical acoustics one speaks of loudness, quality, timbre.
    - Think about musical instrument, concert rooms.
    - When this interaction is not properly managed though problems will occur (collapsing bridges due to external excitation is an extreme example).
- Stage to Tim Näsling, acoustician and guitar maker.

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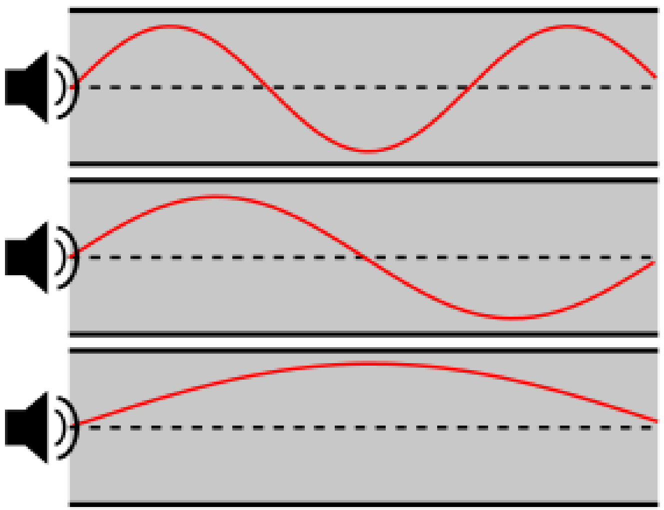
Music instruments: wood-wind



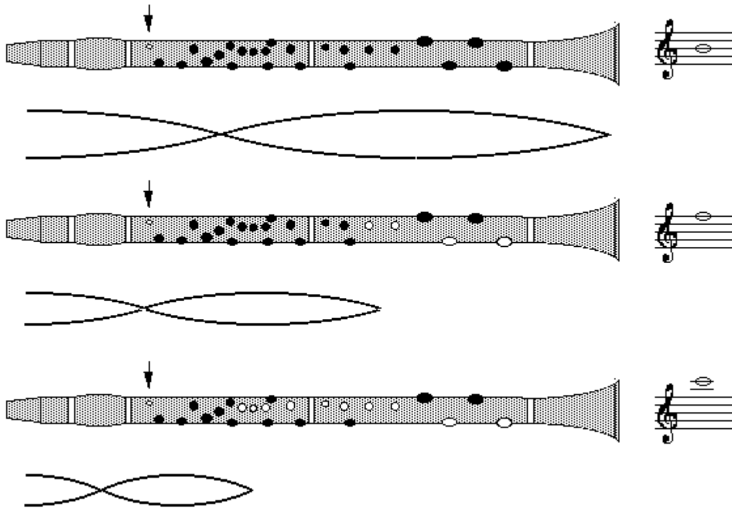
Open-open / Closed-closed:

$\lambda = 2L/n$

$f_n = n \cdot v/2L$

NOTE: Open-closed vary

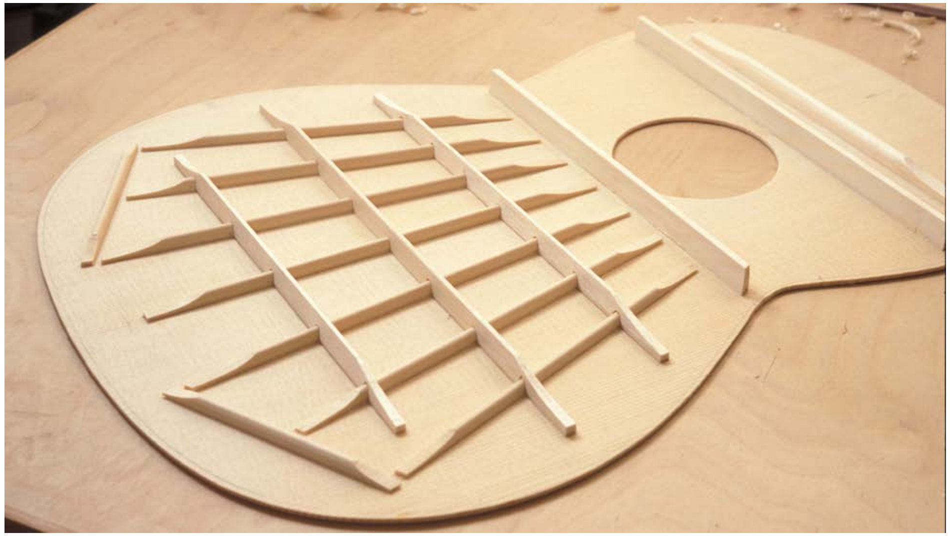
$v = (\text{temperature}/\text{molecular weight})^{1/2}$



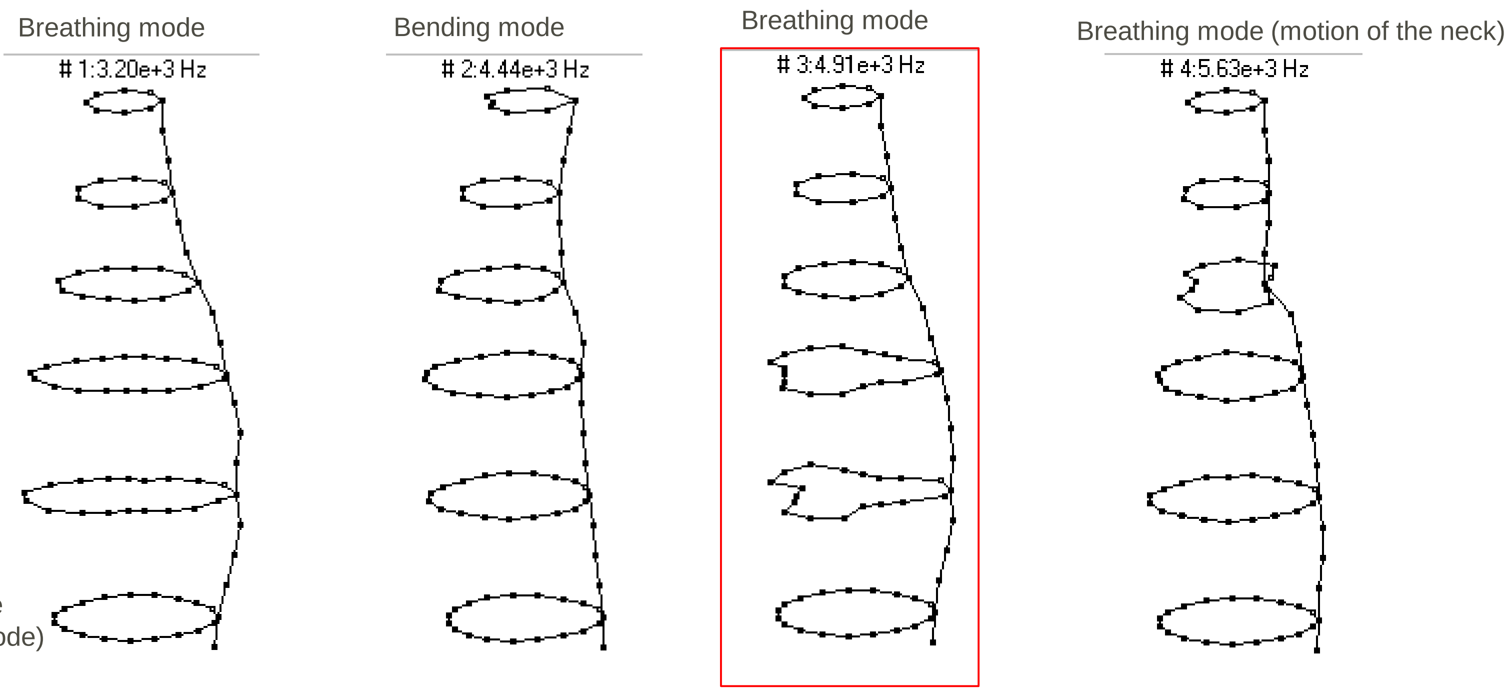
Cover holes → Vary L

Change of v (molecular weight)

Music instruments: soundboards

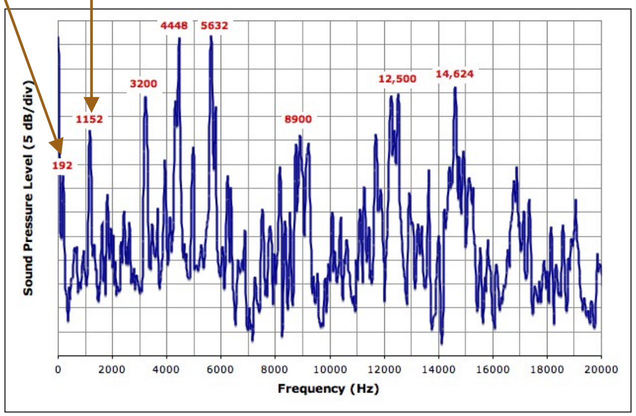


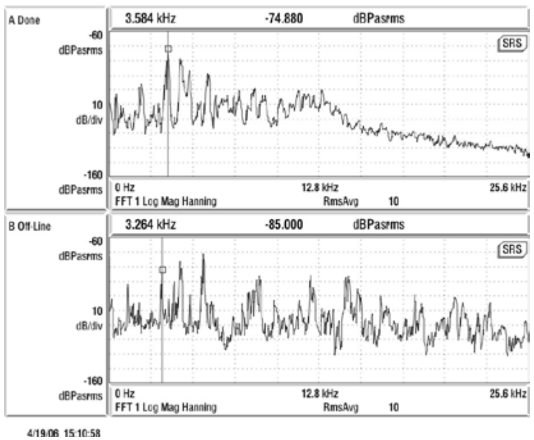
Glass beer bottle



Cavity resonance (Helmholtz)

Cavity resonance (higher mode)





With water

No water

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Standing waves – Pressure profile

Standing waves – Pressure profile

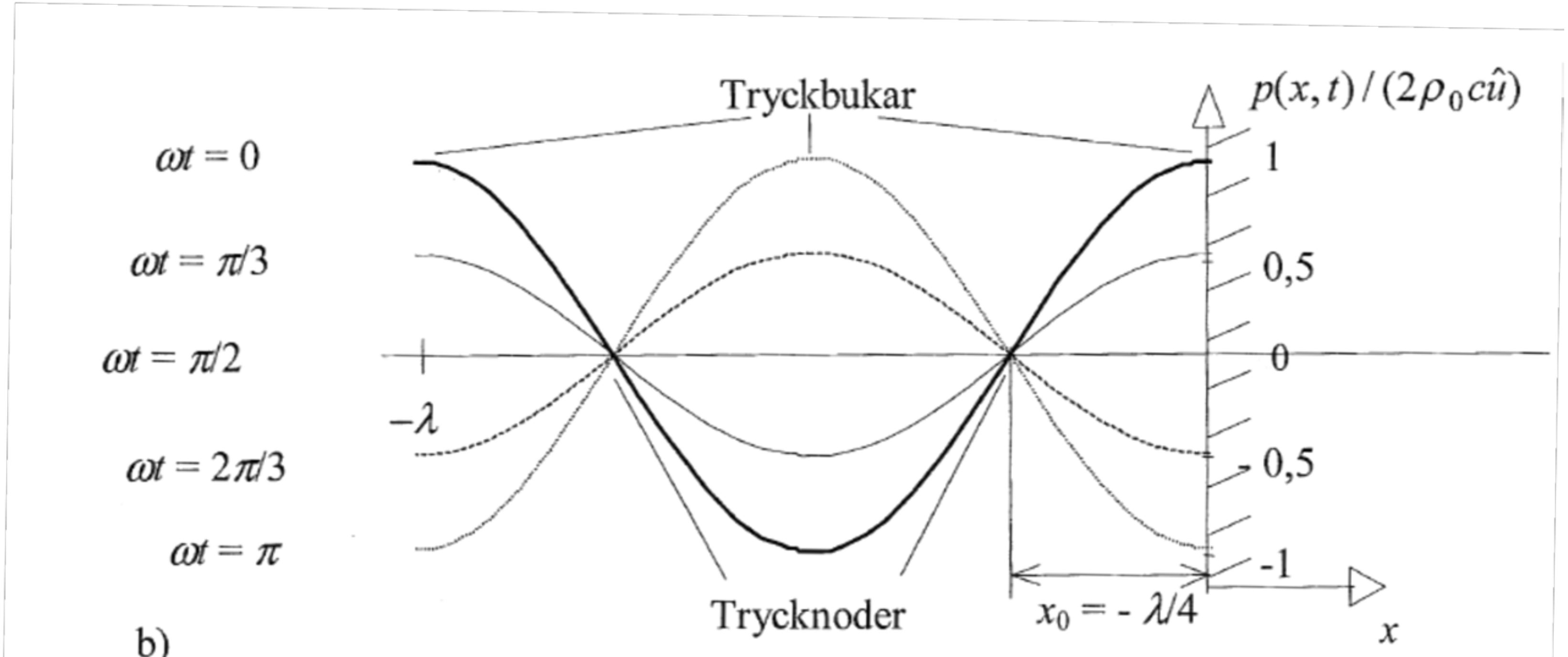

Figure: Ljud och vibrationer - Bodén

- Pressure profile: $p(x,t) = 2\hat{p}_+ \cos(kx)\cdot e^{i\omega t}$
  - Nodes if: $\cos(kx) = 0, \ (x<0)$ $\quad x = -\left(\frac{1}{2}+n\right)\frac{\lambda}{2}$
  - Maximum if: $x = -\lambda/2, -\lambda, -3\lambda/2$ osv. → Measurement by a hard wall

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Transmission and reflection

$$Z_1 = \rho_1 c_1$$

$$p_i(x,t) = \hat{p}_i e^{\mathrm{i}(\omega t - k_1 x)}$$

$$u_i(x,t) = \frac{\hat{p}_i}{\rho_1 c_1} e^{\mathrm{i}(\omega t - k_1 x)}$$

$$p_r(x,t) = \hat{p}_r e^{\mathrm{i}(\omega t + k_1 x)}$$

$$u_r(x,t) = -\frac{\hat{p}_r}{\rho_1 c_1} e^{\mathrm{i}(\omega t + k_1 x)}$$

$$Z_2 = \rho_2 c_2$$

$$p_t(x,t) = \hat{p}_t e^{\mathrm{i}(\omega t - k_2 x)}$$

$$u_t(x,t) = \frac{\hat{p}_t}{\rho_2 c_2} e^{\mathrm{i}(\omega t - k_2 x)}$$

$x = 0$

- $$\hat{p}_i + \hat{p}_r = \hat{p}_t$$
  $$\frac{\hat{p}_i}{\rho_1 c_1} - \frac{\hat{p}_r}{\rho_1 c_1} = \frac{\hat{p}_t}{\rho_2 c_2}$$

- $$\frac{\hat{p}_r}{\hat{p}_i} = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} \equiv R = \frac{1 - \frac{\rho_1 c_1}{\rho_2 c_2}}{1 + \frac{\rho_1 c_1}{\rho_2 c_2}}$$ **Reflection coefficient**
  $$\frac{\hat{p}_t}{\hat{p}_i} = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1} \equiv T = \frac{2}{1 + \frac{\rho_1 c_1}{\rho_2 c_2}}$$ **Transmission coefficient**

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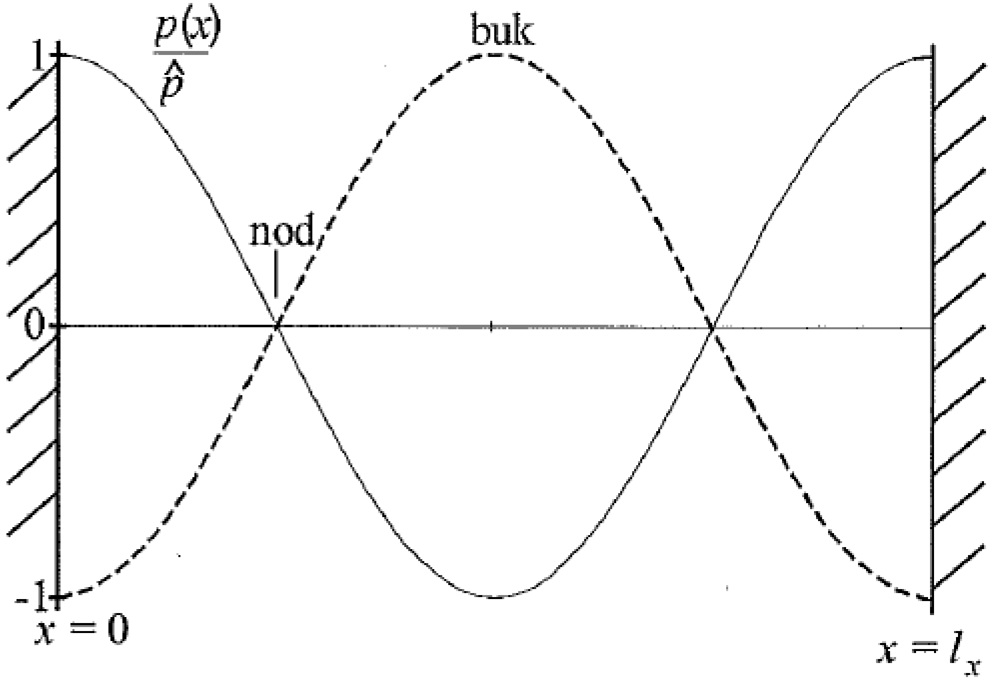
Two hard walls – Pressure function

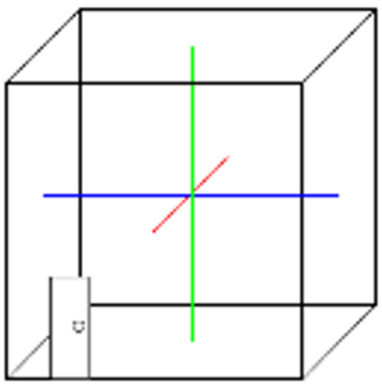
$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \xrightarrow[\text{solution}]{\text{Homogeneous}} \lambda_n = \frac{2\pi}{k_n} = \frac{2\pi L}{n\pi} = \frac{2L}{n} \longrightarrow f_n = \frac{c}{\lambda_n} = \frac{c}{2}\frac{n}{L}$$



Room eigenfrequencies and eigenmodes

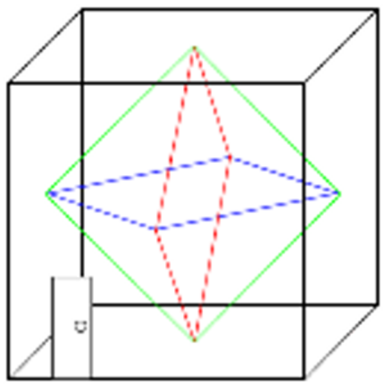
- Eigenfrequencies of a 3D room $f_{n_x,n_y,n_z} = \frac{c}{2}\sqrt{\left(\frac{n_x}{L}\right)^2 + \left(\frac{n_y}{B}\right)^2 + \left(\frac{n_z}{H}\right)^2}$

- Types of modeshapes. Link.



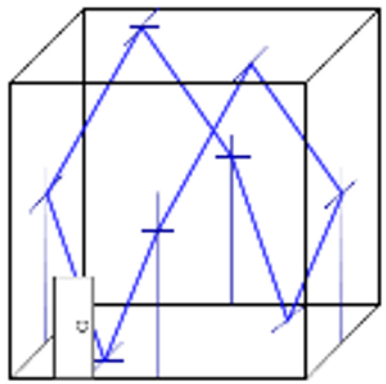
Axial modes 1D

Two n-indexes are 0



Tangential modes 2D

One n-index is 0



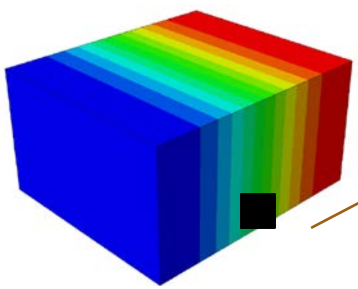
Oblique modes 3D

No n-index is 0

Eigenmode: different ways air in a room can vibrate generating standing waves

Forced response and modes

- Depending on the spatial location of the driving loudspeaker, different modes are excited.
- E.g. modes that are not excited by the loudspeaker in the middle position have a node in that point.
- All modes have a peak or a valley at a corner (hard walls).



(1,0,0)

Source: Carl Hopkins, *Sound Insulation*

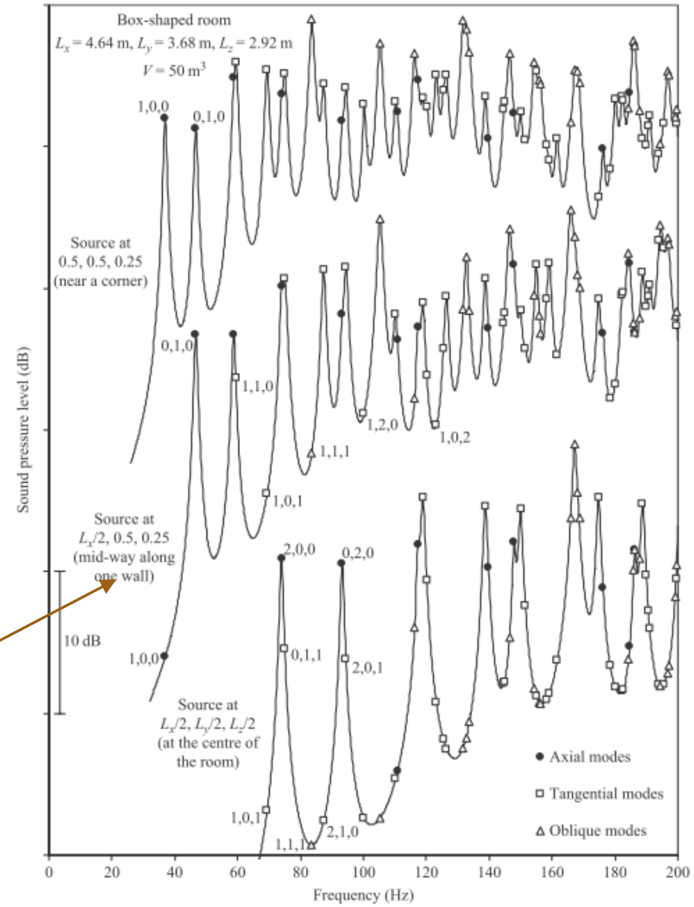


Figure 1.37

Excitation of room modes with three different source positions. Curves for the sound pressure level in the corner position (L_x, L_y, L_z) are shown along with the axial, tangential and oblique mode frequencies to assess which modes are, and which modes are not excited by the source position. Note that the curves have been offset from each other; this allows the relative levels along each individual curve to be assessed, but not the relative levels between different curves.

Mode count in a room

- How many modes we have in a room?

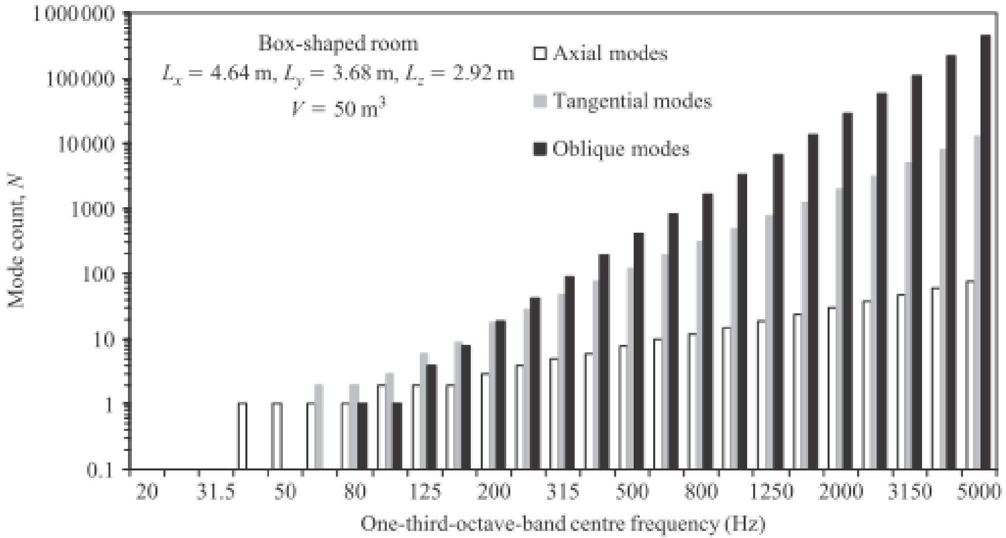


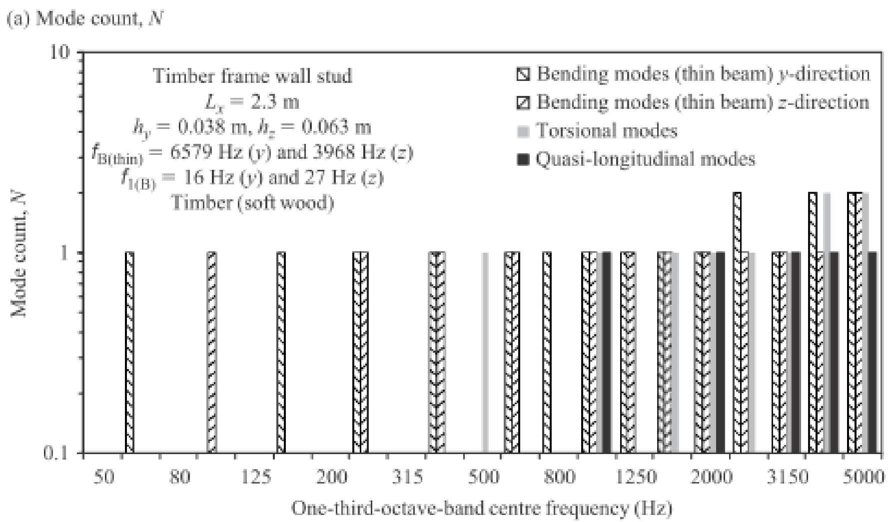
Figure 1.14

Mode count for axial, tangential, and oblique modes in a 50 m^3 box-shaped room.

Source: Carl Hopkins, *Sound Insulation*

Modes in structures - beams

- Each structure has its own eigenfrequencies (modes) – like the string.
- In particular each kind of wave motion has its own eigenfrequencies.
- At lower frequencies only bending modes are present.
    - And not so many modes either



Source: Carl Hopkins, *Sound Insulation*

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Modes in structures - plates

- Each structure has its own eigenfrequencies (modes) – like the string and the beam.
- In particular each kind of wave motion has its own eigenfrequencies.
- At lower frequencies only bending modes matter.

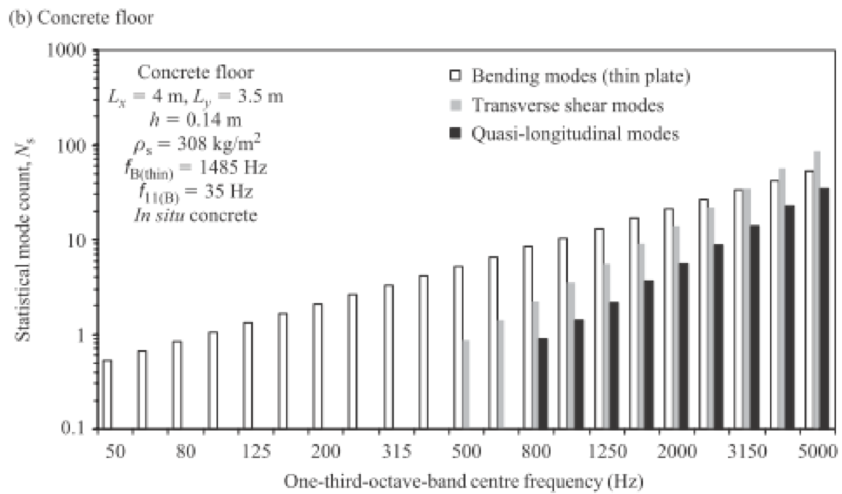

Figure 2.28

Plates: statistical mode counts for a sheet of plasterboard, a concrete floor, and two different masonry walls. Note that values are only shown at and above the frequency band that contains the estimated fundamental mode for each wave type.

Source: Carl Hopkins, *Sound Insulation*

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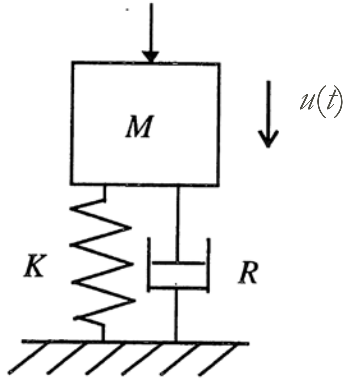
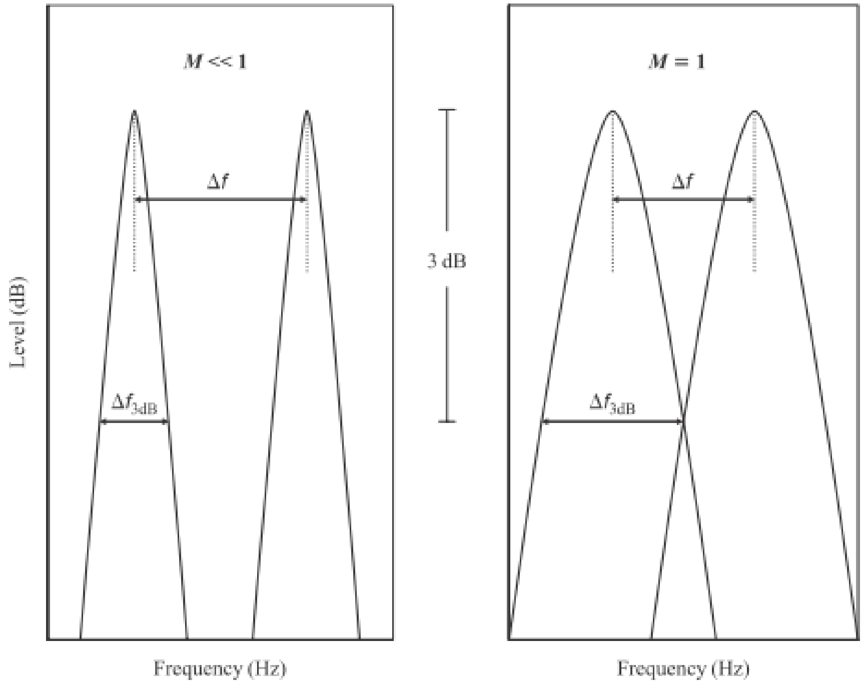
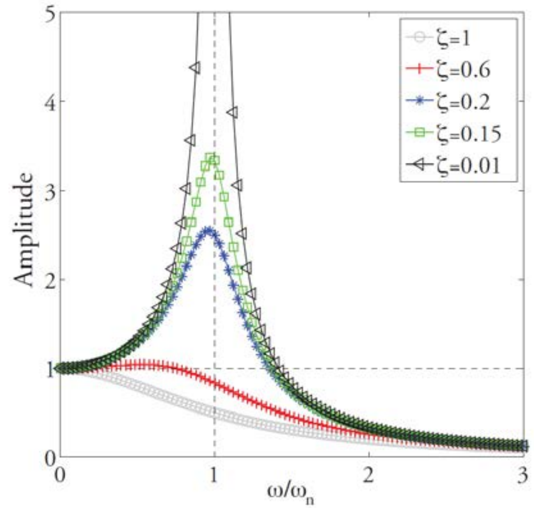
Overlap of modes

- Damping will make modes overlap.

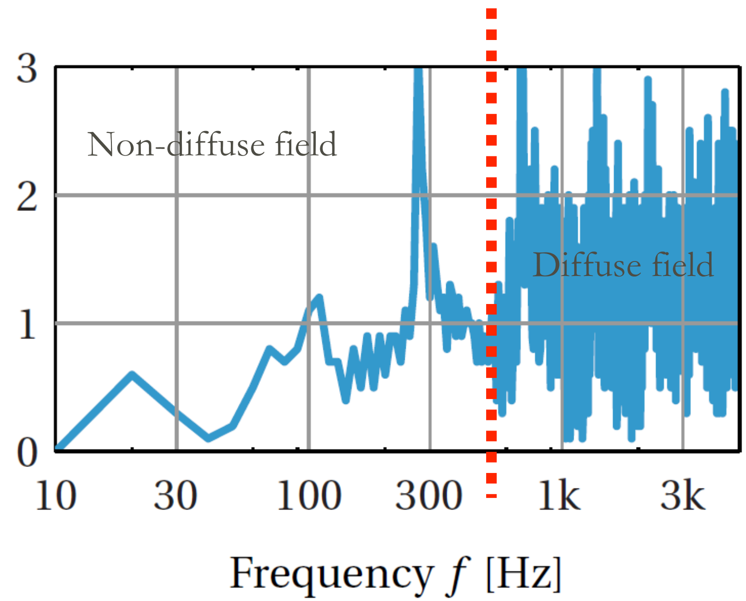




Source: Carl Hopkins, *Sound Insulation*



Diffuse field & Non-diffuse field



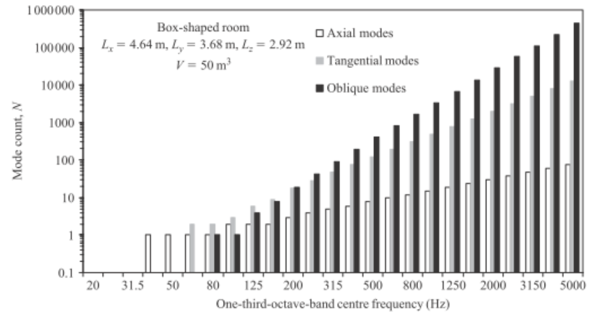
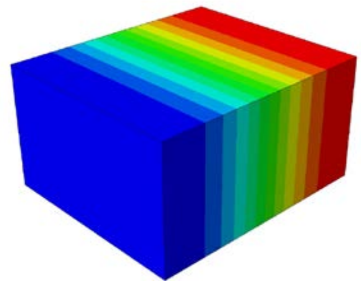


Figure 1.14

Mode count for axial, tangential, and oblique modes in a 50 m^3 box-shaped room.



- Low modal density in the low frequency range → measurement problems
- Higher modal density in high-frequency range
- Limit between both "behaviours" → Schroeder frequency
  - More about this in room acoustics lectures

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Sound propagation

- Definitions – different kinds of acoustic field
- Quantities – SPL, SIL, SWL
- Laws – distance
- Phenomena – wind and temperature gradients, diffraction

Diffuse field

Free field

Sound (acoustic) intensity – definition

- Sound power (i.e. rate of energy) per unit area [W/m²]
  - Instantaneous value: $\vec{I}(t) = p(t)\vec{v}(t)$
  - Vector quantity: energy flow and direction: $\bar{\vec{I}} = \langle pv \rangle = \frac{1}{T}\int_0^T p(t)\vec{v}(t)dt$
  - In a free field (plane waves): $\bar{I} = \frac{\widetilde{p^2}}{\rho c}$; $\bar{I} \propto p^2$
- In decibels… $L_I = 10\log\left(\frac{\bar{I}}{I_{ref}}\right)$; $I_{ref} = 10^{-12}\ {}^{W}/_{m^2}$

NOTE 1: $\bar{I}$ is the magnitude of the time average $\bar{\vec{I}}$

NOTE 2: $p(t)$ is the particle pressure and $\vec{v}(t)$ the particle velocity

NOTE 3: Free field occurs when the sound field is not influenced by any surrounding object or close surfaces

NOTE 4: In a perfectly diffuse sound field the sound intensity is zero

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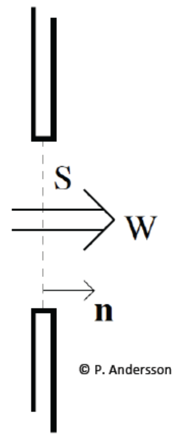
Sound (acoustic) power – definition

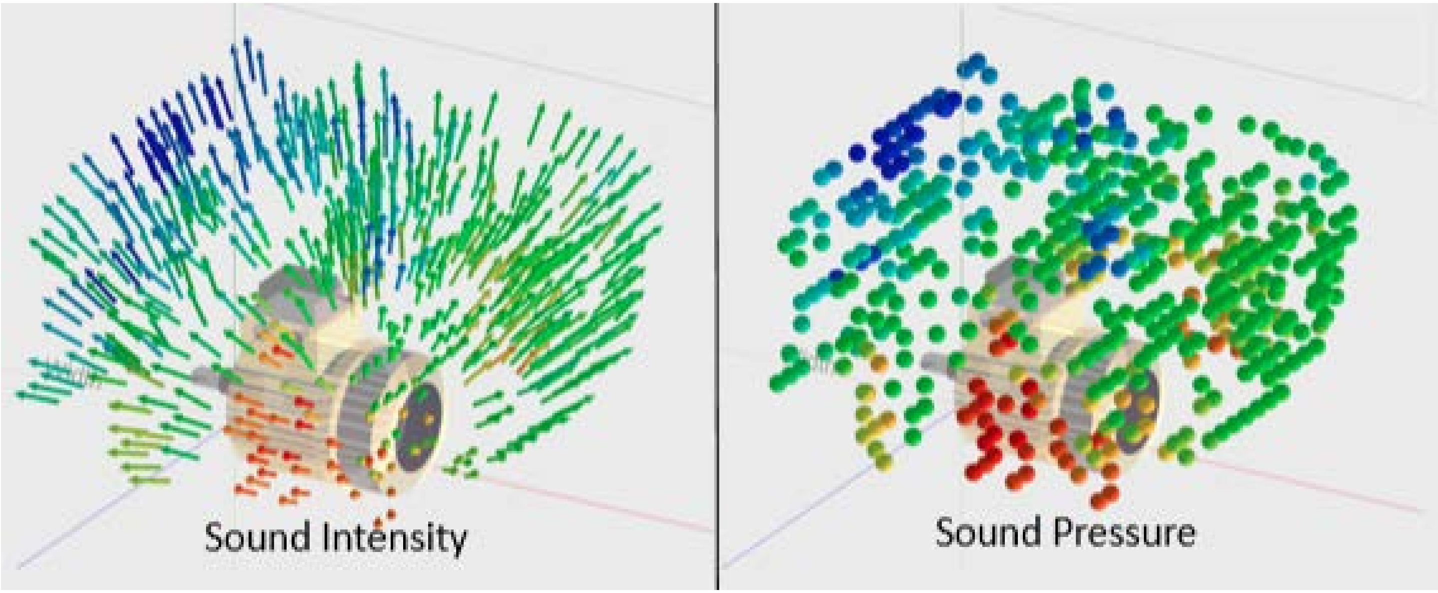
- Rate of energy transported through a surface [W=J/s]
- Instantaneous value: $$W(t) = \int_S \vec{I}(\vec{x},t)\cdot\vec{n}dS = \int_S I_n(\vec{x},t)dS$$



Relation between SPL, SWL, SIL

- Sound Pressure (SPL), Sound Power (SWL), and Sound Intensity (SIL) acoustic quantities that can be expressed in dB. They describe different aspects of sound, and the decibels for each represent different measurement quantities.
  - SPL, [Pa]
    - Amplitude level of sound at a specific location in space (scalar quantity)
    - Dependent on the location and distance to the source
    - Property of the sound field
  - SWL, [W]
    - Rate at which sound is emitted from an object
    - Independent of location or distance
    - Scalar quantity, property of the source
  - SIL, [W/m^2]
    - Sound power flow per unit of area
    - Vector quantity
    - Sound energy quantity

Relation between SPL, SWL, SIL



Amplitudes are the same / Directions are the difference (easier to troubleshot with SI)

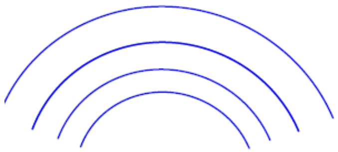
Relation between SPL, SWL, SIL

Receiver ⟷ Sound Pressure Level

$$L_p = 10\ \log_{10}\frac{p^2}{p_o^{\,2}}$$

$$p_o = 2\times10^{-5}\,N/m^2$$

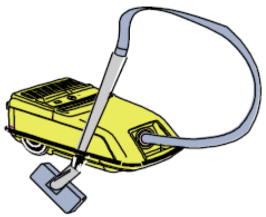
$$= 20\,\mu Pa$$



Path ⟷ Sound Intensity Level

$$L_i = 10\ \log_{10}\frac{I}{I_0}$$

$$I_o = 1pW/m^2$$



Source ⟷ Sound Power Level

$$L_w = 10\ \log_{10}\frac{W}{W_o}$$

$$W_o = 1pW$$

www.bksv.com, 2

Brüel & Kjær
BEYOND MEASURE

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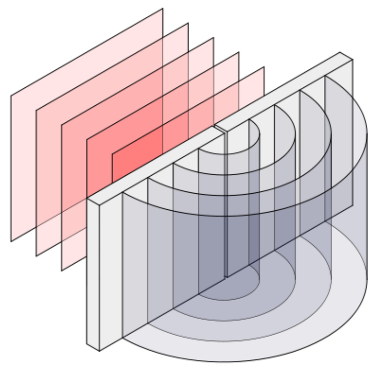
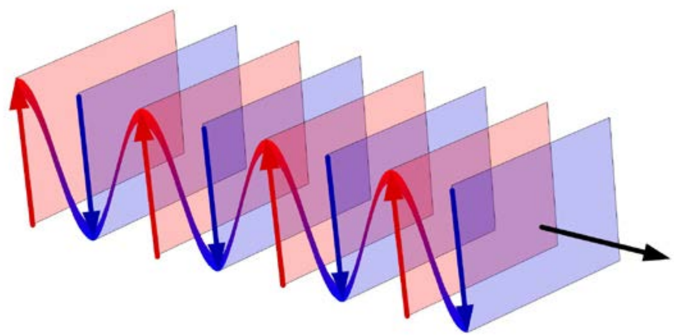
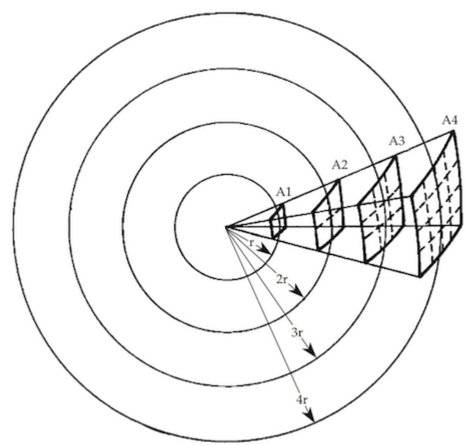
Types of propagation

- Plane: $\quad I \equiv \text{constant}\,;$
- Cylindrical: $\quad I(r) \propto \dfrac{1}{r}; \quad I(r) = \dfrac{\Pi}{2\pi h r}$
- Spherical: $\quad I(r) \propto \dfrac{1}{r^2}; \quad I(r) = \dfrac{\Pi}{4\pi r^2}$







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Distance laws

- Spherical propagation (point source)

$$\Delta L = L(r_2) - L(r_1) = -20\log\left(\frac{r_2}{r_1}\right)$$

Doubling the distance…

$$\Delta L = L(2r_1) - L(r_1) = -6\text{dB}$$

- Cylindrical propagation (line source)

$$\Delta L = L(r_2) - L(r_1) = -10\log\left(\frac{r_2}{r_1}\right)$$

Doubling the distance…

$$\Delta L = L(2r_1) - L(r_1) = -3\text{dB}$$

- Plane wave

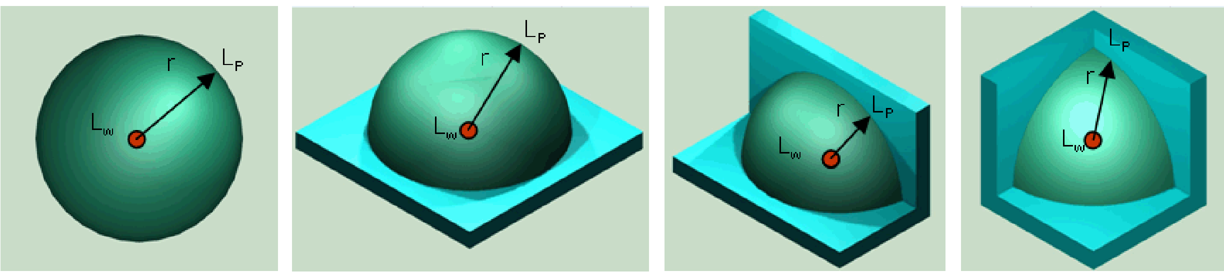
$$\Delta L = L(r_2) - L(r_1) = 0$$

Doubling the distance…

$$\Delta L = L(2r_1) - L(r_1) = 0$$

Emission and directivity factor

- Sound emission
  - Sound power continuously emitted from a sound source
- Sound power level (SWL / L_W / L_Π) or acoustic power
  - We have seen how it is related to sound intensity and sound power.
  - Each source radiates with a certain directivity
    - Described by directivity factor Q



Source: www.sengpielaudio.com

$$L_W = L_p + \left| 10 \log \left(\frac{Q}{4\pi r^2} \right) \right|$$

- Q=1: Full sphere
- Q=2: Half sphere
- Q=3: Quarter sphere
- Q=4: Eighth sphere

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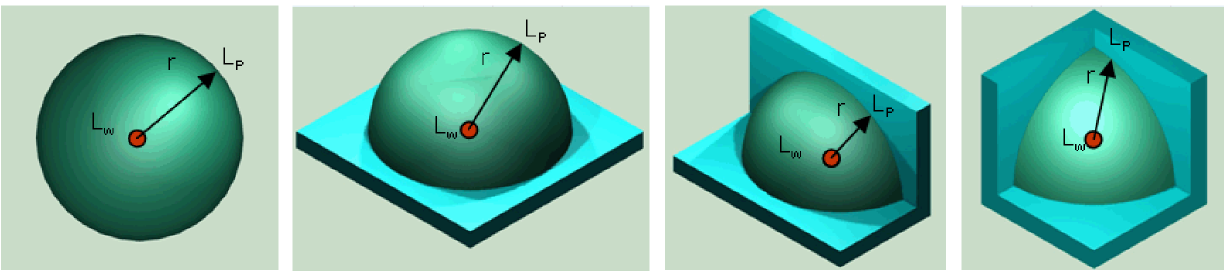
How it goes in practice

- There is an industry with many noise sources.
- New apartment buildings are planned in a adjacent field. Or the industry wants to increase its production capacity increasing its buildings/machineries.
- Acousticians go to the plant and estimate sound power of various sources at short distance – in this way each source is measured in an objective way.
- Go back to the office, and from the estimated sound power compute sound pressure at new distances; perhaps add new sources, or modify existing ones with lower SWL.
- Evaluate calculated SPL with regulations.

$$L_W = L_p + \left| 10 \log \left(\frac{Q}{4\pi r^2} \right) \right|$$



Source: www.sengpielaudio.com

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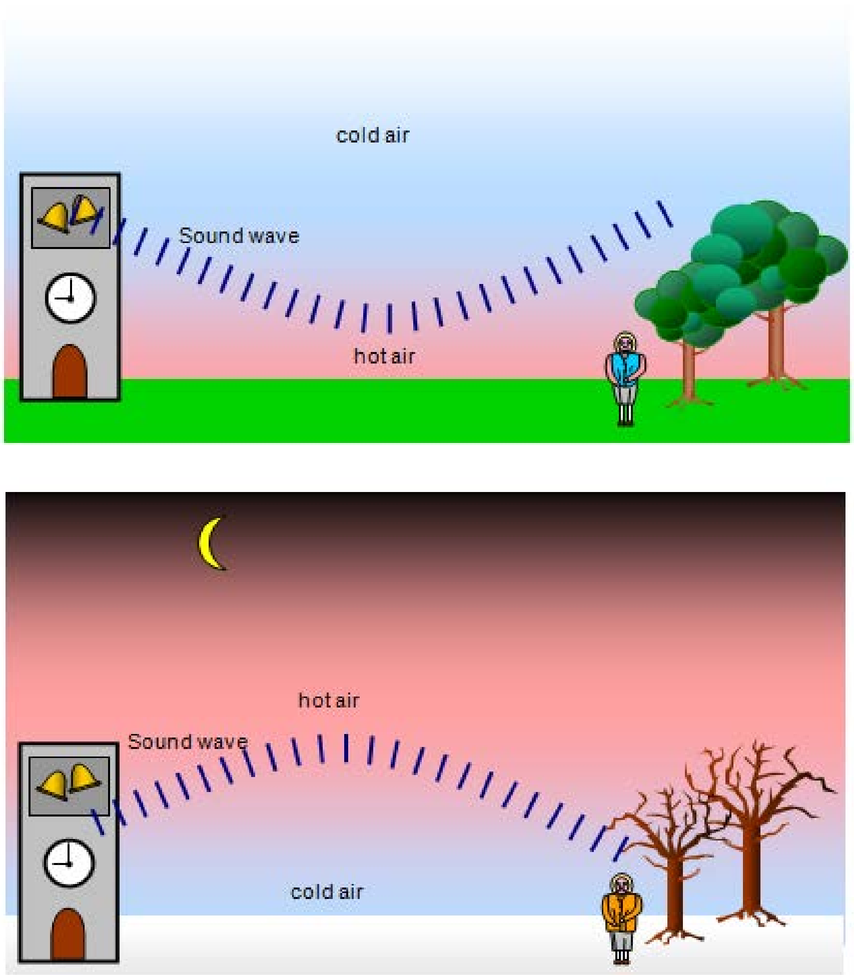
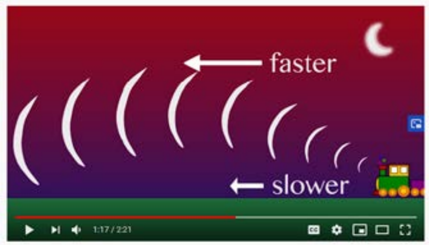
Outdoor sound propagation – Temperature (II)



Source: http://www.schoolphysics.co.uk



https://youtu.be/ZgwEAUHpNrs

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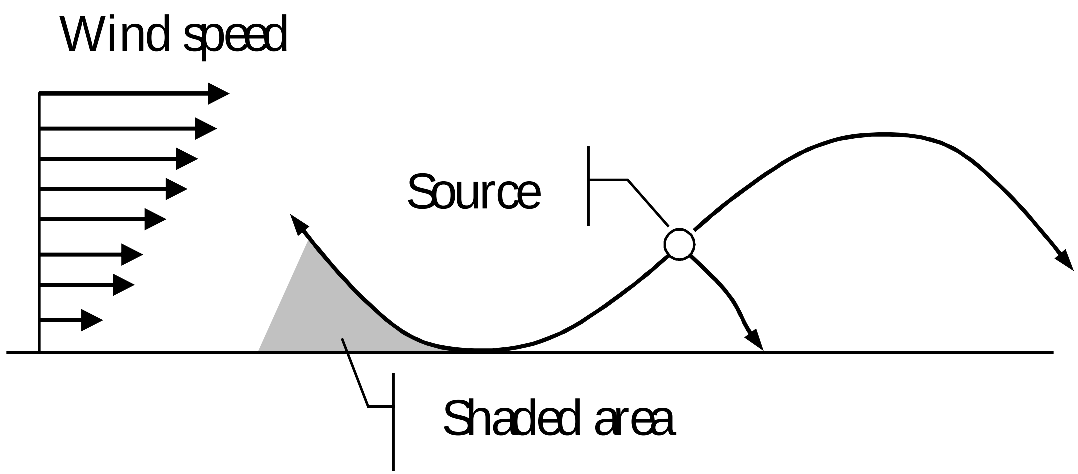
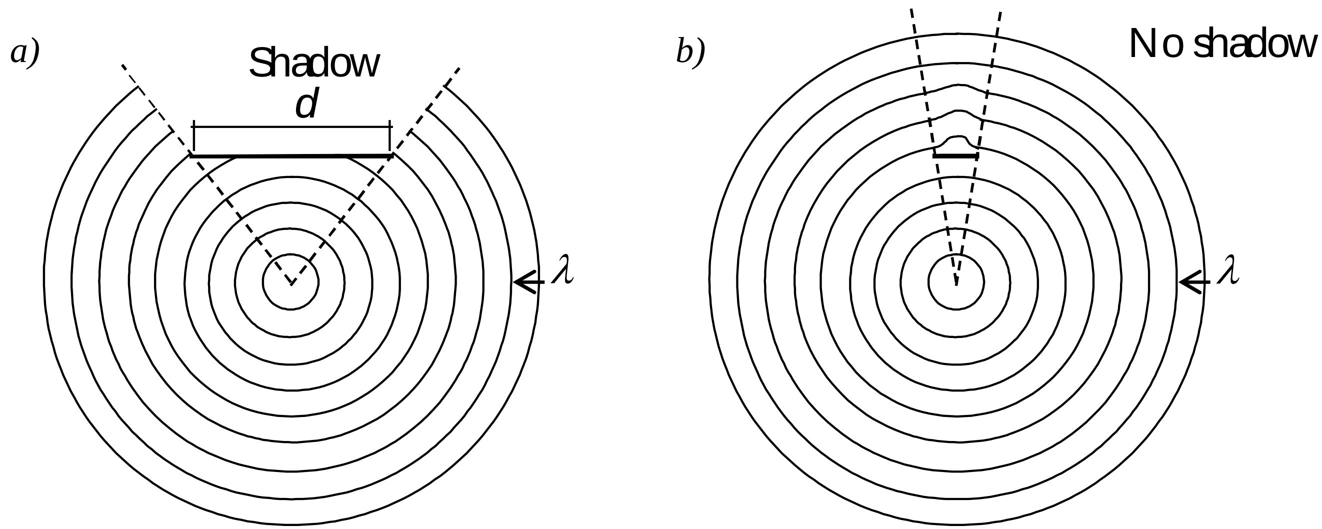
Outdoor sound propagation – Wind

- Generally greater than the temperature dependence
- Upwind / Downwind
- SPL reduction due to turbulence: 4-6 dB/100m
  - Independent of wind direction
  - More obvious the greater the wind speed is



Diffraction – Sound ”bending”

- Om $d > \lambda$ → the obstacle ”exists”
- Om $d < \lambda$ → the sound bends around the obstacle



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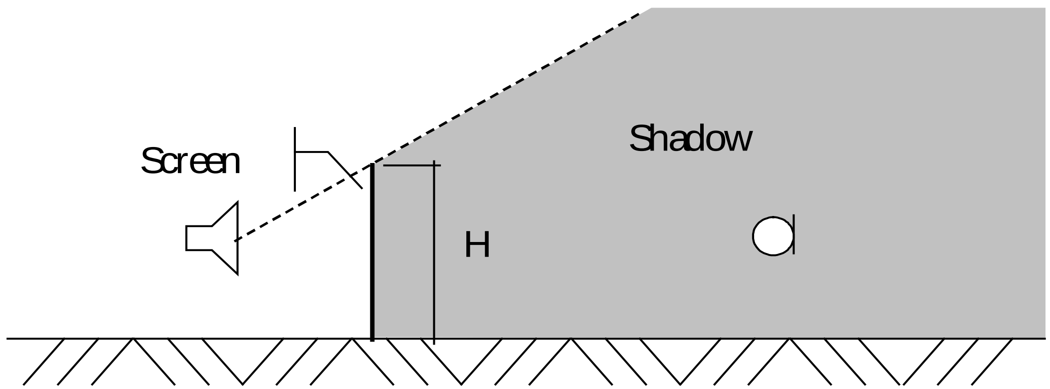
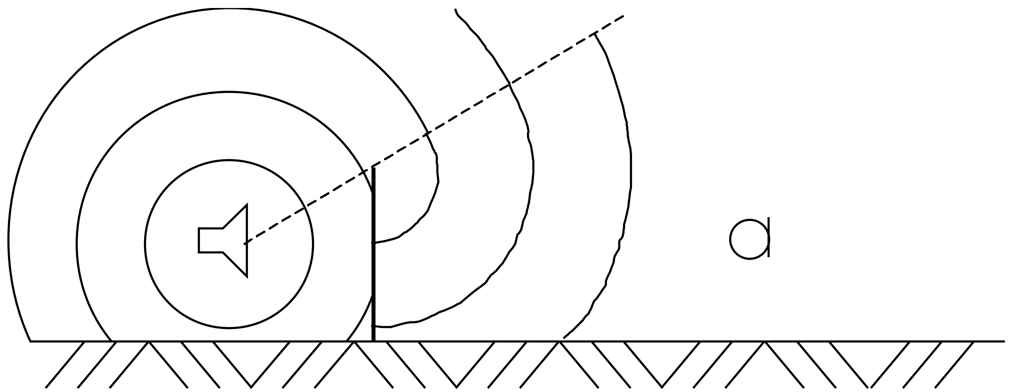
Noise barriers

- $\lambda << H$



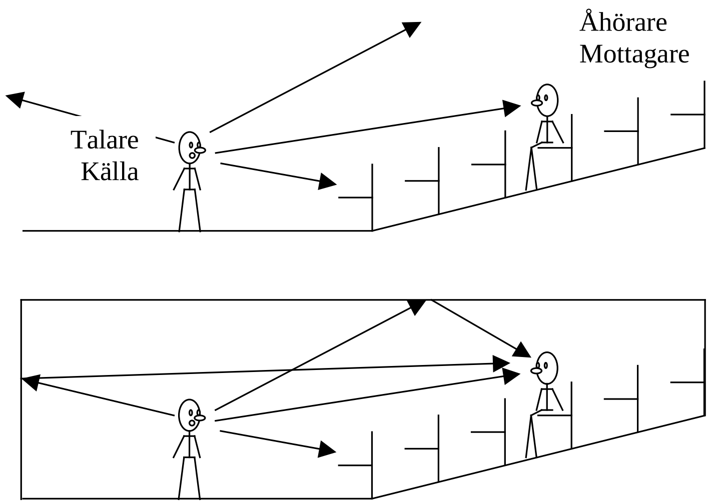
- $\lambda >> H$



MORE ABOUT THIS IN THE TRAFFIC NOISE LECTURE

Indoor sound propagation?

- Indoor sound propagation comprises effects of absorption and reflection.
- Basic concepts on the three types of propagations hold in principle.
    - Office landscape example of cylindrical propagation?
- More on that after the break (room acoustics).



Thank you for your attention!

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