RECORDING

Ljud i byggnad och samhälle (VTAF01) – Waves in air

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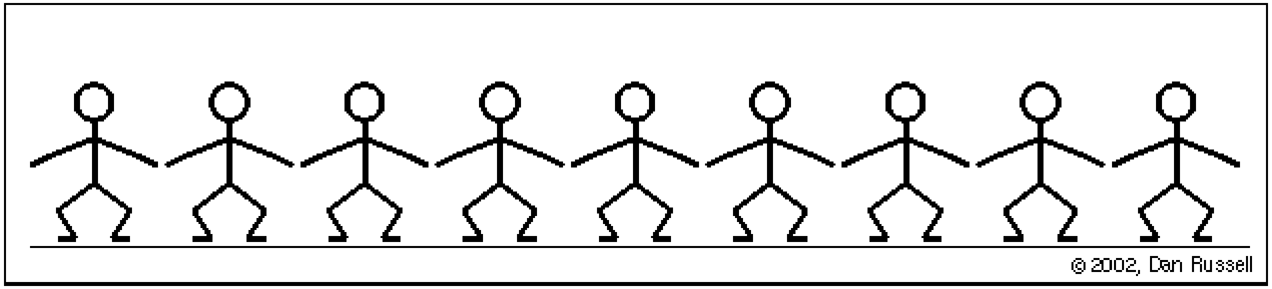
Reminder

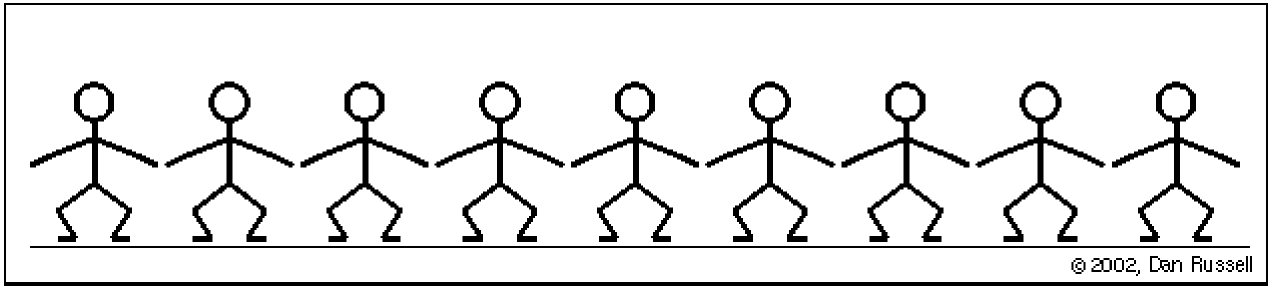


- Almost all animations come from this great website:
  - https://www.acs.psu.edu/drussell/Demos.html

Recap from previous lecture F3

- Recap...

What is a wave then?



- A disturbance or deviation from a pre-existing condition. Its motion constitutes a transfer of information from one point in space to another.
- Time plays a key role – static displacement of a rubber band is a disturbance but not a wave.
  - Wave travels at finite speed (hitting a perfectly rigid rod making the rod moving as a unit is no wave, just rigid body motion)
  - The rod is elastic, the impulse travels from one end to the other.
- All mechanical waves travel in a material medium (unlike e.g. electromagnetic waves)
- Many waves satisfy $c^2\nabla^2 u - \ddot{u}$=0 – but not all!

Derivation of longitudinal wave equations (I)

- General approach to derive equations of motion:

1. Newton's law – dynamic equilibrium
2. Constitutive relations – forces, stresses and strains
   - Relations between two physical quantities in a material

   a. Force – stress

   b. Stress – strain
3. Strain – displacement relation (definition)

General form of a wave equation

One dimension: $\frac{\partial^2 u_x}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u_x}{\partial t^2} = 0$

Three dimensions: $c^2\nabla^2 u - \ddot{u}=0$

$\Delta f = \nabla^2 f = \nabla \cdot \nabla f$ $\quad$ $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$ Laplacian

https://en.wikipedia.org/wiki/Laplace_operator

- It took some physics reasoning and some math but now we have an expression that we can use with most wave types that are relevant in acoustics and vibrations!
  - Not however with the most important structural waves in acoustics, which is a bit special!

Recap - Types of waves in solid media

- Longitudinal waves (∞ medium ≈ beams)
  - Quasi-longitudunal waves (finite ≈ plates)

$$\frac{\partial^2 u_x}{\partial x^2} - \frac{\rho}{E'}\frac{\partial^2 u_x}{\partial t^2} = 0$$

$$c_L = \sqrt{\frac{E}{\rho}}$$

$$c_{qL} = \sqrt{\frac{E'}{\rho}} = \sqrt{\frac{E}{\rho(1-\upsilon^2)}}$$

- Shear waves

$$\frac{\partial^2 u_y}{\partial x^2} - \frac{\rho}{G}\frac{\partial^2 u_y}{\partial t^2} = 0$$

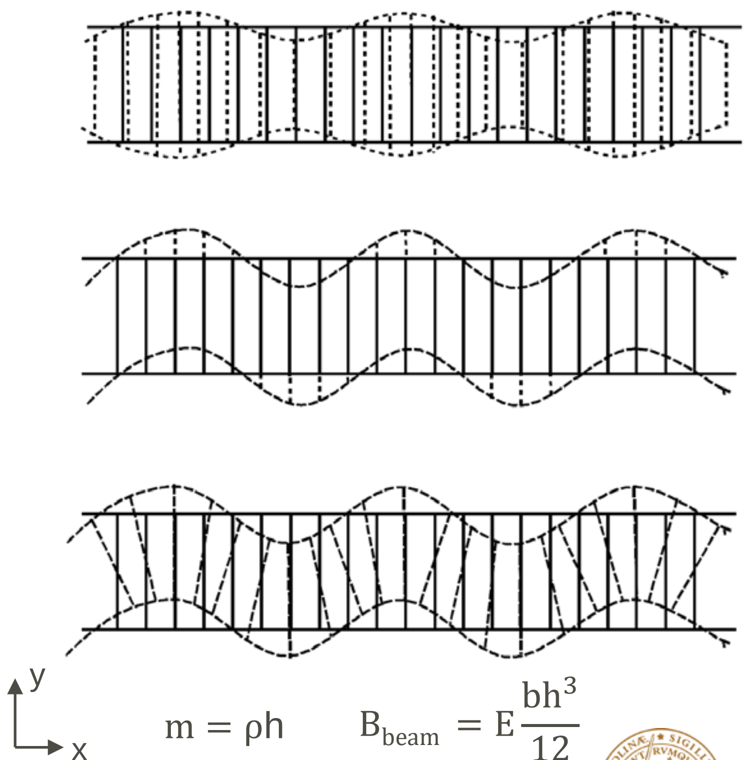
$$c_{sh} = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{2(1+\upsilon)\rho}}$$

- Bending waves (dispersive)

$$B\frac{\partial^4 u_y}{\partial x^4} + m\frac{\partial^2 u_y}{\partial t^2} = 0$$

$$c_{B(\omega)} = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$$

Plate: E, G, ρ, υ, h



$$m = \rho h \qquad B_{beam} = E\frac{bh^3}{12}$$

$$B_{plate} = \frac{Eh^3}{12(1-\upsilon^2)}$$

NOTE: torsional waves (beams and columns) are not addressed here

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Solution to the wave equation

One dimension: $\frac{\partial^2 u_x}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u_x}{\partial t^2} = 0$

- What is the simplest shape of a sound wave?
  - An harmonic shape of sinusoidal shape
    - » $u = A \sin a(x - ct); u = A \cos a(x - ct)$
  - It follows that $u = Ae^{\pm ia(x-ct)}$ is also a solution.
  - As $u = A \ln a(x - ct)$; or $u = A\sqrt{a(x - ct)}$, which are not oscillatory.
- It turns out that any function of the form $u = f(x - ct)$ is solution.
- No assumption is made on f – except on its argument.
  - What does that imply?

Solution to the wave equation

One dimension: $\frac{\partial^2 u_x}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u_x}{\partial t^2} = 0$

- A wave is translated, unchanged in shape, along x (space)
- A given point on a wave is translated unchanged with speed *c*.
- This operation defines propagation!

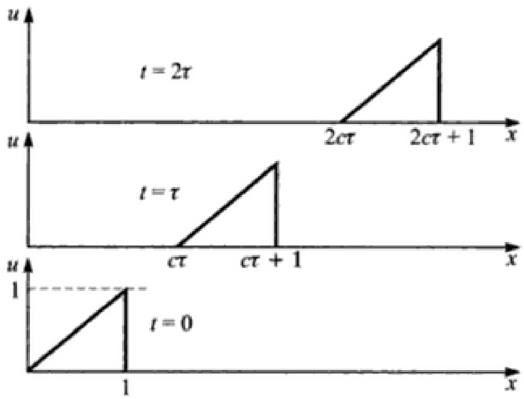

Figure 1.1 Sketches showing the waveform in space when the solution is a section of a ramp function.

Blackstock, Fundamentals of physical acoustics

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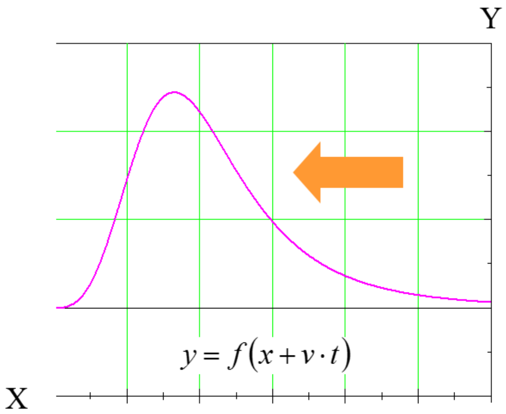
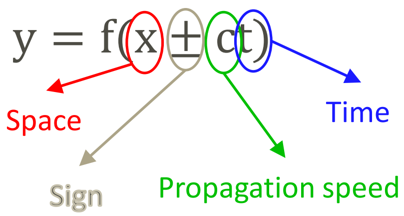
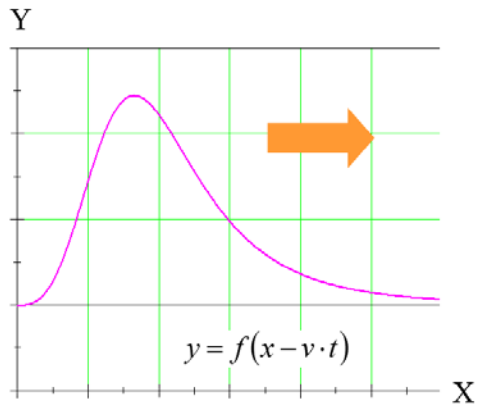
Wave equation solution

- Most general solution is forward and backward travelling wave.
  - d'Alambert's solution







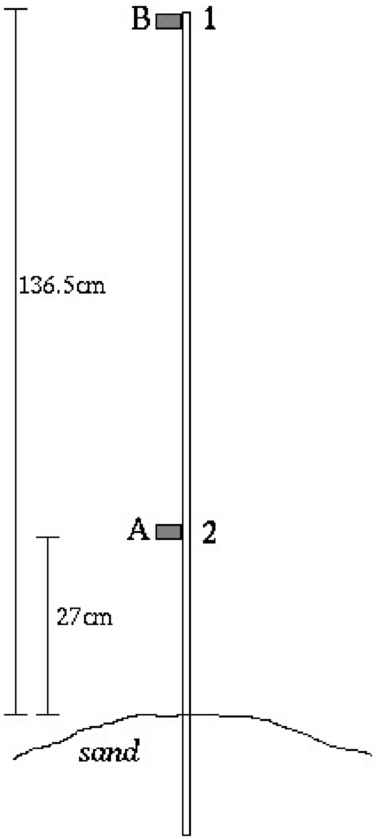
$$y = f\left(x \pm \frac{\omega}{k} t\right) = f\left(\frac{kx \pm \omega t}{k}\right) = f(kx \pm \omega t)$$

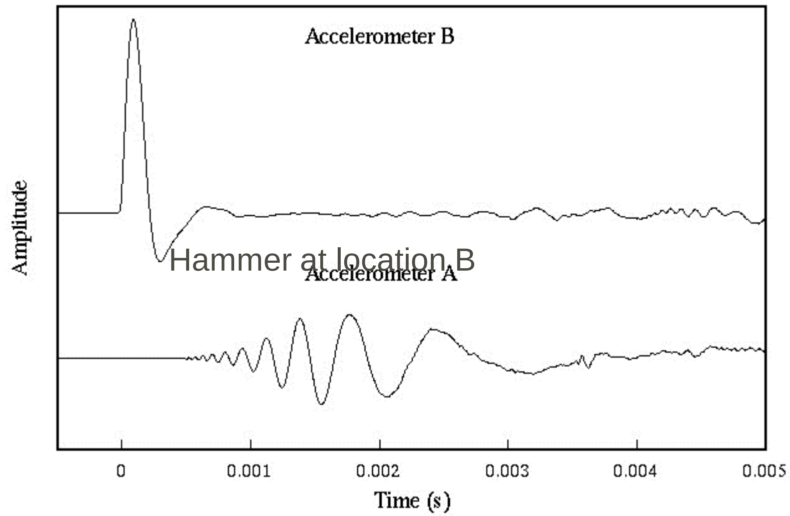
- Alternative forms:

Bending waves

$$B\frac{\partial^4 u_y}{\partial x^4} + m\frac{\partial^2 u_y}{\partial t^2} = 0$$

$$c_{B(\omega)} = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$$





- Force pulse is very clean at location B
- Pulse disperses by the time it reaches location A --- higher frequency waves travel faster and arrive first --- lower frequency waves travel slower and arrive later

https://www.acs.psu.edu/drussell/Demos/Dispersion/Flexural.html

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Bending waves - solution

$$B\frac{\partial^4 u_y}{\partial x^4} + m\frac{\partial^2 u_y}{\partial t^2} = 0$$

- Due to the different form of equations with four-times spatial derivatives, the solution is more complex solutions including *near-field* terms

$$\zeta(x,t) = \hat{\zeta} e^{i(\omega t - k_B x)} . \qquad \zeta(x,t) = (\mathbf{A}e^{ik_B x} + \mathbf{B}e^{-ik_B x} + \mathbf{C}e^{k_B x} + \mathbf{D}e^{-k_B x})e^{i\omega t},$$

- Why near field!?
  - Because they decay rather quickly away from the *boundary* or *load point!*

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Bending waves - solution

- More complex solutions including near-field terms

$$\zeta(x,t) = \hat{\zeta} e^{i(\omega t - k_B x)}. \qquad \zeta(x,t) = (\mathbf{A}e^{ik_B x} + \mathbf{B}e^{-ik_B x} + \mathbf{C}e^{k_B x} + \mathbf{D}e^{-k_B x})e^{i\omega t},$$

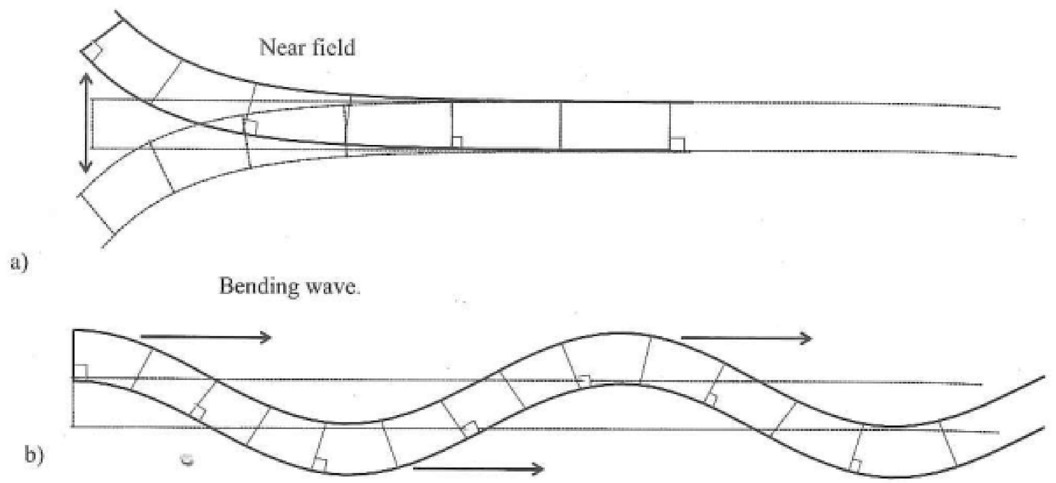

Figure 6-23 Solutions to the bending wave equation near the end of a beam. a) The near field is characterized by an amplitude that decays exponentially with distance from the excitation point. A reasonable engineering approximation would be to ignore the near field at distances greater than 1/3 of the bending wavelength. The near field is significant near boundaries, force application points, and other discontinuities. b) The bending wave can, if the losses are small, spread over large distances.

Source: *Sound and Vibration*, Wallin, Carlsson, Åbom, Bodén, Glav

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Bending waves - solution

- More complex solutions including near-field terms

$$\zeta(x,t) = \hat{\zeta} e^{i(\omega t - k_B x)}.$$

$$\zeta(x,t) = (\mathbf{A}e^{ik_B x} + \mathbf{B}e^{-ik_B x} + \mathbf{C}e^{k_B x} + \mathbf{D}e^{-k_B x})e^{i\omega t},$$

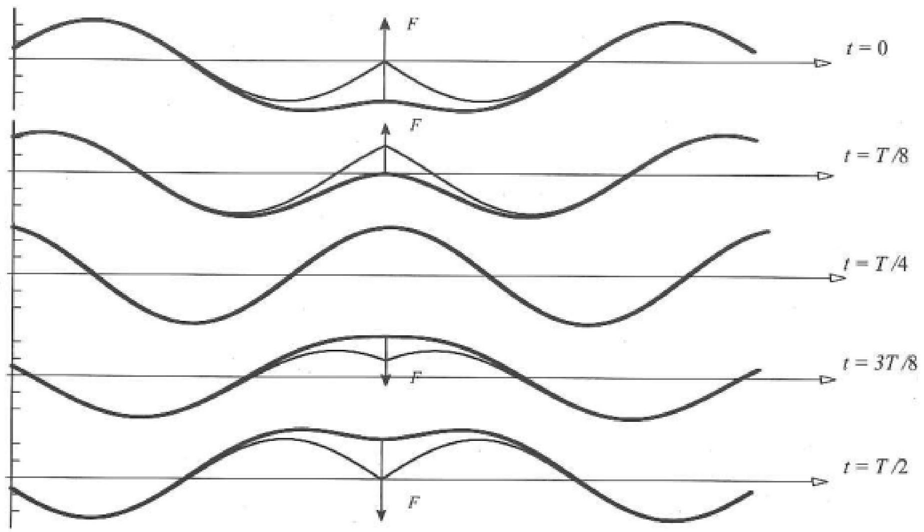


Figure 6-24 The bending wave near field permits a continuous slope. The near field is a consequence of the beam's ability to withstand shear. The string, lacking that ability, instead exhibits a slope discontinuity at a point of force application. Thin line – no near field (string). Thick line – including near field (beam).

Source: *Sound and Vibration*, Wallin, Carlsson, Åbom, Bodén, Glav

Outline

Introduction

Wave propagation in fluids

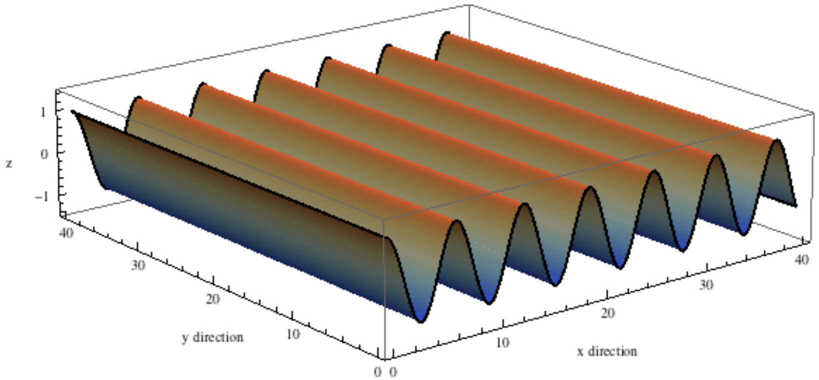
Wave phenomena

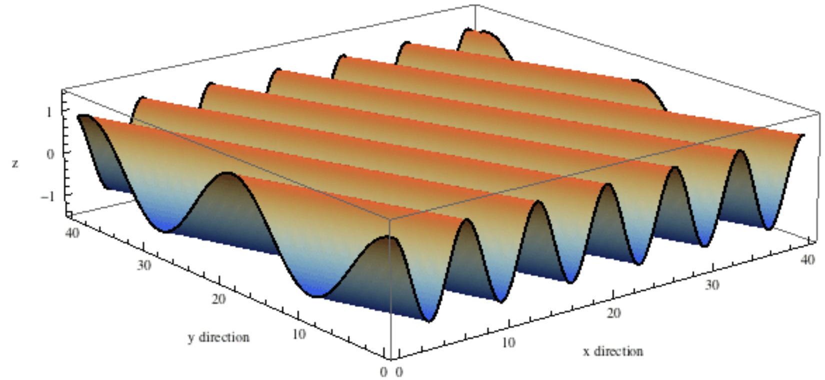
Musical instruments

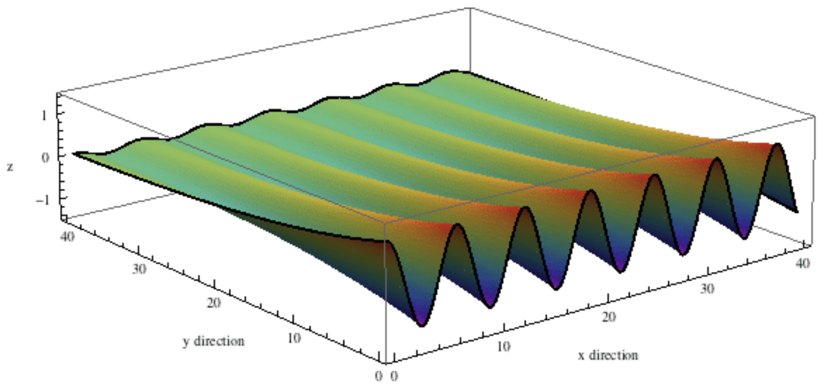
Summary

Propagating waves VS evanescent waves

- Waves may thus propagate or not propagate!







https://www.acs.psu.edu/drussell/Demos/EvanescentWaves/EvanescentWaves.html

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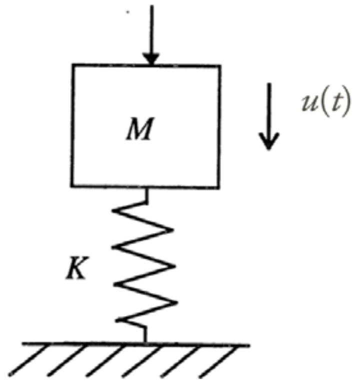
Boundary and initial conditions

- A structure can be mathematically described by
  - Equations of motion
  - Boundary conditions (in space)
  - Initial conditions (in time – irrelevant if harmonic motion $e^{i\omega t}$ is assumed)
- Boundary-value problem – in mathematical language

Do you remember…?



- EoM: $M\ddot{x} + Kx = 0$
- Solution: $x(t) = ae^{i\omega_0 t} + be^{-i\omega_0 t} = A\sin(\omega_0 t) + B\cos(\omega_0 t)$.
- One needs *conditions* to determine A and B.

Boundary conditions (beam in bending)

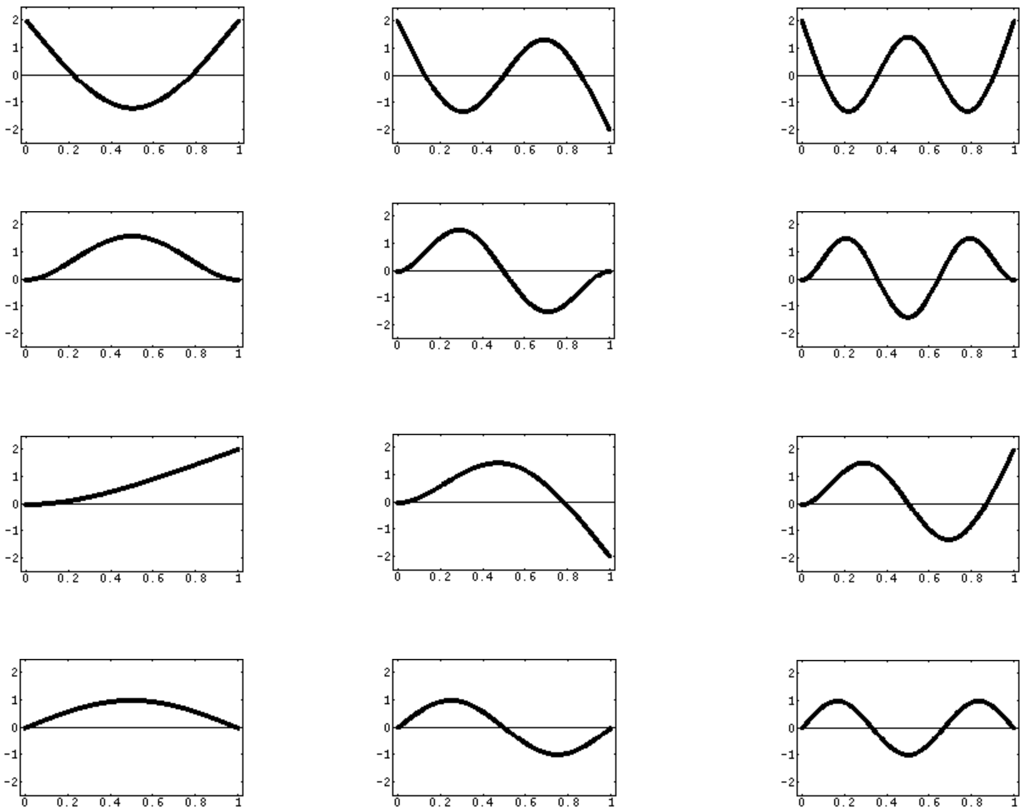
- Structures and the waves in them behave (=look) differently depending on boundary conditions – i.e. How structures are connected at their ends

Free-free

Clamped-clamped

Clamped-free

Simply supported both ends



Recap from previous lecture F2

- End of recap...

Derivation of fluid wave equations

- General approach to derive equations of motion:

1. Newton's law – dynamic equilibrium
2. Constitutive relations – forces, stresses and strains
    - Relations between two physical quantities in a material
    a. Force – stress
    b. Stress – strain
3. Strain – displacement relation (definition)

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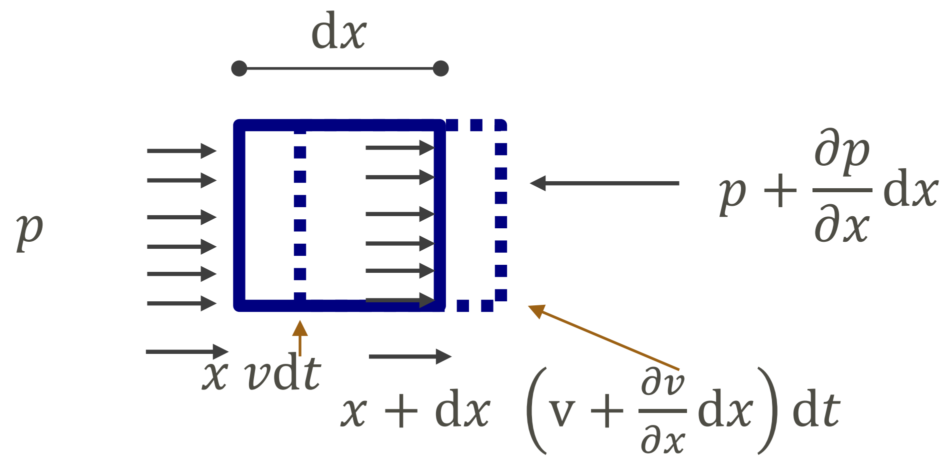
Derivation of longitudinal wave equations



- We take a small fluid element.
- Balance of forces $pS - \left(pS + \frac{\partial p}{\partial x} S\mathrm{d}x\right) = ma = -\rho S\mathrm{d}x\,\frac{\partial v}{\partial t}$
- Hooke's law. $p = -D\varepsilon = -D\,\frac{\partial u}{\partial x}$.
- Some more equations and assumptions…
  - Adiabatic compression! (no heat is transferred, liquid is inviscid)
  - Small changes of sound pressure!

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Waves in fluid media

- Sound waves: longitudinal waves
  - Pressure as field variable

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \longrightarrow p(x,t) = \widehat{p_\pm}\cos(\omega t \pm kx) = \widehat{p_\pm}e^{-i(\omega t \pm kx)}$$

$$c_{medium} = \sqrt{\frac{D}{\rho}}, \qquad c_{air} = \sqrt{\frac{\gamma P_0}{\rho(T = 0°C)}}\left(1 + \frac{T}{2 \cdot 273}\right) = 331.4\left(1 + \frac{T}{2 \cdot 273}\right), \qquad k = \frac{2\pi}{\lambda}$$

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Waves in fluid media

- Solution: $p(x,t) = \hat{p}_+ \cos(\omega t - kx)$ or $p(x,t) = \hat{p}_+ e^{\mathrm{i}(\omega t - kx)}$
  - Pressure as field variable

– In one of the passages of the derivation, the relation between pressure and particle velocity appears, $\frac{\partial p}{\partial x} = -\rho \frac{\partial v}{\partial t}$ (check dimensions!)

  – $v = -\int \frac{1}{\rho} \frac{\partial p}{\partial x} \mathrm{d}t = -\frac{1}{\rho} \int -\mathrm{i}k \hat{p}_+ e^{\mathrm{i}(\omega t - kx)} \mathrm{d}t =$

  $= \frac{\mathrm{i}k}{\rho} \frac{1}{\mathrm{i}\omega} \hat{p}_+ e^{\mathrm{i}(\omega t - kx)} = \frac{1}{\rho c} \hat{p}_+ e^{\mathrm{i}(\omega t - kx)}$.

- Impedance $Z \equiv \frac{p}{v} = \rho c$ – real number (in general is complex though).
  - High Z, high sound pressure to achieve a certain velocity.

Waves in fluid media

- Sound waves: longitudinal waves
  - Pressure as field variable

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \longrightarrow p(x,t) = \widehat{p_\pm}\cos(\omega t \pm kx) = \widehat{p_\pm} e^{-i(\omega t \pm kx)}$$

  - Velocity as field variable

$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2} \longrightarrow v(x,t) = \frac{1}{\rho c}\widehat{p_\pm}\, e^{-i(\omega t \pm kx)}$$

Comparing both equations: $Z \equiv \frac{p_\pm}{v_\pm} = \pm\rho c$ (acoustic impedance)

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Fluid-structure interaction – coincidence

- We have looked at structural waves
- We have looked at fluid waves

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

- Structural waves and fluid waves interact – this is key phenomenon in acoustics.
- Complex phenomenon
  - A powerful approach to study it is dispersion curves
- Walls
- (Glass bottles)
- Musical instruments

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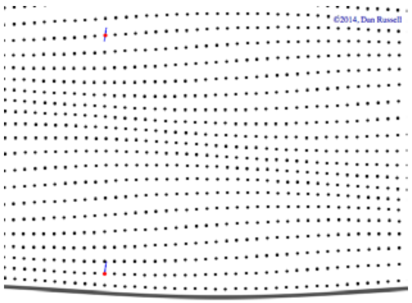
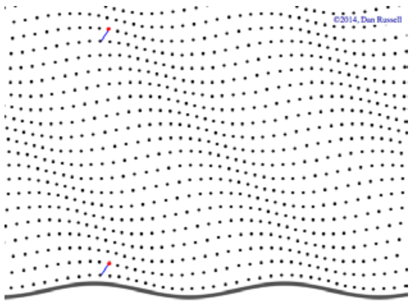
Coincidence

- Animations – look at the red particle and its trajectory!
- Bending wave in the plate which create wave in the air next to it.

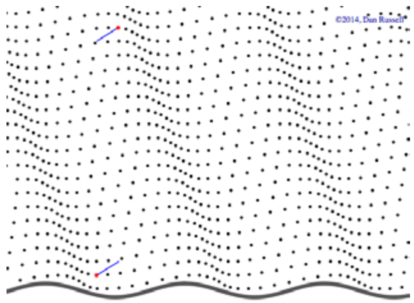
$c_{plate} = 5\ c_{fluid}$ (plane wave)



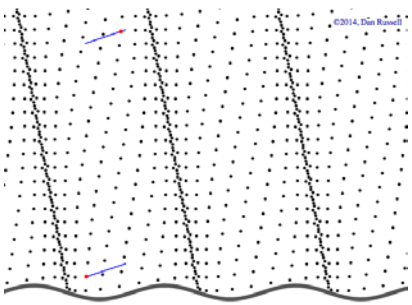
$c_{plate} = 1.5\ c_{fluid}$ (plane wave)



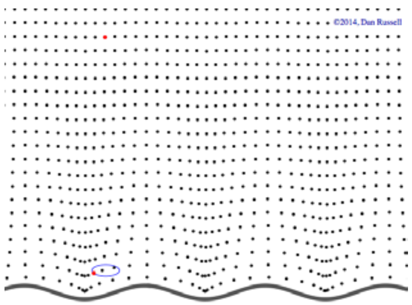
$c_{plate} = 1.1\ c_{fluid}$ (plane wave)



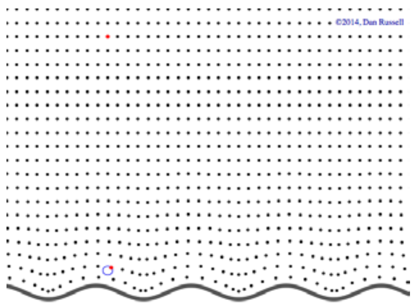
$c_{plate} = 1.03\ c_{fluid}$ (plane wave)



$c_{plate} = 0.95\ c_{fluid}$ (plane wave)



$c_{plate} = 0.75\ c_{fluid}$ (plane wave)

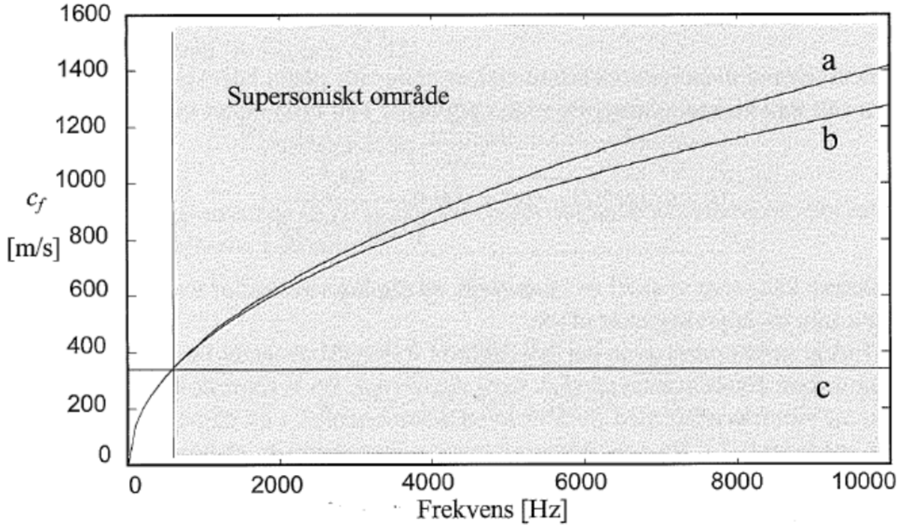


https://www.acs.psu.edu/drussell/Demos/EvanescentWaves/EvanescentWaves.html

Coincidence

- Dispersion curves

$$c_{B(\omega)} = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$$



Figur 6-28 Fashastigheten för en cirkulärcylindrisk stålbalk med diametern 5 cm. a) Enligt Bernoulli-Eulerteori, b) enligt Timoshenkoteori. c) Fashastigheten för en kompressionsvåg i luft. Den frekvens där böjvågens fashastighet är lika med ljudhastigheten i det omgivande mediet kallas koincidensfrekvens.

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Coincidence

- Speed in structure < speed in air

$$\lambda = \frac{c}{f}$$

- wavelength in structure < wavelength in air

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

- Wavenumber in structure > wavenumber in air

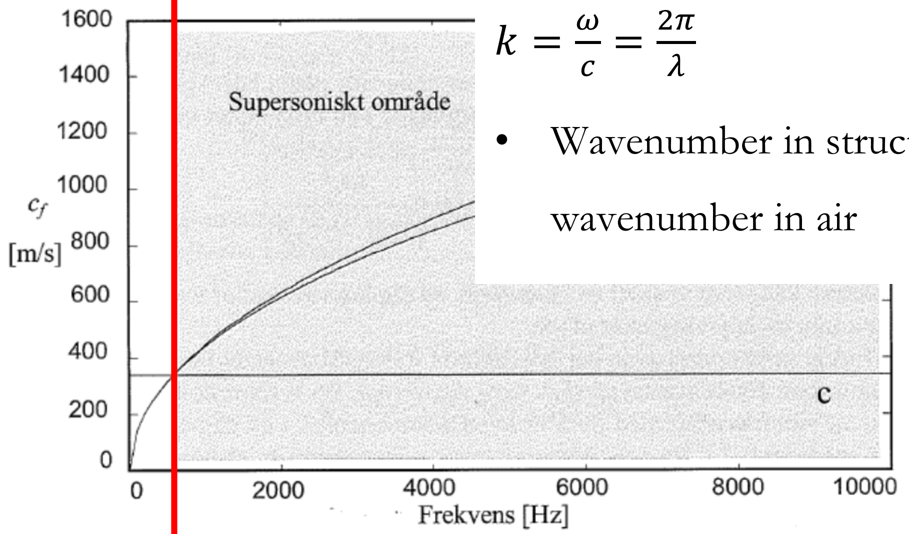
- Speed in structure > speed in air

$$\lambda = \frac{c}{f}$$

- wavelength in structure > wavelength in air

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

- Wavenumber in structure < wavenumber in air



Figur 6-28 Fashastigheten för en cirkulärcylindrisk stålbalk med diametern 5 cm. a) Enligt Bernoulli-Eulerteori, b) enligt Timoshenkoteori. c) Fashastigheten för en kompressionsvåg i luft. Den frekvens där böjvågens fashastighet är lika med ljudhastigheten i det omgivande mediet kallas koincidensfrekvens.

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Coincidence

- For an infinte plate (x,z), with bending wave wavenumber k_B, the pressure (x,y) is:

$$p(x,y)=\frac{\rho_0 c_0 \hat{u}}{\sqrt{1-\frac{k_{\mathrm{B}}^2}{k^2}}}\cdot e^{\mathrm{j}(\omega t-k_{\mathrm{B}}x)}\cdot e^{\mathrm{j}\sqrt{k^2-k_{\mathrm{B}}^2}\,y}.$$

- The pressure wave is propagating only if $k > k_B$
  - If $k < k_B$ (wavelength in plate is **smaller** than wavelength in air), we have a near field.
  - If $k > k_B$ (wavelength in plate is **larger** than wavelength in air) we have a propagating wave

Coincidence

- Radiation efficiency
  - Ratio of *how much sound* is radiated by a structure over its vibration velocity at its surface

$$\sigma = \frac{W_{\text{rad}}}{\rho_0 c_0 S \left\langle \tilde{u}^2 \right\rangle},$$

- For an finite plate (x,y), sound radiation behave like this:

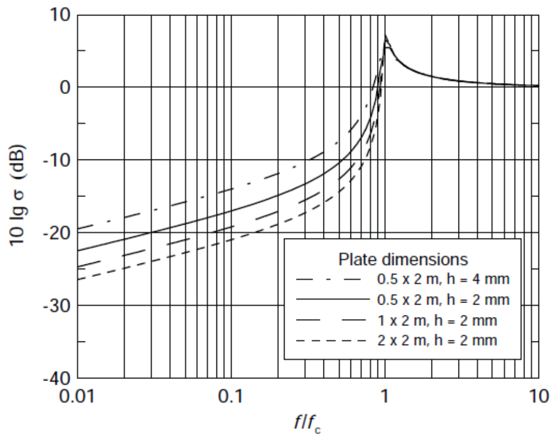
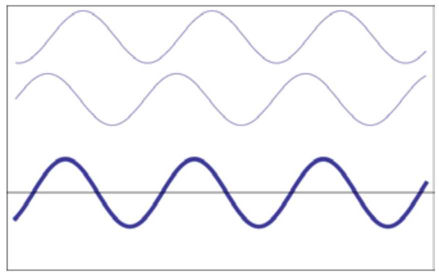


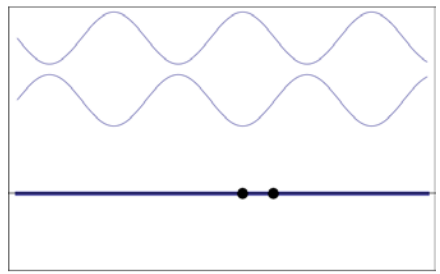
Figure 6.15 Radiation index by resonant radiation from plates of steel or aluminium. Calculated from expressions by Leppington et al. (1982).

Some more wave phenomena…

- Superposition of two waves

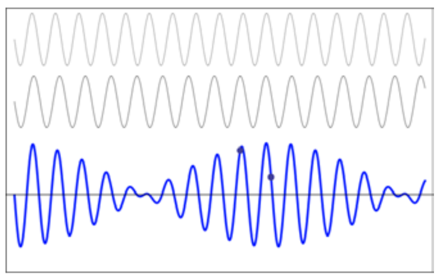
- Constructive and destructive interference (phase shift)





- Standing waves

- Beats



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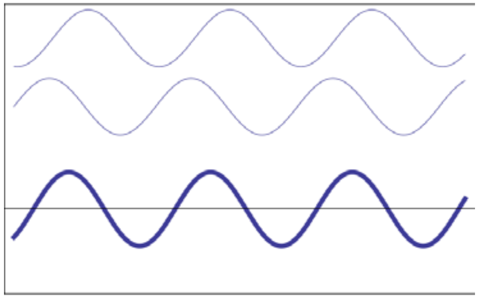
Wave phenomena

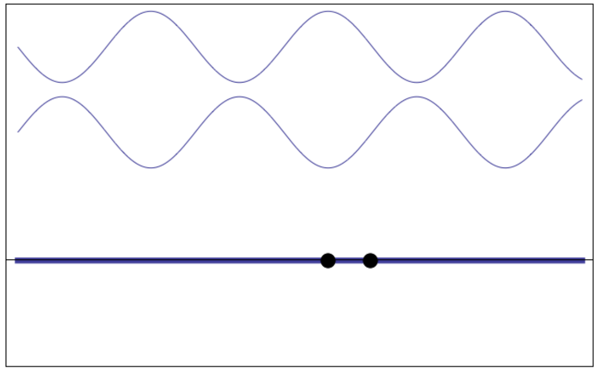
- Interferences: constructive / destructive

Constructive/destructive depending on Φ

$$y_1(x,t) = \hat{y}\cos(\omega t - kx)$$
$$y_2(x,t) = \hat{y}\cos(\omega t - kx + \theta)$$

$$y(x,t) = y_1(x,t) + y_2(x,t) = 2\hat{y}\cos\left(\frac{\theta}{2}\right)\sin(\omega t - kx + \theta)$$





Source: Dan Russell

- Standing waves (coherent source)

$$y_-(x,t) = \hat{y}\cos(\omega t - kx)$$
$$y_+(x,t) = \hat{y}\cos(\omega t + kx)$$

$$y(x,t) = y_-(x,t) + y_+(x,t) = 2\hat{y}\sin(kx)\cos(\omega t)$$

Two travelling waves of same frequency, type and fixed phase relation propagating in opposite directions

Position-dependent amplitude oscillating according to cos(ωt)

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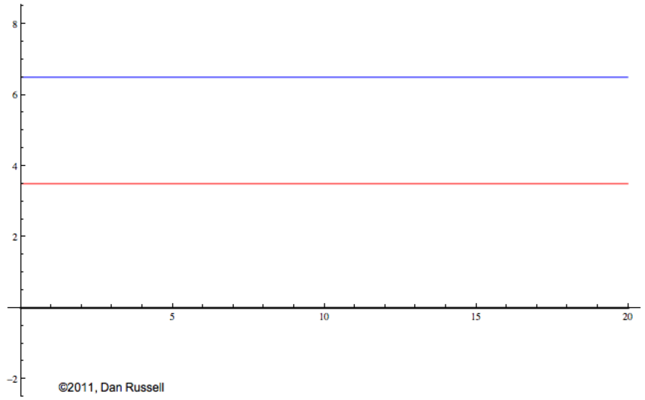
Wave phenomena

- What happens when waves are travelling in a structure and interact with each other?
    - A sound exciting waves in a wall
    - An hammer exciting waves in a plate
- The waves in the structure will travel, bounce on boundaries, travel back, interact with each and create phenomena of constructive and destructive interference.



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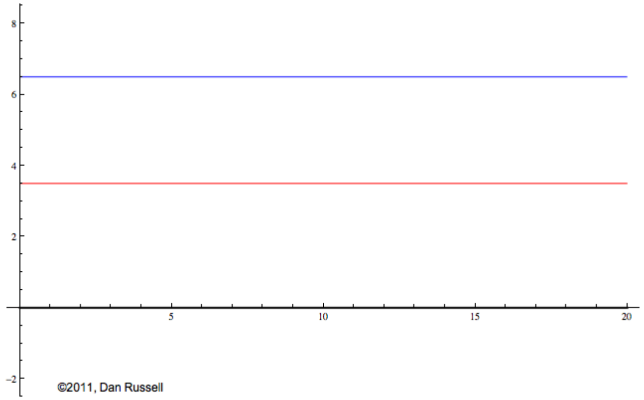
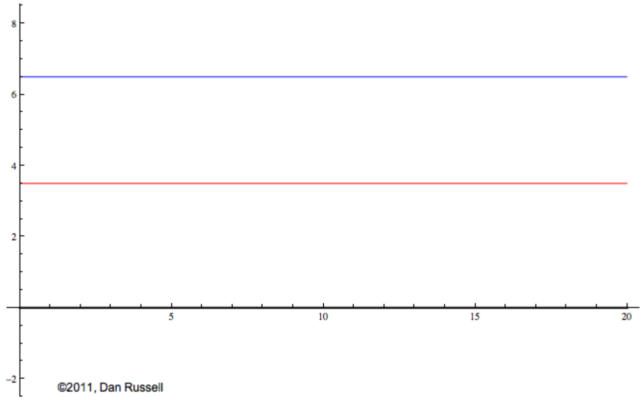
Wave phenomena



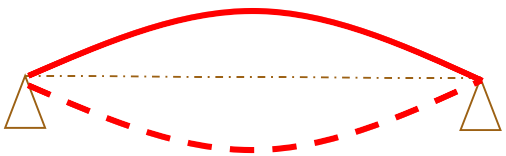
- A pattern in space is created from these reflections resulting in waves interfering constructively and destructively with each other.
- This pattern is called mode (eigenmode)
- The frequency at which this pattern occurs is natural frequency (eigenfrequency)

Wave phenomena

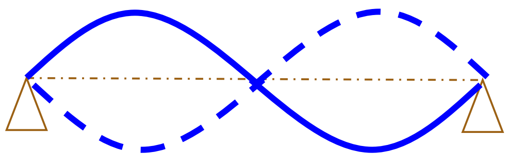


- Just as for the SDOF mass-spring system, natural frequencies and the corresponding spatial pattern of natural modes are intrinsic characteristics of any structure – structure *like* to move at those frequencies.

Standing waves in a string



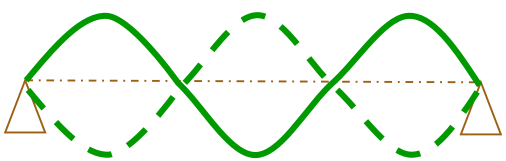
$\lambda=2L$
$f_1=c/2L$ (natural frequency)
Fundamental eigenfrequency / 1st harmonic



$\lambda=L$
$f_2=2f_1$ (natural frequency)
Second eigenfrequency / 2nd harmonic

In general:

$\lambda=2L/n$
$f_n=n \cdot c/2L$

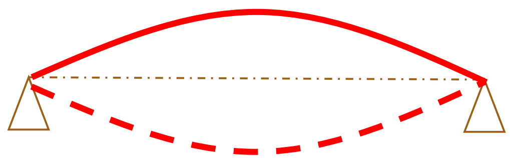


$\lambda=(2/3)L$
$f_3=3f_1$ (natural frequency)
Third eigenfrequency / 3rd harmonic

Eigenmode: different ways a string (structure in general) can vibrate generating standing waves
Examples: 1 / 2 / 3 / 4.

Eigenfrequencies in a string

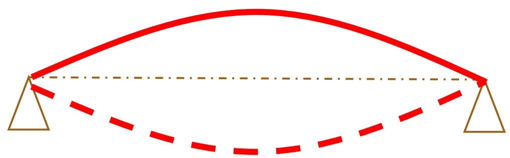
- $u_{xx} - \frac{1}{c^2} u_{tt} = 0$; solution: $u(x,t) = \mathrm{Re}\left(\tilde{u}(x) e^{i\omega t}\right)$.



- $\frac{\partial^2 \tilde{u}}{\partial x^2} + k^2 \tilde{u} = 0$; $k^2 = \omega^2 \frac{\rho}{E} = \frac{\omega^2}{c^2}$
- We know that this a solution: $\tilde{u}(x) = A \sin kx + \mathrm{B} \cos kx$.
- Second-order spatial derivative requires two boundary conditions.
  - Fixed boundaries. At the two ends there is not motion.
  - $x = 0 \rightarrow \tilde{u}(x = 0) = 0 = A \cdot 0 + B = \boldsymbol{B} = \mathbf{0}$.
  - $x = L \rightarrow \tilde{u}(x = L) = 0 = A \sin kL = 0$, which is true
    - if A=0 – trivial solution
    - If sinus is zero. $kL = \frac{\pi}{L}; \frac{2\pi}{L}; \frac{3\pi}{L}; \dots; \frac{n\pi}{L}$. With n a natural number.

Eigenfrequencies in a string

- If $\omega = 2\pi f = kc$ then $\mathrm{f} = \frac{kc}{2\pi}$
- $f_{n=1} = \frac{\pi}{L}\frac{c}{2\pi} = \frac{c}{2L}$
- $\lambda_{n=1} = \frac{c}{f_{n=1}} = 2L;\ \lambda_n = \frac{2L}{n}$.
- $u(x,t) = \mathrm{Re}\left(\tilde{u}(x)e^{i\omega t}\right) = \mathrm{Re}\left(\sin\frac{n\pi}{L}x\, e^{i\omega t}\right)$.
- This procedure can be done for *any* structure with *any* boundary conditions.
  - But sometimes it can be very hard or practically impossible due to lack of information.



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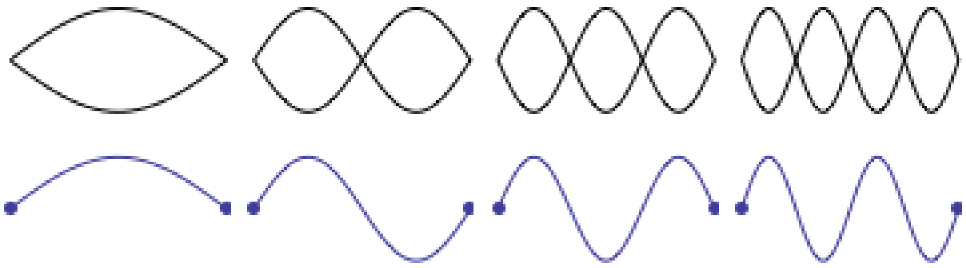
Standing waves in a string



$$y(x,t) = y_m \sin\left(\frac{n\pi}{L}x\right)\cos(2\pi f t)$$

https://youtu.be/-gr7KmTOrx0

Watch this too! https://youtu.be/QcoQvzNQp6Q

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Modes in structures - plates

- Standing waves in a plate

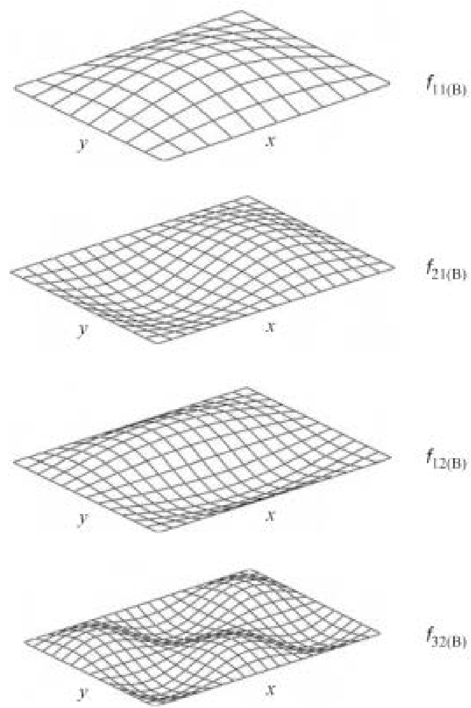

Figure 2.32

Mode shapes for bending modes on a plate with simply supported boundaries.

Source: Carl Hopkins, *Sound Insulation*

<u>Eigenmode</u>: different ways a plate (structure in general) can vibrate generating standing waves

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Modes in structures - plates

- Standing waves in a plate (change in geometry – i.e. Boundary conditions – will change shape of mdoes)

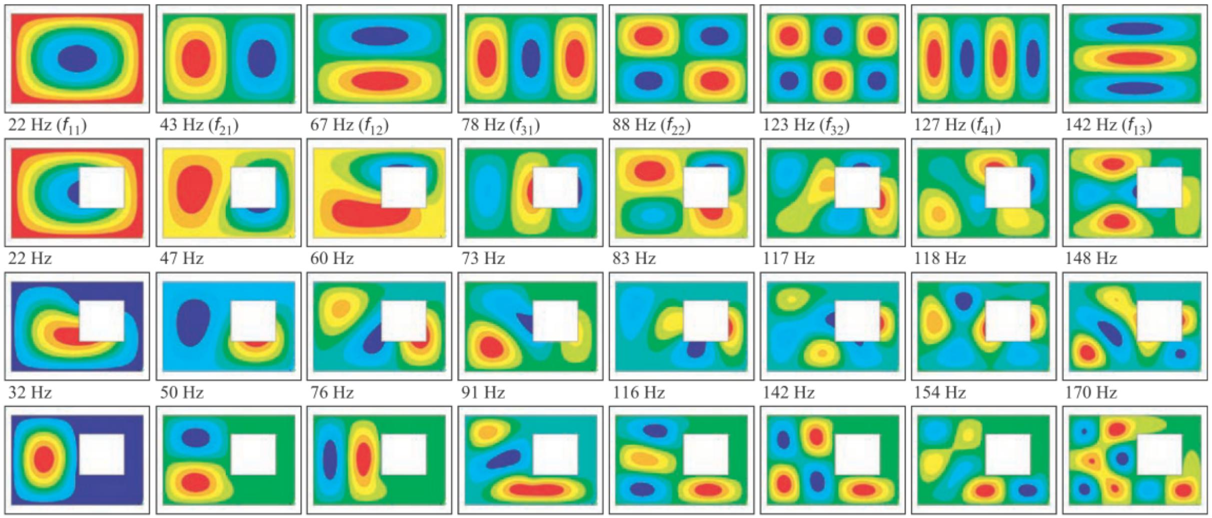


Figure 2.33

Mode shapes for a wall with simply supported boundaries, with and without a window opening.
Row 1: Wall without an opening.
Row 2: Wall with an opening that has free boundaries.
Row 3: Wall with an opening where the top boundary is simply supported and other boundaries are free.
Row 4: Wall with an opening that has simply supported boundaries.
In each row the eight lowest mode frequencies are shown. For all modes except those in the first column, red and dark blue indicate maximum displacement in opposite directions.
Wall properties: $x = 3.5$ m, $y = 2.4$ m, $h = 0.1$ m, $\rho = 600$ kg/m^3, $c_L = 1900$ m/s, $\nu = 0.2$.
Reproduced with permission from Hopkins (2003).

Source: Carl Hopkins, *Sound Insulation*

Eigenmode: different ways a plate can vibrate generating standing waves

Standing waves in a plate

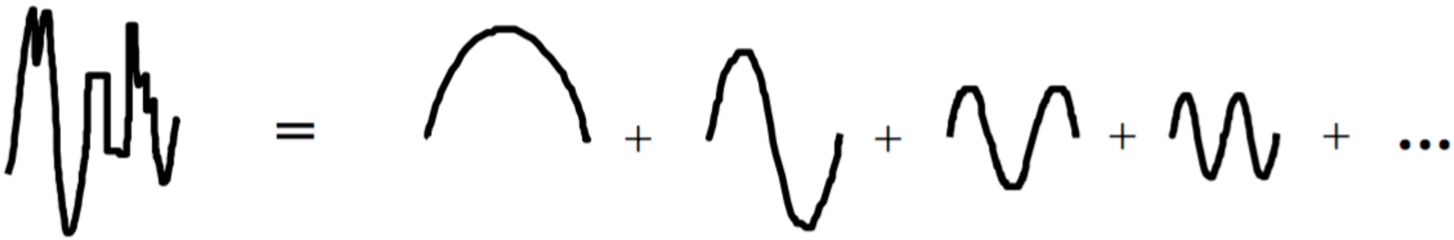
- Chladni plates i.e. visualisation of modes in plates

https://youtu.be/wYoxOJDrZzw

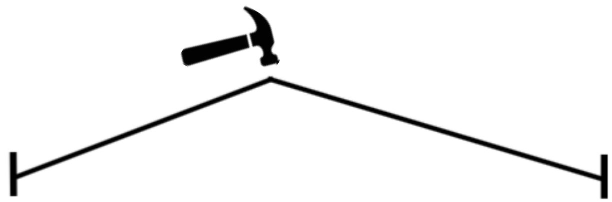
Musical instruments

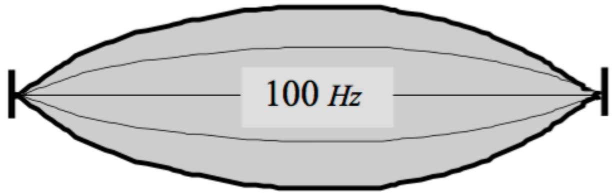
- Systems' eigenfrequencies are derived without external forces – the homogeneous solution.
- Interesting phenomena happens on the other hand when external forces with their own driving frequencies interact with systems' eigenfrequencies – i.e. resonance phenomena happen – the particular solution.
- Systems eigenfrequencies, and accordingly systems' response to sound and vibrations, will therefore sustain, maintain and add character to external driving frequencies. In musical acoustics one speaks of loudness, quality, timbre.
  - Think about musical instrument, concert rooms.
  - When this interaction is not properly managed though problems will occur (collapsing bridges due to external excitation is an extreme example).
- Stage to Tim Näsling, acoustician and guitar maker.

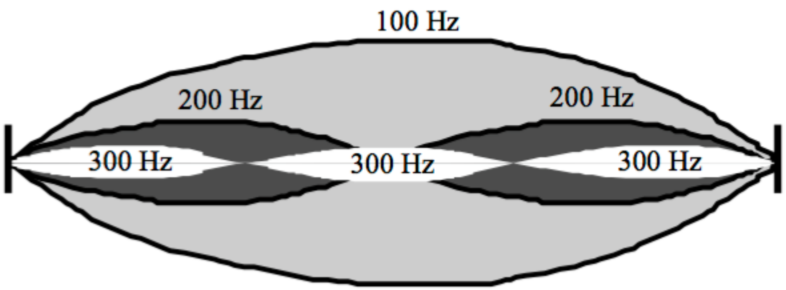
Standing waves and higher harmonics (I)

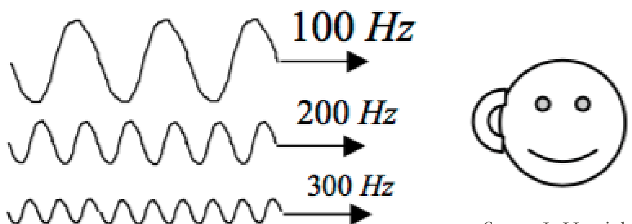


Any motion = sum of motion of all the harmonics (Fourier series)









Source: J. Hetricks

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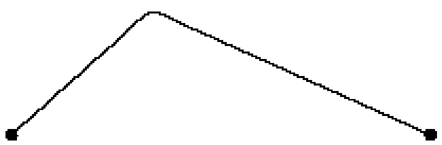
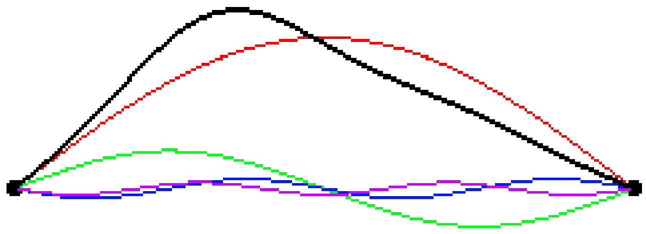
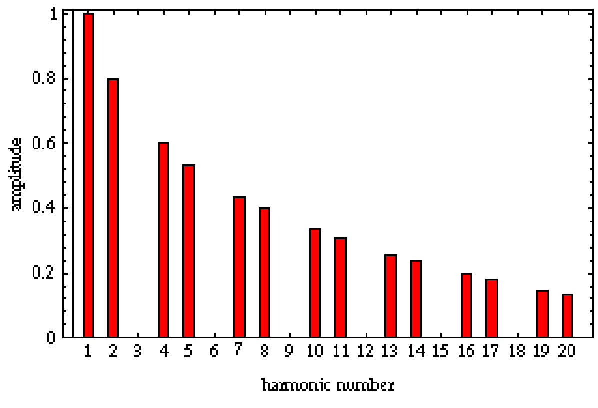
Standing waves and higher harmonics (II)

- The vibration of fixed-fixed string plucked at a distance 1/3 of the length







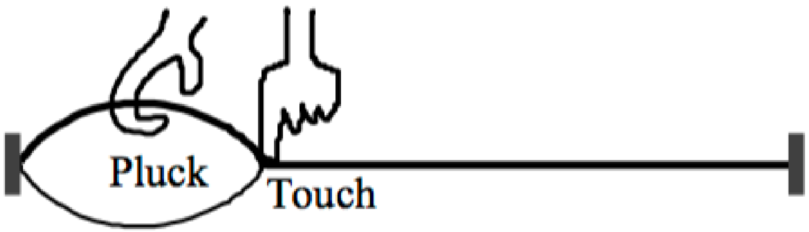
- Y(x,t) = 0 for x=L/3

Music instruments: string (e.g. violin)

$\lambda=2L/n$

$f_n=n\cdot v/2L$

$v=(\text{tension}/\text{mass-length})^{1/2}$



When ones plays → Changes L

Knobs (tune) → Vary tension

To discuss: Piano?

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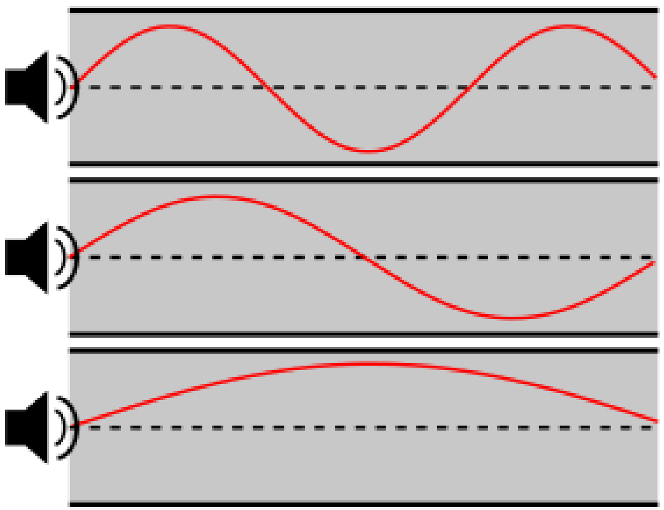
Music instruments: wood-wind



Open-open / Closed-closed:

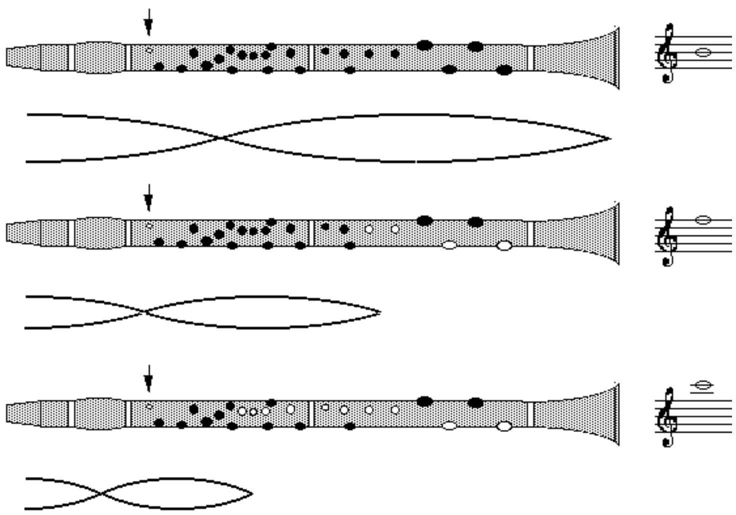
$\lambda = 2L/n$

$f_n = n \cdot v/2L$

NOTE: Open-closed vary

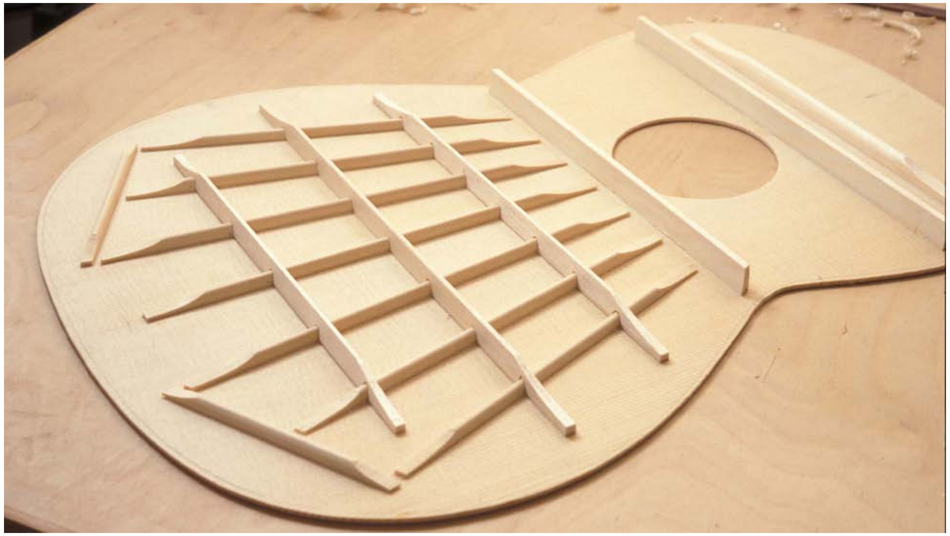
$v = (\text{temperature}/\text{molecular weight})^{1/2}$

Change of v (molecular weight)



Cover holes → Vary L

Music instruments: soundboards

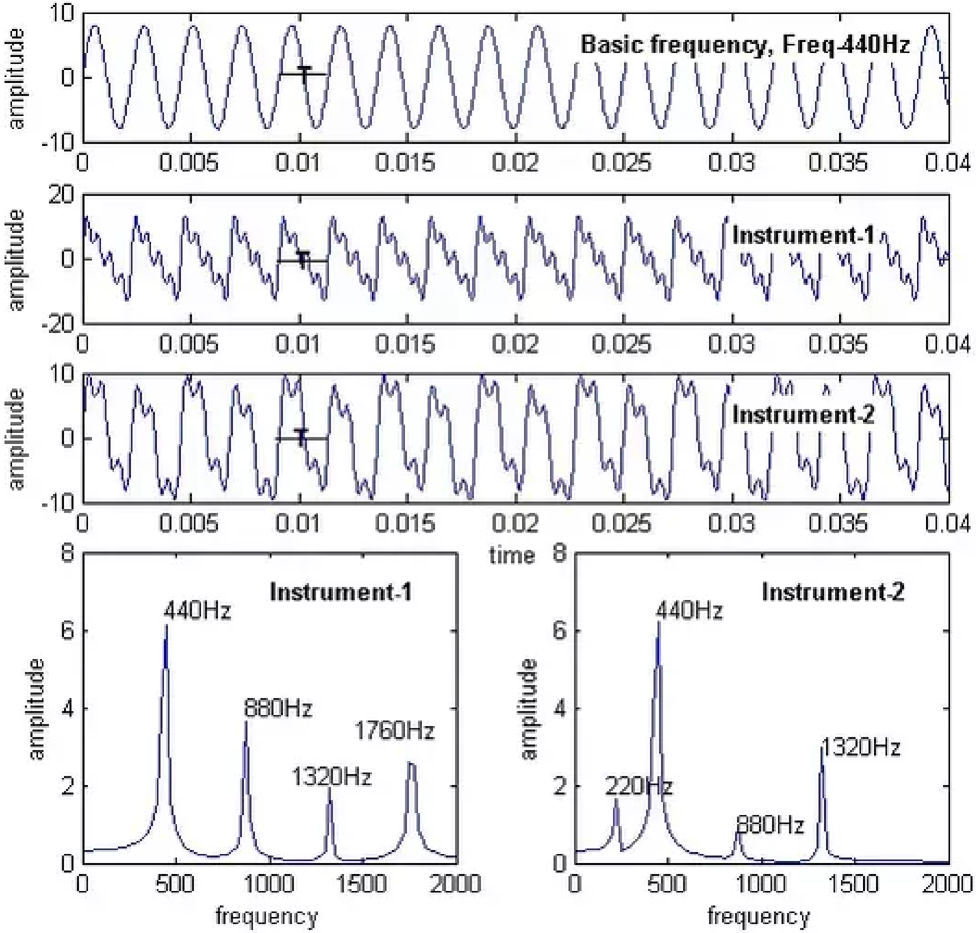


Standing waves and timbre

- Characteristics of sound:
  - Loudness (amplitude)
  - Pitch (frequency)
  - Quality or Timbre
    - » "Cocktail" characteristic of every instrument/source
    - » Different combination of higher harmonics
    - » What makes us distinguish one instrument from another

- To discuss: helium & voice
  - Change of molecular weight (i.e. v) → natural frequency goes up

Standing waves and timbre



Source: Quora

LINK TO SOUND

Thank you for your attention!

mathias.barbagallo@construction.lth.se

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