RECORDING

Ljud i byggnad och samhälle (VTAF01) – SDOF

MATHIAS BARBAGALLO
DIVISION OF ENGINEERING ACOUSTICS, LUND UNIVERSITY

… recap from last lecture

- Course introduction
- Log and complex numbers – why and how
- Why study acoustics and what is acoustics
- Description of a sound wave
- Key concepts
  - Decibel – summation of sources
  - Frequency weighting
  - Frequency bands
  - Noise metrics

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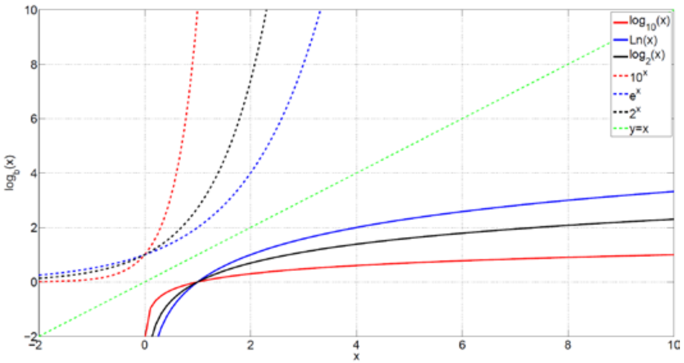
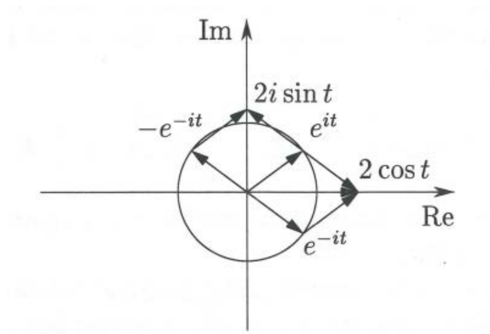
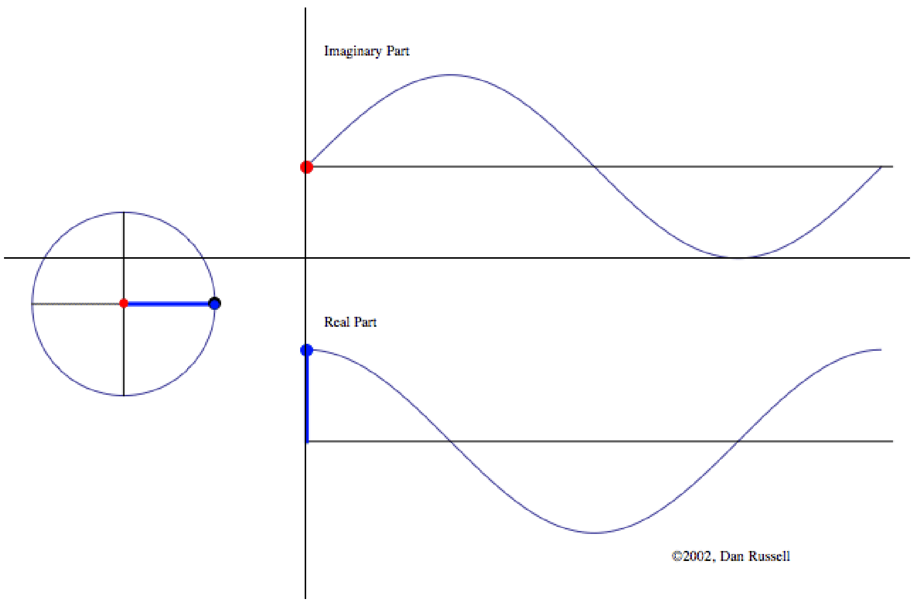
… recap from last lecture



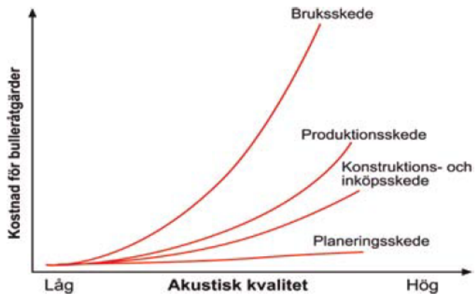




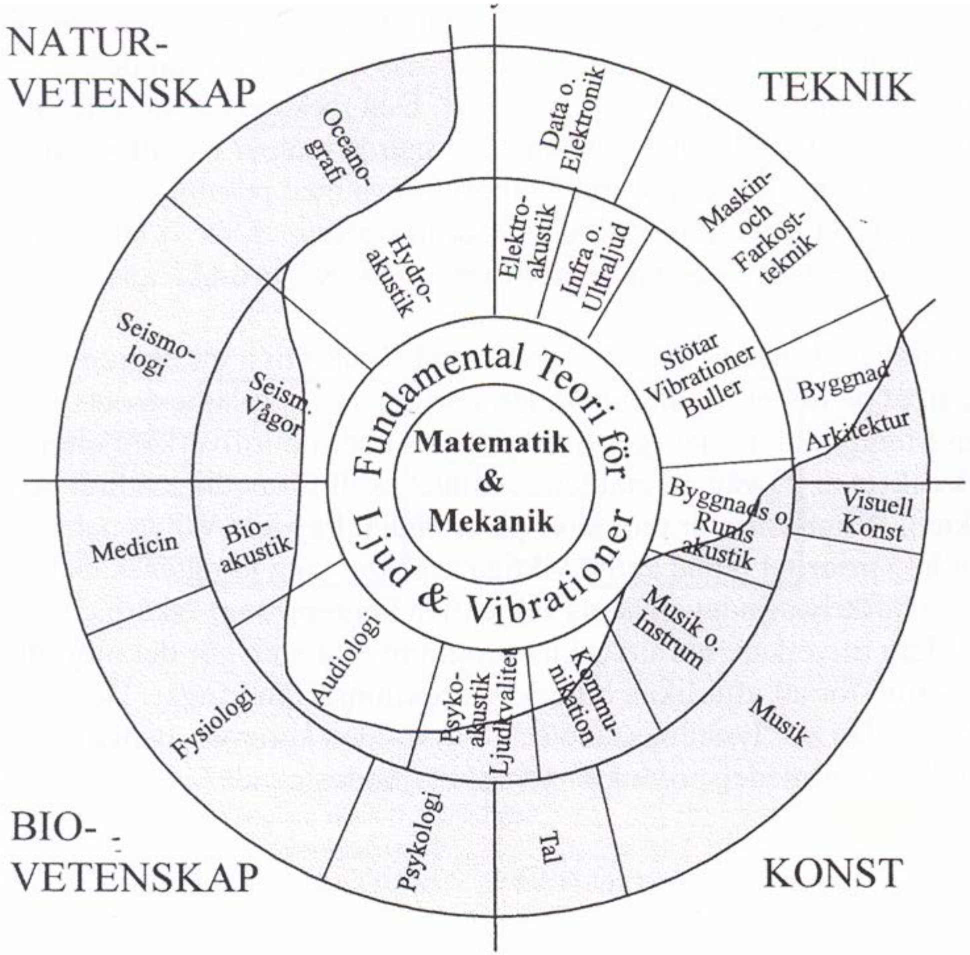
A sinus and a cosinus which relate via the complex plane and Euler's equations…

… recap from last lecture

Figur 1.1
Samband mellan akustisk kvalitet och kostnad för bulleråtgärder.
Källa: SOU 1993:65



… recap from last lecture



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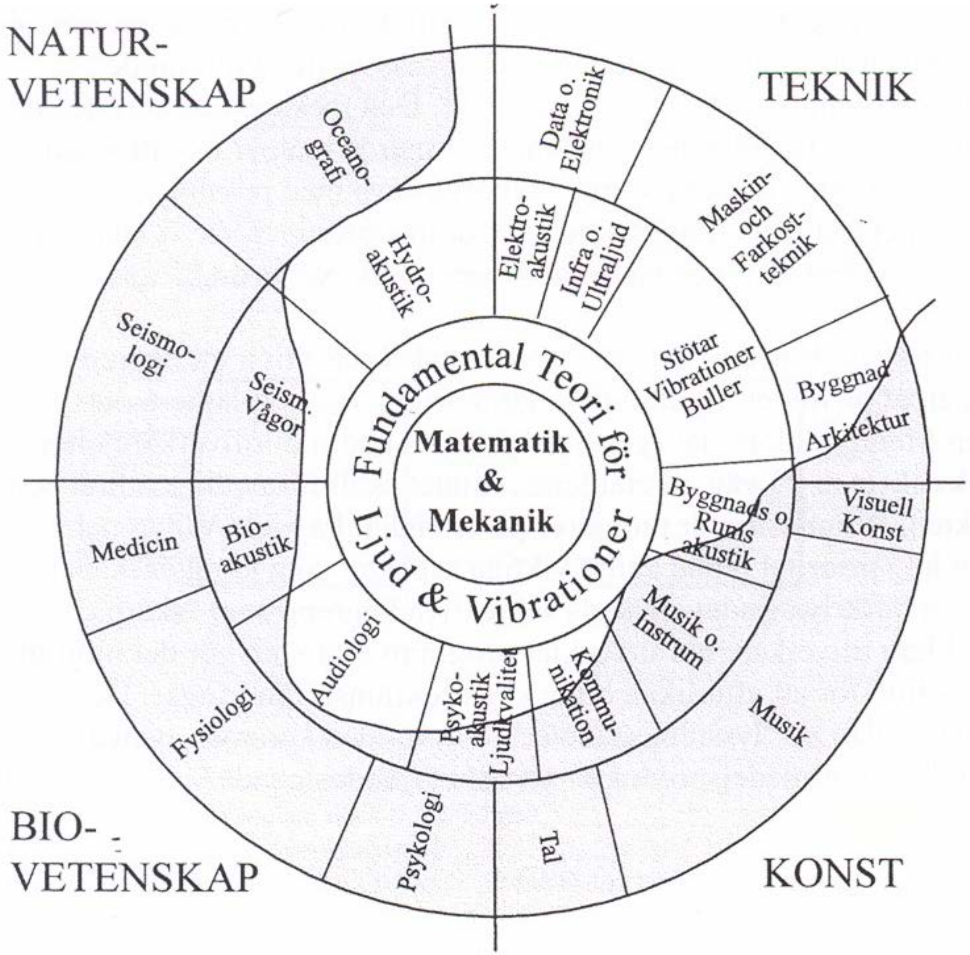
… recap from last lecture



Time & frequency domains

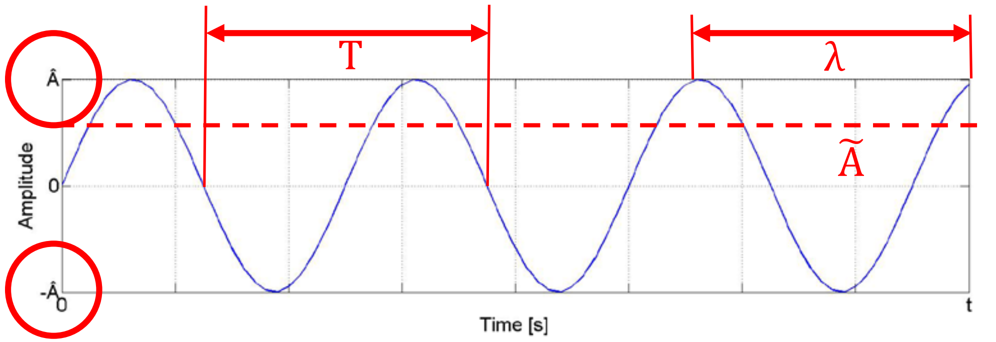
Harmonic signal: $y(t) = \hat{A}\sin(\omega t) = \hat{A}\cos(\omega t + \varphi) = \hat{A}\sin(2\pi f \cdot t)$

- Amplitude: $\hat{A}$
- Period [s]: $T = {}^{1}/_{f}$
- Frequency [Hz]: $f = {}^{1}/_{T}$
- Wavelength [m]: $\lambda = cT = {}^{c}/_{f}$
- Propagation Speed [m/s]: $c = f\lambda$

  NOTE: $c \neq v$

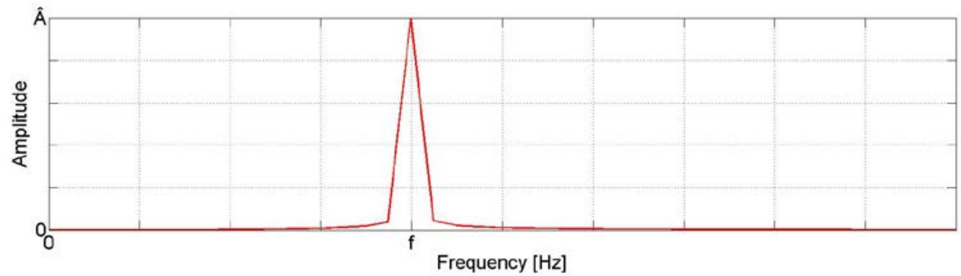
- Effective value (RMS):

$$A_{RMS} = \tilde{A} = \sqrt{\frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} y^2(t)\,dt}\,, \quad \tilde{A}_{\substack{\text{harmonic}\\\text{signal}}} = {}^{\hat{A}}/_{\sqrt{2}}$$



FFT

How much a time-signal resembles sine waves with varying f



- Frequency domain

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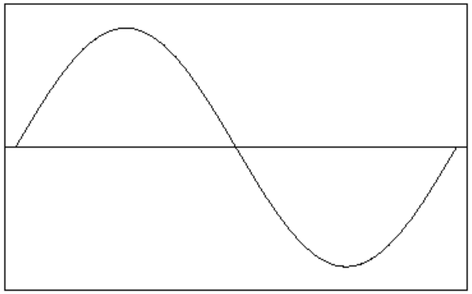
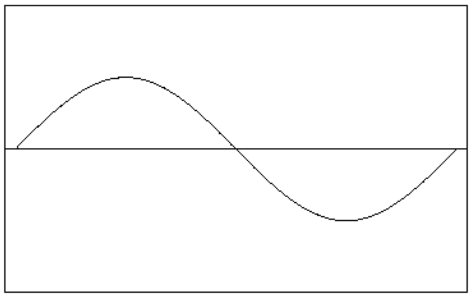
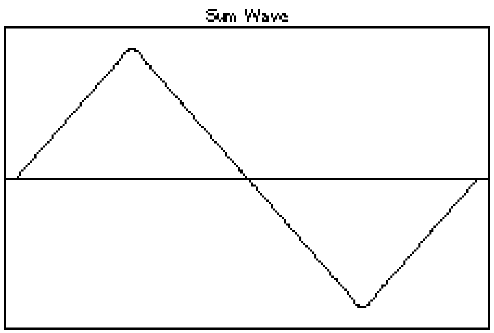
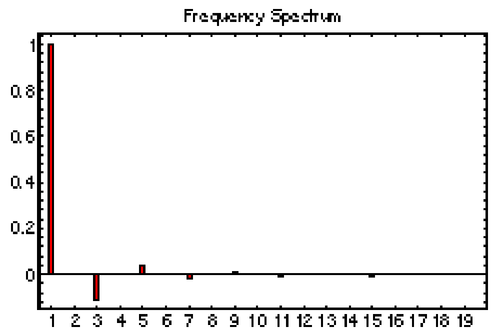
Time & frequency domains

- **Fourier Transform**: decomposes a signal into its constituent frequencies.









DEF: The decibel (dB) & SPL

- Logarithmic way of describing a ratio
  - Ratio: velocity, voltage, acceleration…
  - Need of a reference

- Sound pressure level (SPL / L_p)

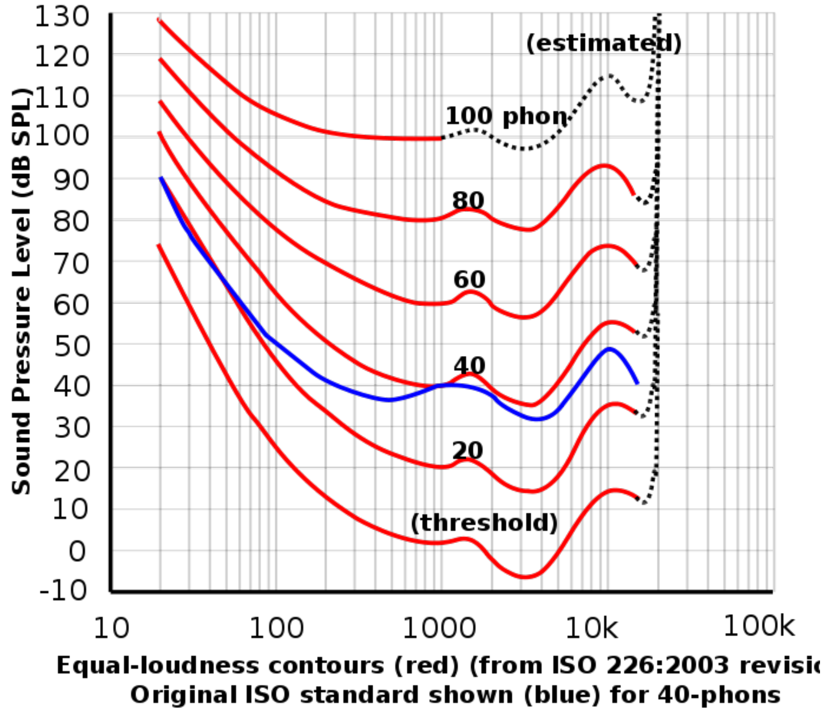
$$L_p = 10\log\left(\frac{\tilde{p}^2}{p_{ref}^2}\right) = 20\log\left(\frac{\tilde{p}}{p_{ref}}\right)$$

$\tilde{p} = \tilde{p}(f) \equiv \text{RMS pressure}$
$p_{ref} = 2\cdot10^{-5}\ \text{Pa} = 20\ \mu\text{Pa}$
$p_{atm} = 101\,300\ \text{Pa}$
$p_{tot}(t) = p_{atm} \pm p(t)$

  - $\tilde{p}$ measured with microphones
  - Frequency response of human hearing changes with amplitude

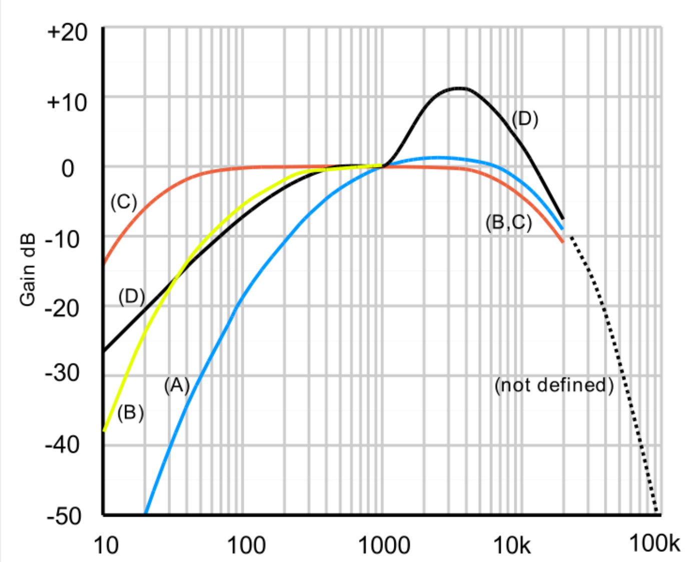
Frequency weightings

- Frequency response of human hearing changes with amplitude
- How to relate the objective measure to the subjective experience of sound?

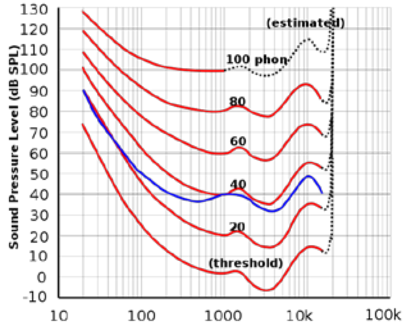


Frequency weightings (II)

- Filters and calculation



A-weighting (blue), B (yellow), C (red), and D-weighting (blk)

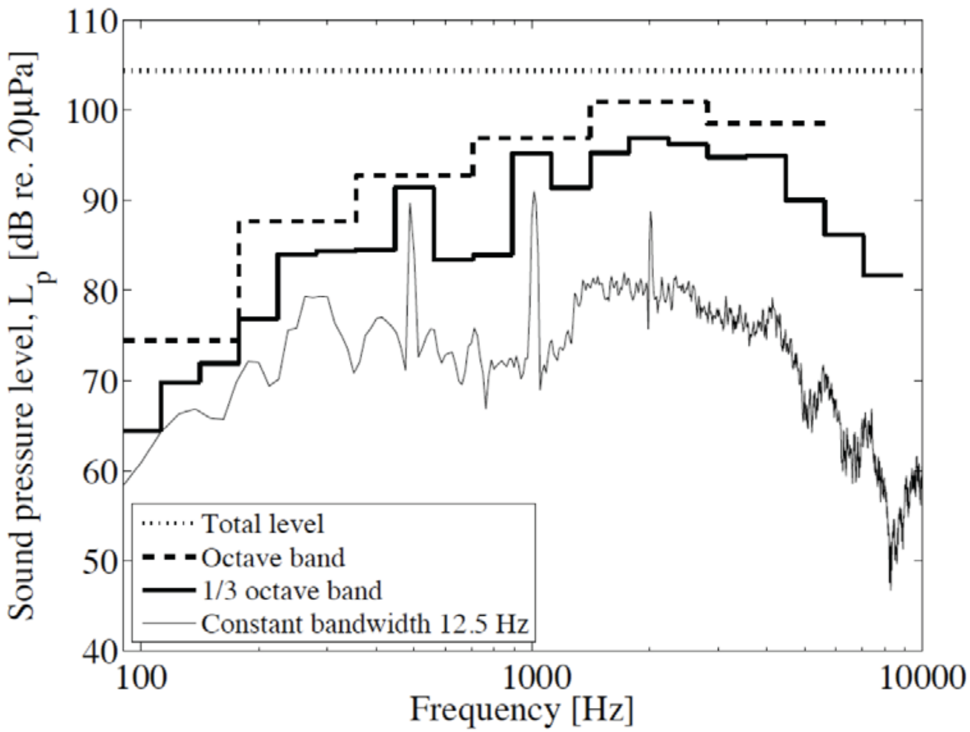


Equal-loudness contours (red) (from ISO 226:2003 revisi
Original ISO standard shown (blue) for 40-phons

Frekvens [Hz]	A-filter [dB]	B-filter [dB]	C-filter [dB]
10	-70.4	-38.2	-14.3
12.5	-63.4	-33.2	-11.2
16	-56.7	-28.5	-8.5
20	-50.5	-24.2	-6.2
25	-44.7	-20.4	-4.4
31.5	-39.4	-17.1	-3.0
40	-34.6	-14.2	-2.0
50	-30.2	-11.6	-1.3
63	-26.2	-9.3	-0.8
80	-22.5	-7.4	-0.5
100	-19.1	-5.6	-0.3
125	-16.1	-4.2	-0.2
160	-13.4	-3.0	-0.1
200	-10.9	-2.0	0
250	-8.6	-1.3	0
315	-6.6	-0.8	0
400	-4.8	-0.5	0
500	-3.2	-0.3	0
630	-1.9	-0.1	0
800	-0.8	0	0
1000	0	0	0
1250	0.6	0	0
1600	1.0	0	-0.1
2000	1.2	-0.1	-0.2
2500	1.3	-0.2	-0.3
3150	1.2	-0.4	-0.5
4000	1.0	-0.7	-0.8
5000	0.5	-1.2	-1.3
6300	-0.1	-1.9	-2.0
8000	-1.1	-2.9	-3.0
10000	-2.5	-4.3	-4.4
12500	-4.3	-6.1	-6.2
16000	-6.6	-8.4	-8.5
20000	-9.3	-11.1	-11.2

Frequency bands

- A sound in the frequency domain may be looked at in several ways.
  - Narrow bands;
  - Third-octave bands;
  - Octave bands;
  - Total value.



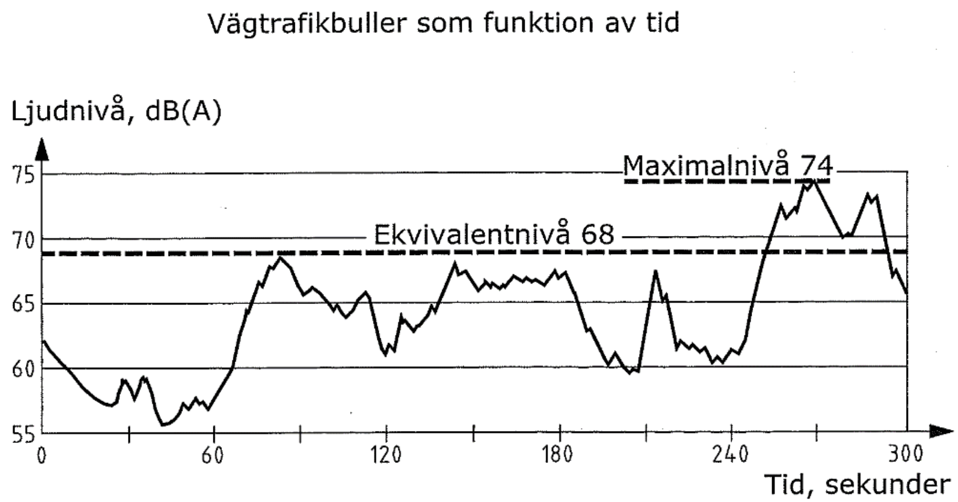
© P. Andersson

$$L_{oct} = 10\log\left(\sum_{i=1}^{3} 10^{\frac{L_{p,i}}{10}}\right)$$

Noise metrics

- There are different noise metrics that we can use, e.g.:
  - Equivalent level
  - Maximum level

$$L_{eq,T} = 10\log\left(\frac{1}{T}\int_0^T \frac{p^2(t)}{p_{ref}^2}dt\right) = \boxed{10\log\left(\frac{1}{T}\int_0^T 10^{\left(\frac{L_p(t)}{10}\right)}dt\right)}$$



Figur 4:9. Ljudnivåns variation under 5 minuter på en livligt trafikerad innerstadsgata. Ekvivalentnivå 68 dB(A), maximalnivå 74 dB(A).

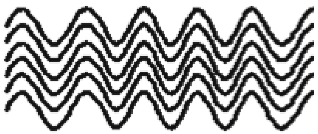
Ex: Calculate the $L_{eq,8h}$ that corresponds to 100 dBA for 15 min.

Summation of noise

- The total RMS pressure:

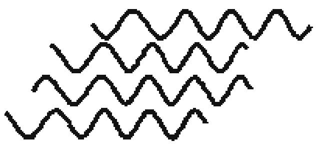
$$\tilde{p}_{tot}^2 = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t}\left[p_1(t)+p_2(t)\right]^2 dt = \tilde{p}_1^2 + \tilde{p}_2^2 + \frac{2}{\Delta t}\int_{t_0}^{t_0+\Delta t} p_1(t)\,p_2(t)\,dt$$

- Types of sources
  - Correlated (or coherent)
    - » Constant phase difference, same frequency
    - » Interferences (constructive/destructive)



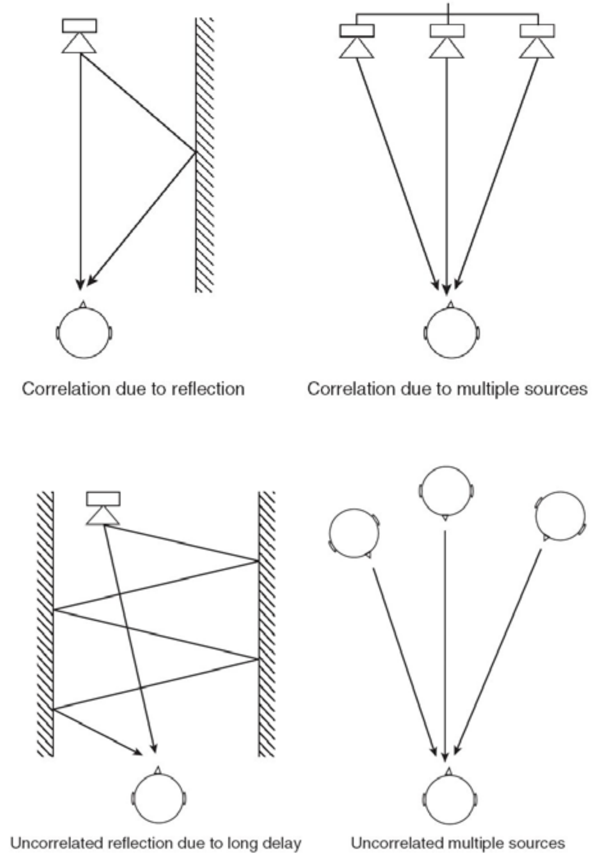
$$L_{p,tot} = 20\log\left(\sum_{n=1}^{N} 10^{\frac{L_{p,n}}{20}}\right)$$

  - Uncorrelated (or uncoherent)



$$L_{p,tot} = 10\log\left(\sum_{n=1}^{N} 10^{\frac{L_{p,n}}{10}}\right)$$

For uncorrelated sources, the 3rd term vanishes



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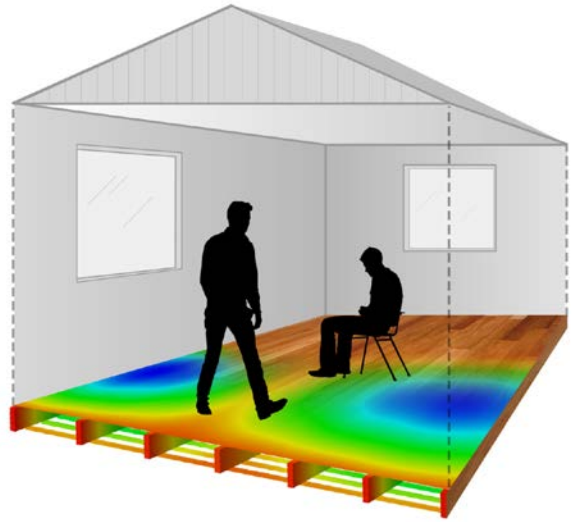
Introduction

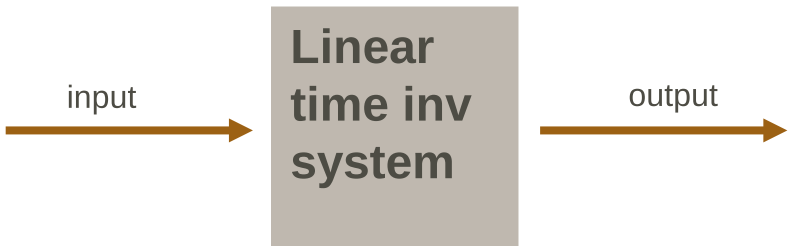
- A very broad definition…
  - Acoustics: *what can be heard…*
  - Vibrations: *what can be felt…*
- Coupled "problem"
  - Hard to draw a line between both domains
- Nuisance to building users
  - Comprise both noise and vibrations



Source: J. Negreira (2016)

Linear system

- What effect does an input signal has on an ouput signal?
  - What effect does a force on a body has on its velocity?
- A way to answer it using theory of linear time-invariant systems.



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Linear system

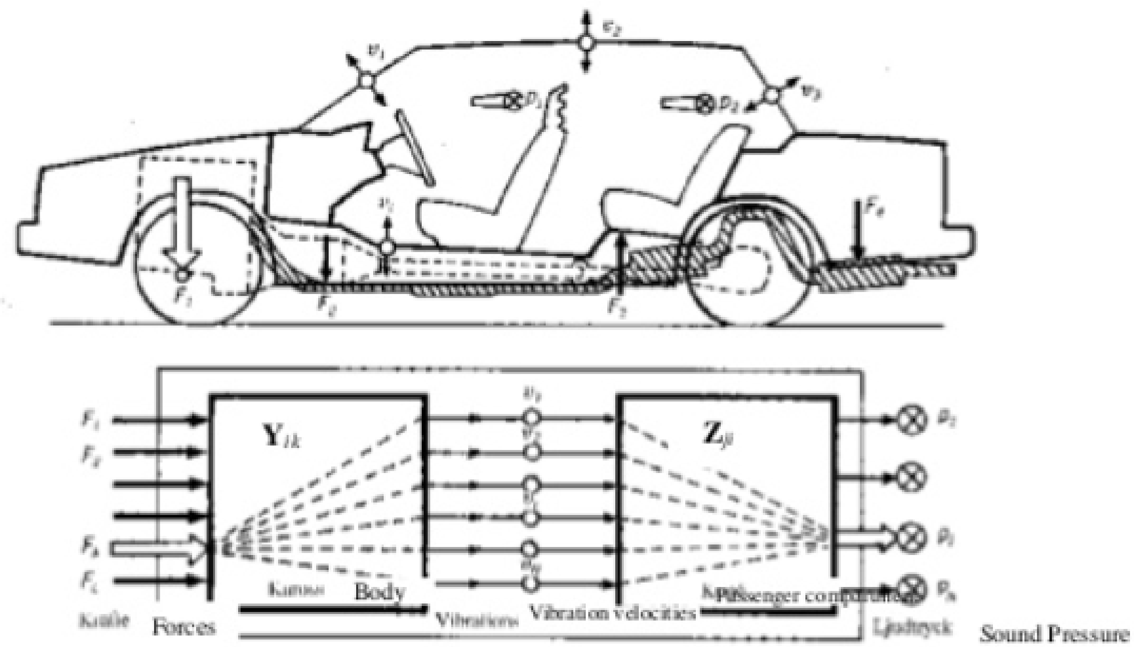
- Linear time-invariant systems
  - Mathematically: relation input/output described by linear differential equations.
  - Characteristics:
    - » Coefficients independent of time.
    - » Superposition principle.
      - $a(t) \rightarrow c(t); b(t) \rightarrow d(t) \Rightarrow a(t) + b(t) \rightarrow c(t) + d(t)$
    - » Homogeneity principle: $\alpha a(t) \rightarrow \alpha \mathrm{b(t)}$
    - » Frequency conserving:
      - a(t) comprises frequencies f1 and f2; b(t) comprises f1 and f2.

Linear systems

- Linear oscillations shall be considered
  - i.e. yielding linear oscillation between exciting forces and resulting motion
- Typically linearity is achieved when involved quantities can be regarded as small variations about an average value
  - Sound!
  - In acoustics we can typically describe relations between inputs and outputs with linear systems and with linear differential equations with time-constant parameters.

Linear systems

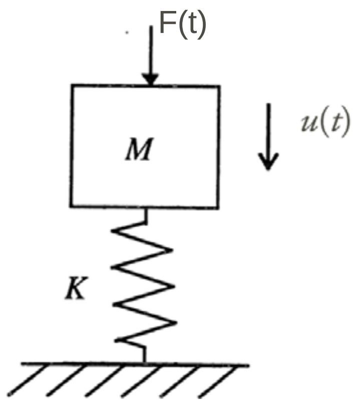
- Really useful and powerful concept



(Picture: Volvo Technology Report, nr 1 1988) [1]

Single-degree-of-freedom

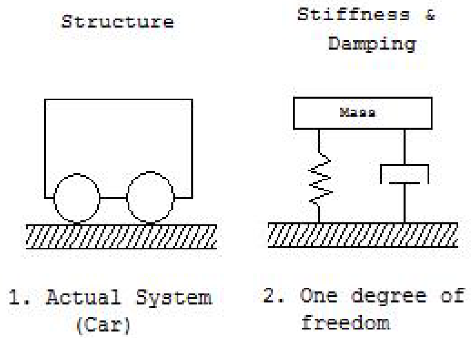
- SDOF simplest linear system?



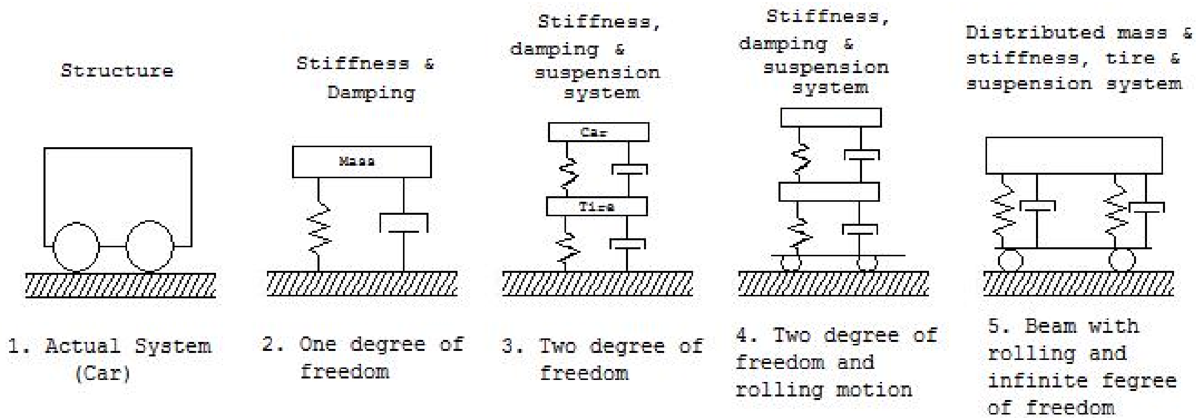
- A mass. A stiffness.
- Input force. Resulting displacement/velocity/accelleration.
- Isn't that perhaps too simple to be useful?

Single-degree-of-freedom

- One can do a lot with SDOF



- And one can use many SDOF to have a multiple degree-of-freedom system (MDOF).



https://commons.wikimedia.org/wiki/File:Structureandideal2.jpg

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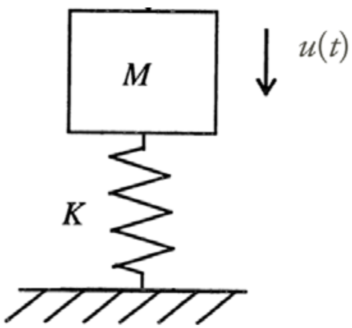
Equations of Motions of a mass-spring system



- Forces in the system:
  - Newton's law: M$\ddot{x}$
  - Hooke's law: $-Kx$
- Forces shall balance each other:
  - M$\ddot{x} = -Kx$
  - M$\ddot{x} + Kx = 0$
- We got Equations of Motions (EoM) of the system!

Free vibrations

- System is excited at a certain time – BANG!
- No more external forces after that excitation!
  - $M\ddot{x} + Kx = 0$
- What is the solution? i.e. how is the system moving?
- EoM is ordinary differential equations
  - We know the solutions from basic math courses
  - It is exponential functions (they appear again!)
  - $x(t) = ae^{\lambda t}$
  - Plug-in that solution in EoM!

Free vibrations

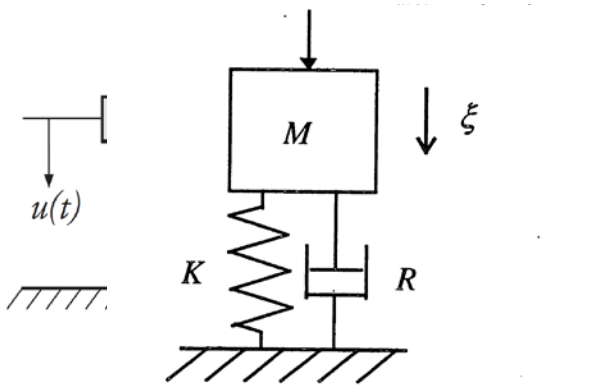
- EoM: $M\ddot{x} + Kx = 0$
- $M\lambda^2 ae^{\lambda t}$+K $ae^{\lambda t}$=0 (mathematically eigenvalue problem)
  - $\lambda^2 + \frac{K}{M} = 0$
    - » $\lambda_1 = \mathrm{i}\sqrt{\frac{K}{M}} = \mathrm{i}\omega_0$; $\lambda_2 = -\mathrm{i}\sqrt{\frac{K}{M}} = -\mathrm{i}\omega_0$.
- What are those solutions λ? (mathematically eigenvalues)
  - K has dimensions N/m = [kg m/s^2]/m = kg/s^2
  - M has dimensions of kg.
  - Then λ is 1/s = Hz. Frequency!
- $x(t) = ae^{\mathrm{i}\omega_0 t} + be^{-\mathrm{i}\omega_0 t}$=$\mathrm{A}\sin(\omega_0 t) + \mathrm{B}\cos(\omega_0 t)$.
- Oscillatory motion!

Analysis of dimensions is very helpful to check for mistakes and understand what is going on!

Free vibrations

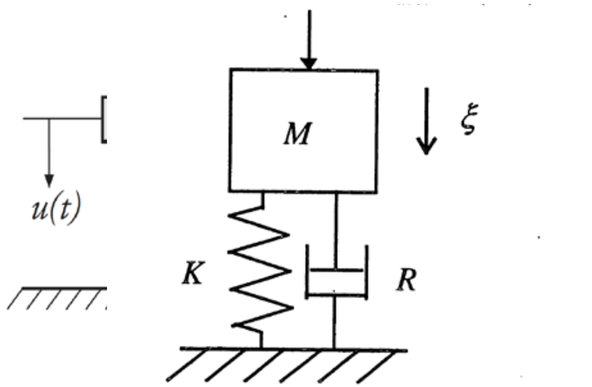
- $\sqrt{\frac{K}{M}} = \omega_0$ is the (angular) frequency with which that system oscillates.
  - It is the *natural* frequency of the system – eigenfrequency.
- We have not solved the problem because we have constants A and B.
  - $\mathrm{x(t)} = \mathrm{Asin}(\omega_0 t) + \mathrm{Bcos}(\omega_0 t)$.
  - We need initial conditions at t=0
  - Displacement $\mathrm{x}(t = 0)$ equals *something*.
  - Velocity $\dot{x}(t = 0)$ equals *something*.
  - Two conditions; two equations; two unknowns (A, B): OK!

Free vibrations with damping



- Forces shall balance each other:
    - $m\ddot{x} = -Kx - \mathrm{R}\dot{x}$
    - $m\ddot{x} + \mathrm{R}\dot{x} + Kx = 0$
- Same solution trial $x(t) = ae^{\lambda t}$

Free vibrations with damping



- $\lambda_1 = -\mathrm{R} + \mathrm{i}\omega_R = \mathrm{i}\omega_0 \; ; \lambda_2 = -\mathrm{R} - \mathrm{i}\omega_R \; ; \omega_R = \sqrt{\omega_0^2 - R^2}.$
- $x(t) = ae^{-\mathrm{R}t+\mathrm{i}\omega_R t} + be^{-\mathrm{R}t-\mathrm{i}\omega_R t} =$

$$= \left[\mathrm{A}\sin(\omega_R t) + \mathrm{B}\cos(\omega_R t)\right] e^{-\mathrm{R}t}$$

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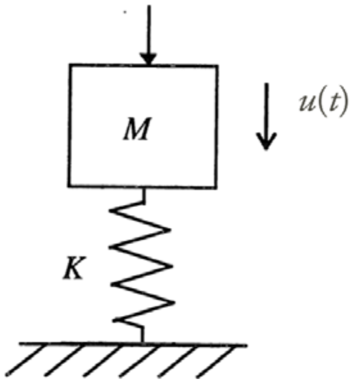
Forced motion – Undamped SDOF

- We had an external force. Forced motion.

$F(t) = F_{driv} \cdot \cos(\omega t)$



$$M\ddot{u}(t) + Ku(t) = F(t)$$

Inertial force — Elastic force — Applied force

  - $u(t)$ obtained by solving the EoM together with the initial conditions
    - » Solution = Homogeneous + Particular (homogeneous = free vibration!)

$$u(t) = u_p(t) + u_h(t)$$

$F(t) = F_{driv} \cdot \cos(\omega t)$ $\quad$ $F(t) = 0$

Forced motion – Undamped SDOF

- Eigenfrequency/Natural frequency: the frequency with which the system oscillates when it is left to free vibration after setting it into movement

$$\omega_0 = \sqrt{\frac{K}{M}} \qquad\qquad f_0 = \frac{1}{2\pi}\cdot\sqrt{\frac{K}{M}}$$

  – Expressed in angular frequency [rad/s] or Hertz [1/s=Hz]

- Homogeneous solution: $u_h(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t) = u_0\cos(\omega_0 t) + \dfrac{v_0}{\omega_0}\sin(\omega_0 t)$

  F(t)=0

  $u(t=0) = u_0\,;\quad \dot{u}(t=0) = v_0$

  Initial conditions

- Particular solution: $u_p(t) = u_0\cos(\omega t) = \dfrac{F_{driv}}{K}\cdot\dfrac{1}{\left[1-\left(\dfrac{\omega}{\omega_0}\right)^2\right]}\cdot\cos(\omega t)$

  "Static solution"

  If $\omega \approx \omega_0$ → **Resonance**

  Displacement response factor (R_d)

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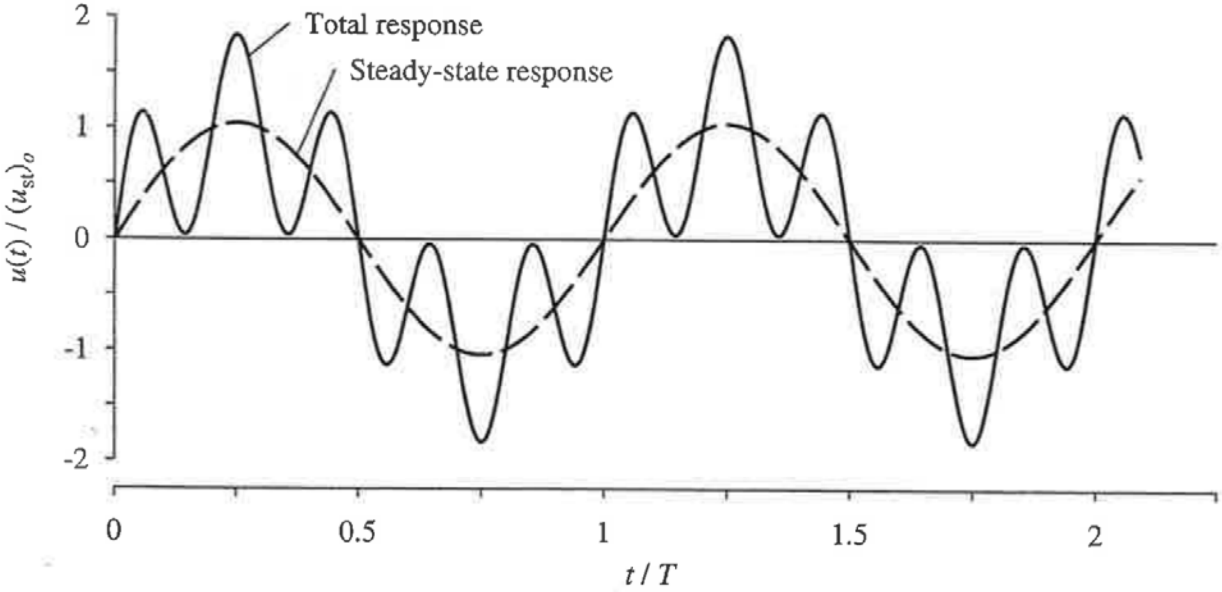
Forced motion – Undamped SDOF

$$u(t) = u_p(t) + u_h(t)$$



$$u_{total}(t) = \boxed{u_0 \cos(\omega_0 t) + \frac{v_0}{\omega_0} \sin(\omega_0 t)} + \boxed{\frac{F_{driv}}{K} \cdot \frac{1}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]} \cdot \cos(\omega t)}$$

Homogeneous

Particular

$$\omega_0 = \sqrt{\frac{K}{M}}$$

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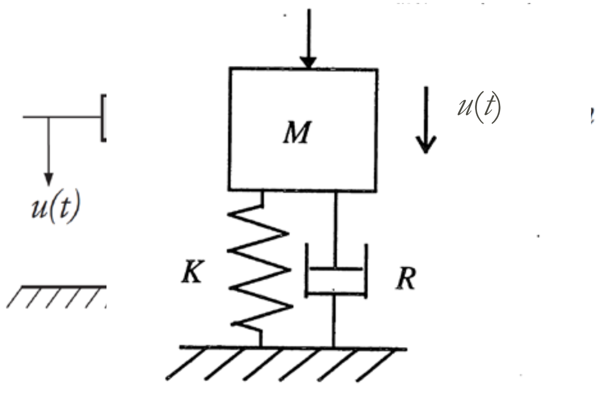
Forced motion – Damped SDOF

- Mass-spring-damper system (e.g. a floor)

$F(t) = F_{driv} \cdot \cos(\omega t)$



$$\underbrace{M\ddot{u}(t)}_{\text{Inertial force}} + \underbrace{R\dot{u}(t)}_{\text{Damping force}} + \underbrace{Ku(t)}_{\text{Elastic force}} = \underbrace{F(t)}_{\text{Applied force}}$$

  - $u(t)$ obtained by solving the EoM together with the initial conditions
    - » Solution = Homogeneous + Particular

$$u(t) = u_p(t) + u_h(t)$$

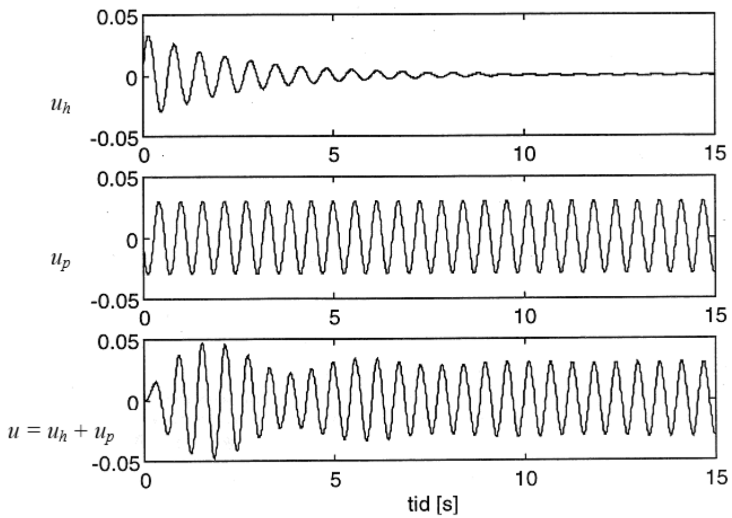
$F(t) = F_{driv} \cdot \cos(\omega t)$ $\qquad$ $F(t) = 0$

NOTE: Damping is the energy dissipation of a vibrating system

SDOF – Eigenfrequency with/without damping

- Remember! The natural frequency is the frequency with which the system oscillates when it is left to free vibration after setting it into movement
  - Undamped: $\omega_0 = \sqrt{\frac{K}{M}}$
  - Damped: $\omega_d = \omega_0 \sqrt{1-\zeta^2}$

NOTE: The natural frequency is not influenced very much by moderate viscous damping (i.e. <0.2)

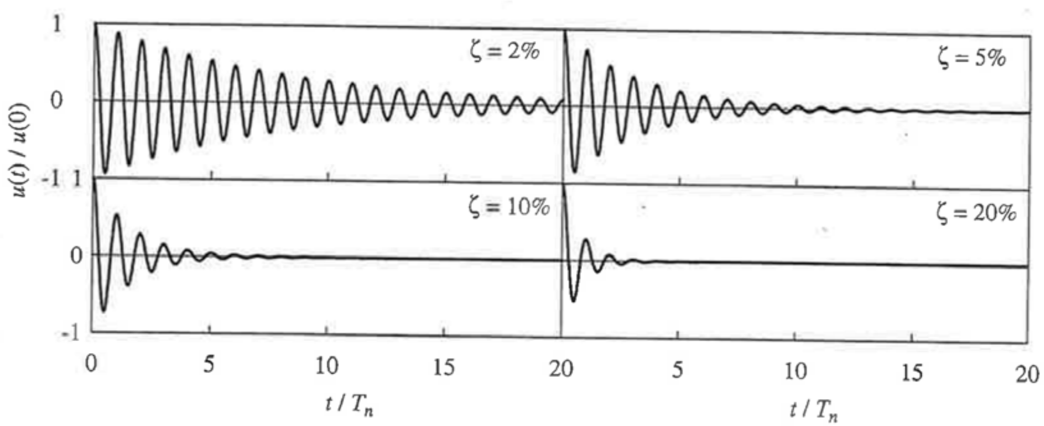


Figur 7 Homogen, parikulär och total lösning

SDOF – Eigenfrequency with/without damping

- Remember! The natural frequency is the frequency with which the system oscillates when it is left to free vibration after setting it into movement
  - Undamped: $\omega_0 = \sqrt{\dfrac{K}{M}}$
  - Damped: $\omega_d = \omega_0 \sqrt{1-\zeta^2}$

NOTE: The natural frequency is not influenced very much by moderate viscous damping (i.e. ζ<0.2)



Various behaviours for realistic levels of damping

Damped SDOF – Homogeneous solution

- Solution yielded when *F(t)=0*
- Solved with help of the initial conditions (B_1 and B_2)
- Composed of:
  - Decaying exponential part
  - Harmonically oscillating part

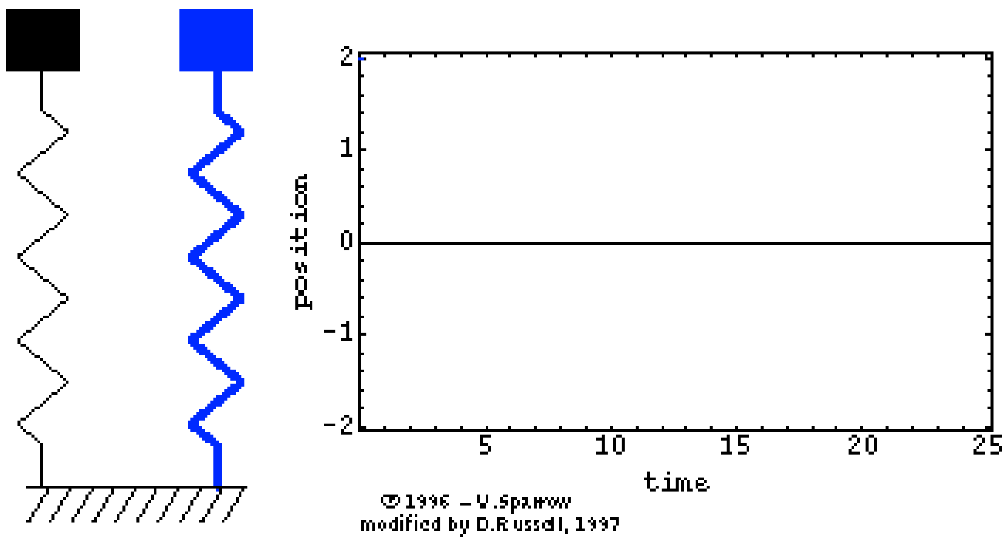
$$u_h(t) = e^{-\frac{\eta}{2}\omega_0 t}\left(A_1 e^{i\omega_d t} + A_2 e^{-i\omega_d t}\right) = \boxed{e^{-\frac{\eta}{2}\omega_0 t}}\boxed{\left(B_1 \sin(\omega_d t) + B_2 \cos(\omega_d t)\right)}$$

$$\eta = \frac{R}{\sqrt{MK}} = 2\zeta \qquad \omega_d = \omega_0 \sqrt{1 - \left(\frac{\eta}{2}\right)^2}$$

<u>NOTE:</u> B_1 and B_2 calculated from the initial conditions

SDOF – Homogeneous solution

- Function of damping
  - Responsible for the system's energy loss
  - Example



Without damping
With damping

Damped SDOF – Particular solution

- Solution showing the displacement under the driving force:
  - For example: $F(t) = F_{driv} \cdot cos(\omega t)$

- The solution has the form: $u_p(t) = D_1 \sin(\omega t) + D_2 \cos(\omega t)$

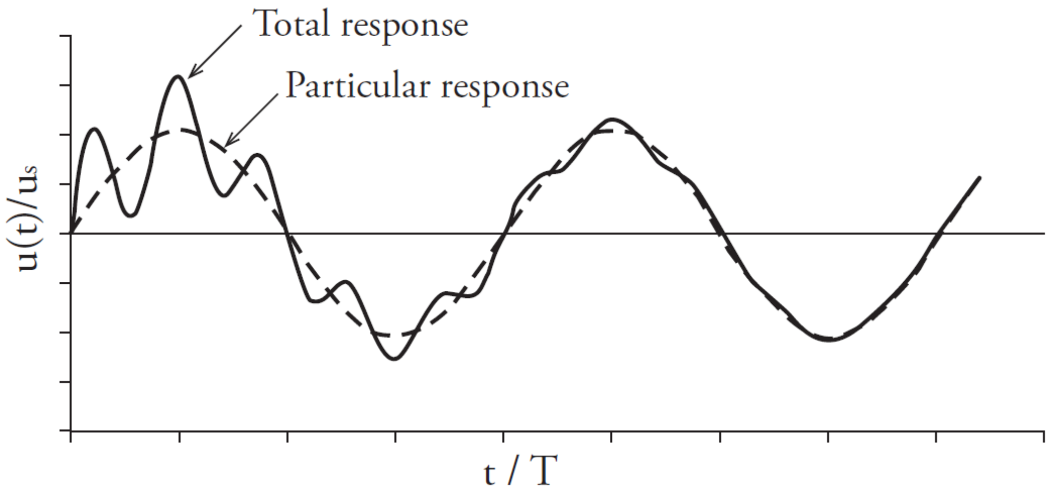
Which gives the solution

$$D_1 = \frac{R\omega}{\left(K - M\omega^2\right)^2 + \left(R\omega\right)^2} \cdot F_{driv}$$

$$D_2 = \frac{K - M\omega}{\left(K - M\omega^2\right)^2 + \left(R\omega\right)^2} \cdot F_{driv}$$

Damped SDOF – Total solution

- Total solution = homogeneous + particular
  - The homogeneous solution vanishes with increasing time. After some time: $u(t) \approx u_p(t)$



Total response of a damped system subjected to a harmonic force,

$$u_{total}(t) = \underbrace{\boxed{e^{-\frac{\eta}{2}\omega_0 t}\left(B_1 \sin(\omega_d t) + B_2 \cos(\omega_d t)\right)}}_{\text{Homogeneous}} + \underbrace{\boxed{D_1 \sin(\omega t) + D_2 \cos(\omega t)}}_{\text{Particular}}$$

$$\omega_d = \omega_0 \sqrt{1-\zeta^2}$$

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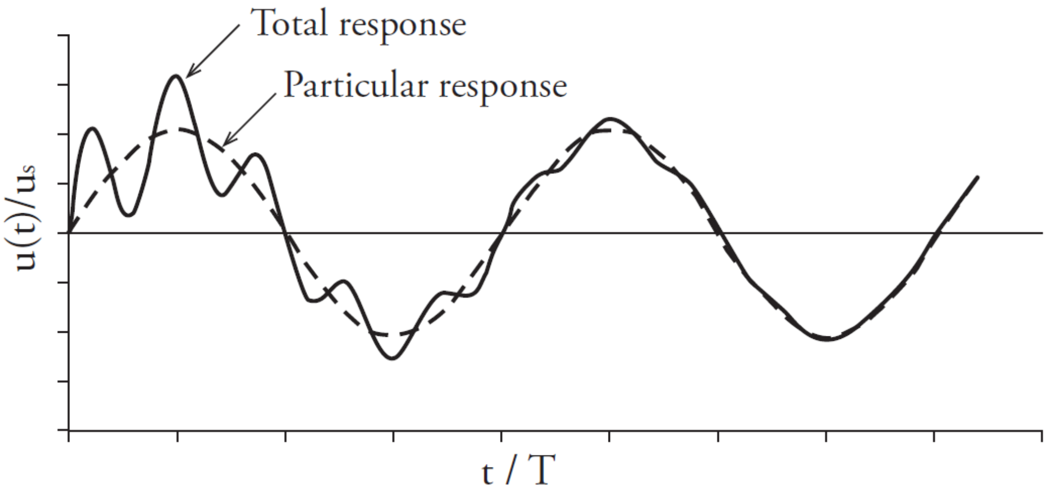
Damped SDOF – Total solution

- Total solution = homogeneous + particular
  - The homogeneous solution vanishes with increasing time. After some time: $u(t) \approx u_p(t)$



Total response of a damped system subjected to a harmonic force,

$$u_{total}(t) = u_p(t) + u_h(t) = \left[\frac{\left(\frac{F_{driv}}{K}\right) \cdot \cos(\omega t - \alpha)}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2 \cdot \left(\frac{\eta}{2}\right) \cdot \left(\frac{\omega}{\omega_0}\right)\right)^2}} \right] + e^{-\frac{\eta}{2}\omega_0 t}\left(B_1 \sin(\omega_d t) + B_2 \cos(\omega_d t)\right)$$

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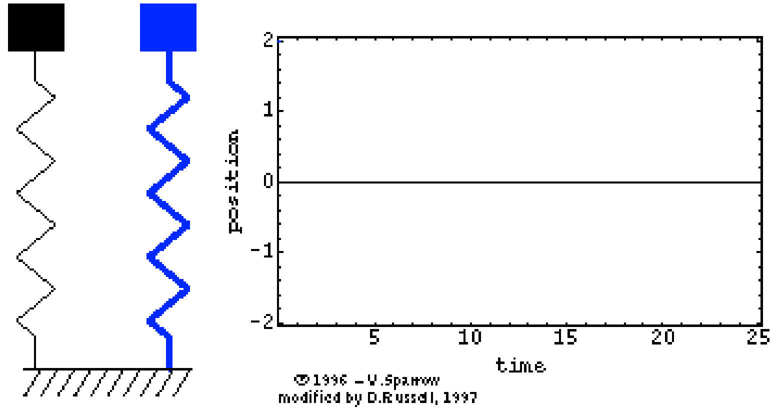
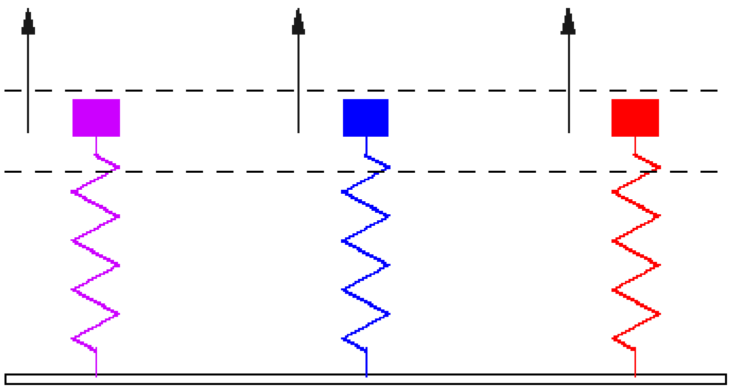
SDOF – Driving frequencies

- Ex:
  - Without damping
  - With damping



- Same natural frequency f=1
- Different driving frequency

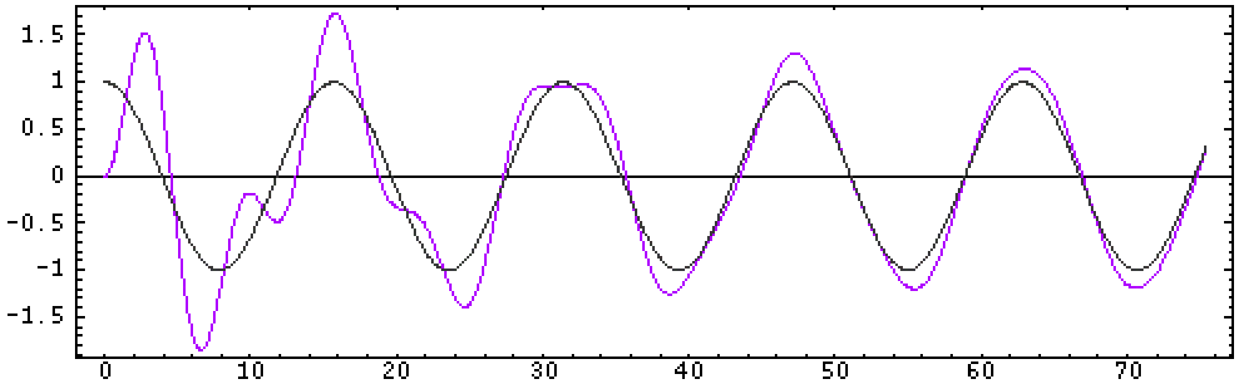


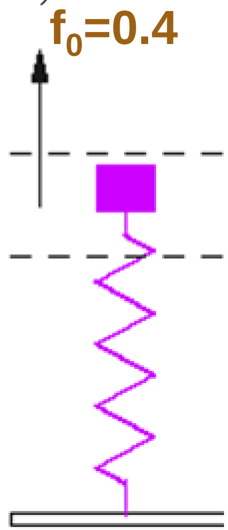
f_0=0.4, f_0=1.01, f_0=1.6

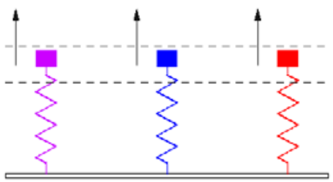
$\omega < \omega_0$ $\omega = \omega_0$ $\omega > \omega_0$

SDOF – Low frequency excitation ($\omega < \omega_0$)

- The spring dominates (increase stiffness to reduce motion)
  - Force and displacement in phase







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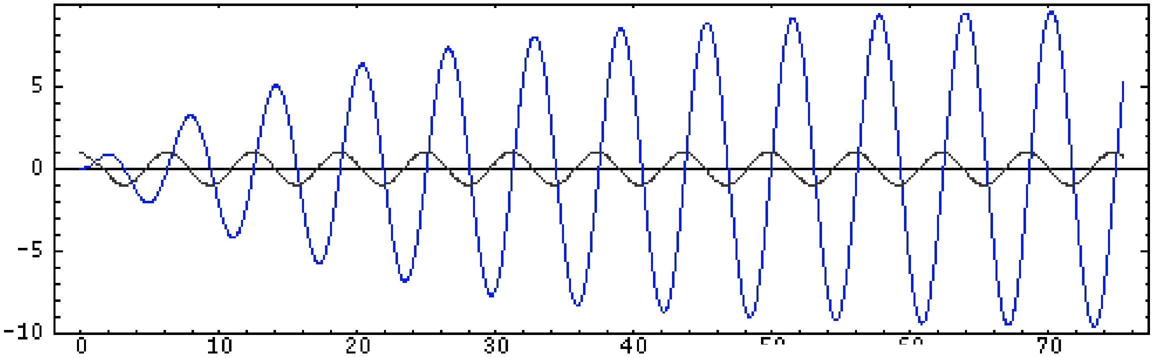
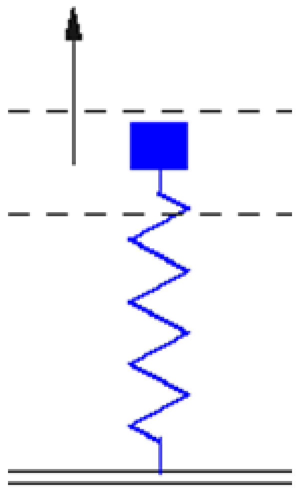
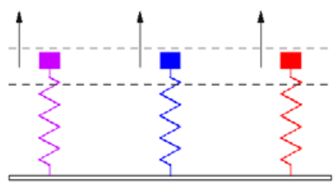
SDOF – Excitation at resonance freq. ($\omega = \omega_0$)

- Damping dominates (increase damping to reduce motion)
  - Phase difference = 90° or π
- If no (or little) damping is present:
  - The system collapses



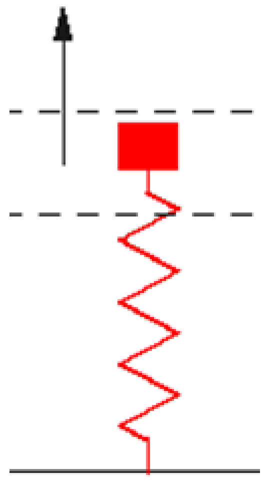


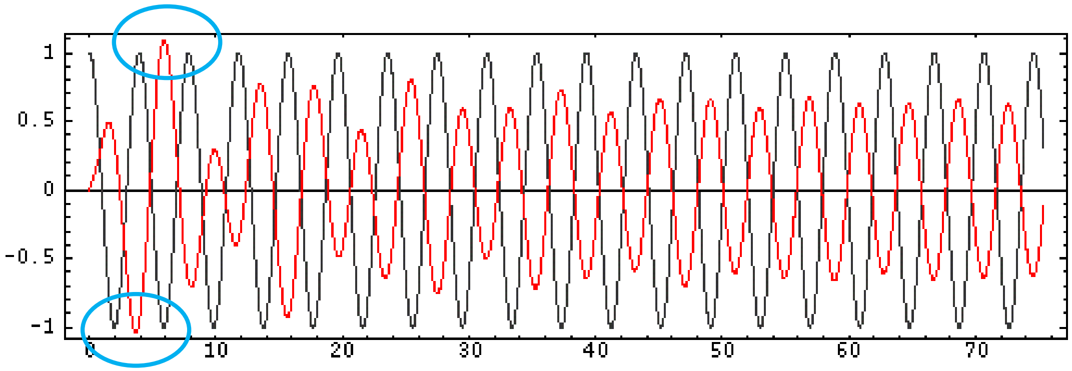


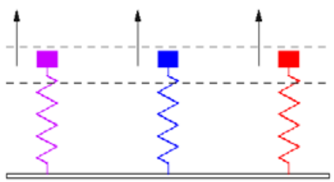
SDOF – High frequency excitation ($\omega > \omega_0$)

- The mass dominates (increase mass to reduce motion)
- Force and displacement in counter phase:
  - Phase difference = 180° or π

f_0=1.6

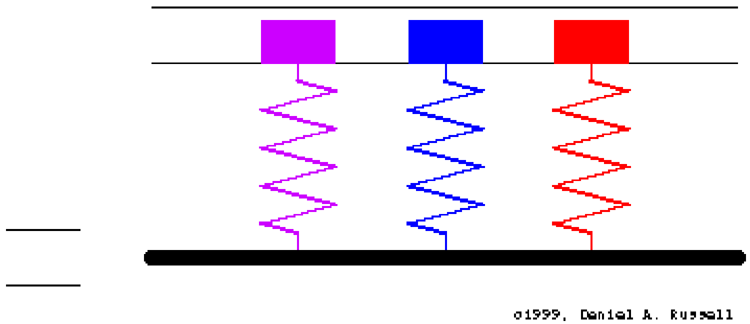






SDOF – Driving frequencies

- Same driving frequency, different natural frequencies
    - That is, different stiffness and/or mass.



Driving f = 1.001 Hz

$f_1 = 1.6$ $f_2 = 1$ $f_3 = 0.63$

SDOF – Complex representation (Freq. domain)

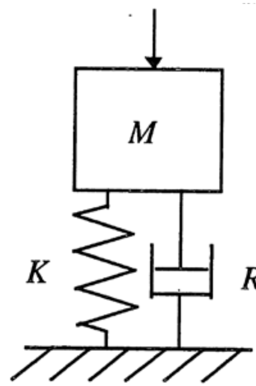
- Euler's formula: $e^{i\varphi} = \cos(\varphi) + i\sin(\varphi)$

- Then: $F(t) = F_{driv}\cos(\omega t) = \mathrm{Re}\left[F_{driv}e^{i\omega t}\right]$

  $u(t) = u_0\cos(\omega t - \varphi) = \mathrm{Re}\left[ue^{i\varphi}e^{i\omega t}\right] = \mathrm{Re}\left[\tilde{u}(\omega)e^{i\omega t}\right]$

- Differenciating: $\dot{u}(t) = \mathrm{Re}\left[i\omega \cdot \tilde{u}(\omega)e^{i\omega t}\right]$

  $\ddot{u}(t) = \mathrm{Re}\left[-\omega^2 \cdot \tilde{u}(\omega)e^{i\omega t}\right]$



- Substituting in the EOM: $M\ddot{u}(t) + R\dot{u}(t) + Ku(t) = F_{driv}\cos(\omega t)$

NOTE: This is the particular solution in complex form for a damped SDOF system. In Acoustics, most of the times, we are interested in the particular solution, which is the one not vanishing as time goes by.

$$\tilde{u}(\omega) = \frac{F_{driv}}{(K - M\omega^2) + Ri\omega}$$

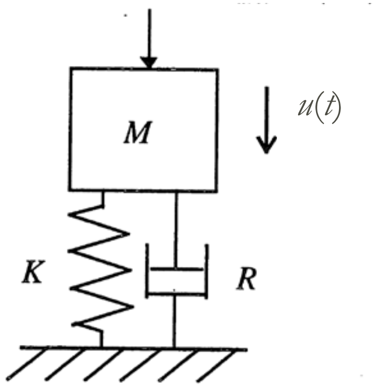
NOTE: Differential equation became second order equation with time-harmonic ansats!

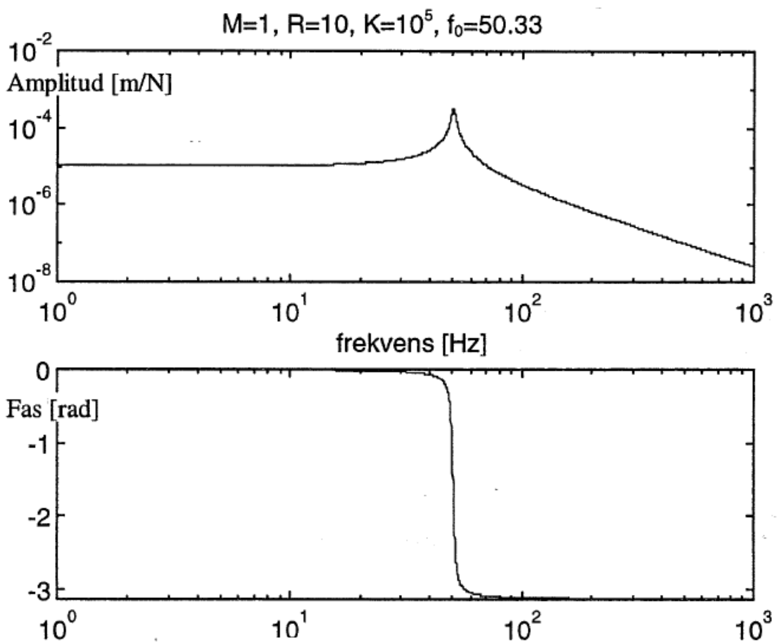
If the system is excited with $\omega_0^2 = K/M$
→ Resonance (dominated by damping)

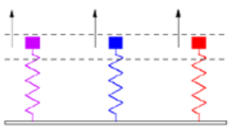
SDOF – Frequency response function

- Output / Input $$\frac{\tilde{u}(\omega)}{F_{driv}} = \frac{1}{(K - M\omega^2) + Ri\omega}$$
- Kvoten är ett komplext tal och heter överföringsfunktion

$F(t) = F_{driv} \cdot \cos(\omega t)$



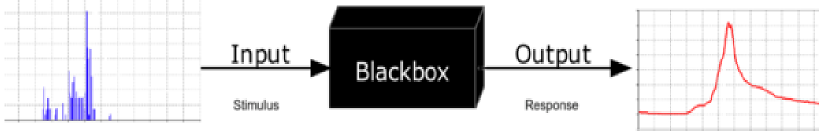




SDOF – Frequency response functions (FRF)

- In general, FRF = transfer function, i.e.:

$$H_{ij}(\omega) = \frac{\tilde{s}_i(\omega)}{\tilde{s}_j(\omega)} = \frac{\text{output}}{\text{input}}$$



  - Contains system information
  - Independent of outer conditions
  - **Frequency domain relationship between input and output of a linear time-invariant system**

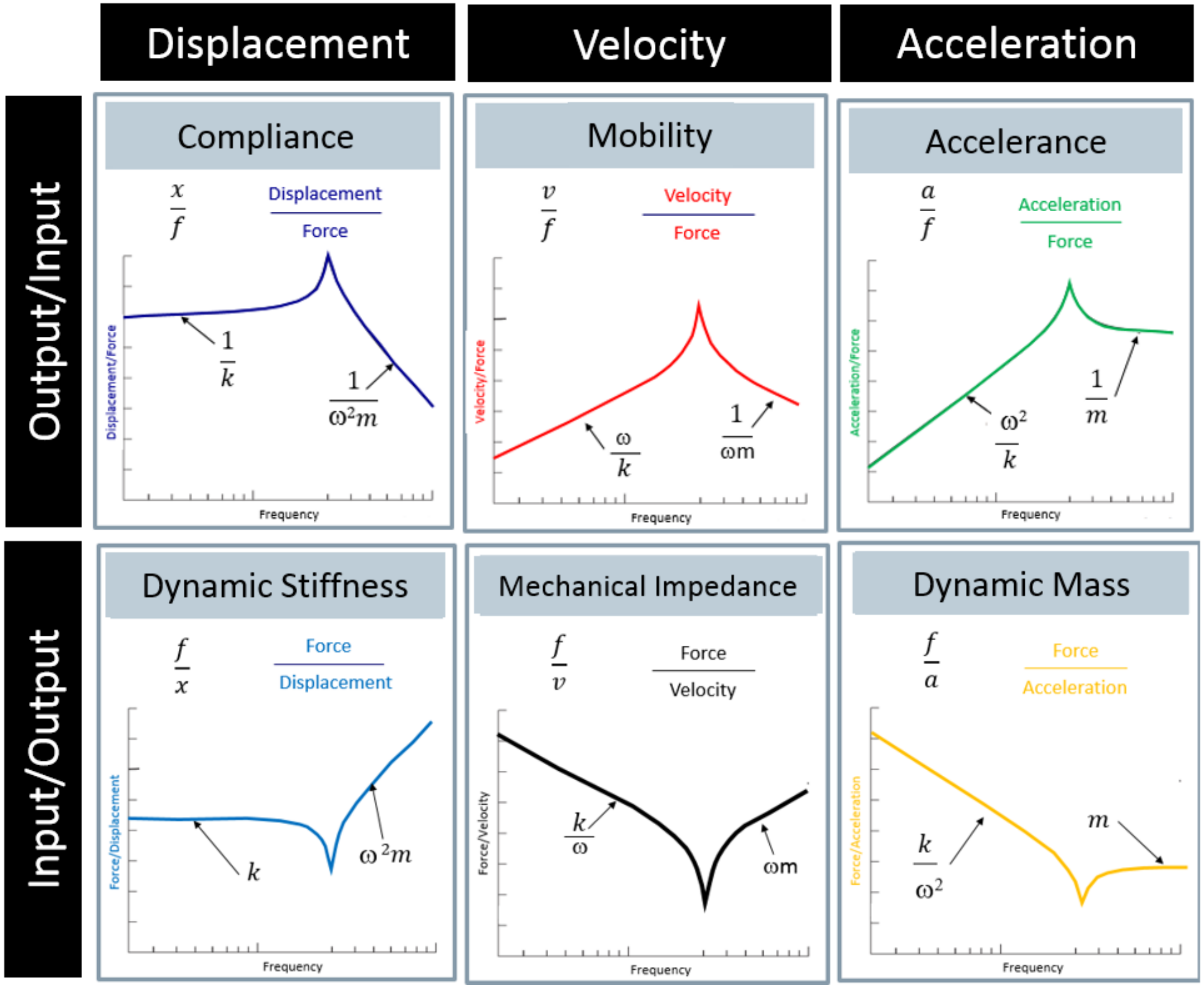
- Different FRFs can be obtained depending on the measured quantity

Measured quantity	**FRF**	
Acceleration (a)	Accelerance = $N_{dyn}(\omega)$ = a/F	Dynamic Mass = $M_{dyn}(\omega)$ = F/a
Velocity (v)	Mobility/admitance = $Y(\omega)$ = v/F	Impedance = $Z(\omega)$ = F/v
Displacement (u)	Receptance/compliance = $C_{dyn}(\omega)$= u/F	Dynamic stiffness = $K_{dyn}(\omega)$ = F/u

$$C_{dyn}(\omega) = \frac{\tilde{u}(\omega)}{F_{driv}(\omega)} = \frac{1}{(K - M\omega^2) + Ri\omega}$$

$$K_{dyn}(\omega) = C_{dyn}(\omega)^{-1} = -M\omega^2 + Ri\omega + K$$

SDOF – Frequency response functions (FRF)



Source: https://community.plm.automation.siemens.com/t5/Testing-Knowledge-Base/The-FRF-and-its-Many-Forms

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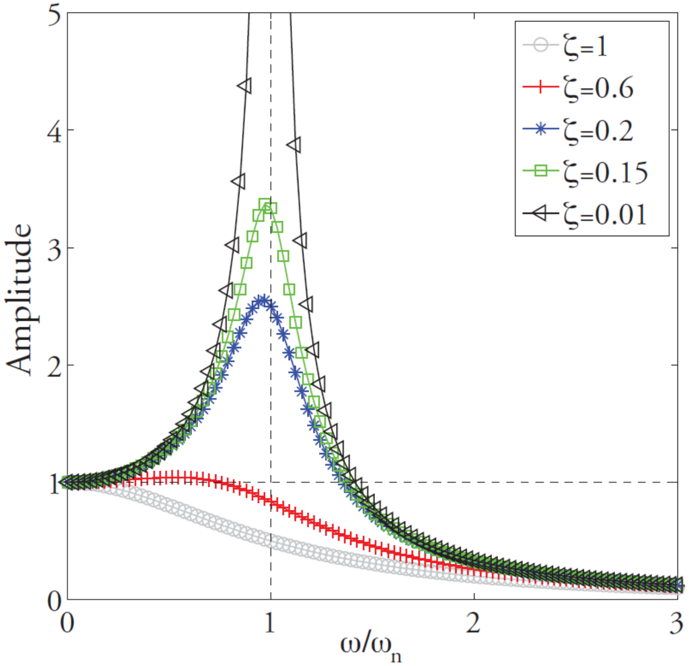
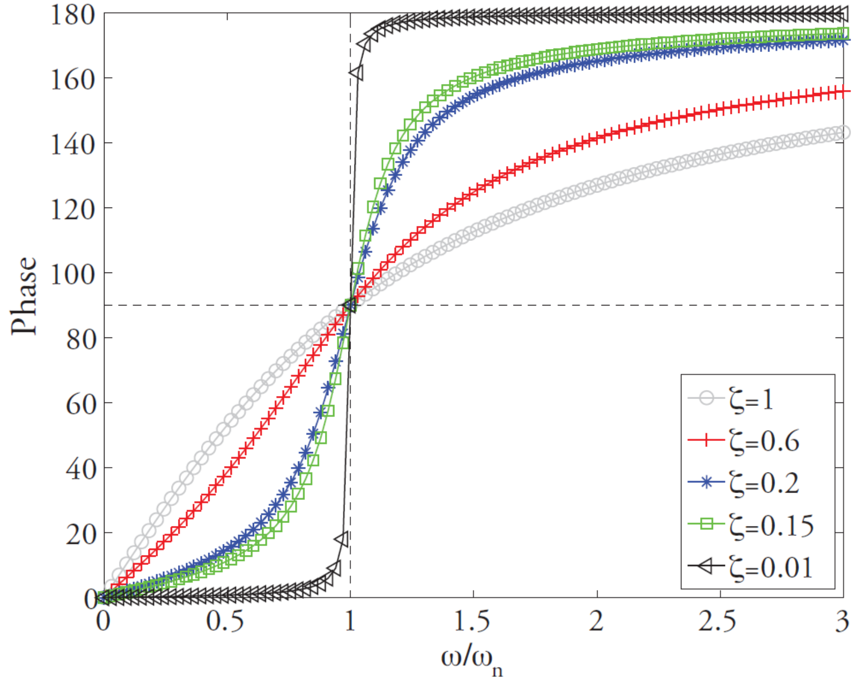
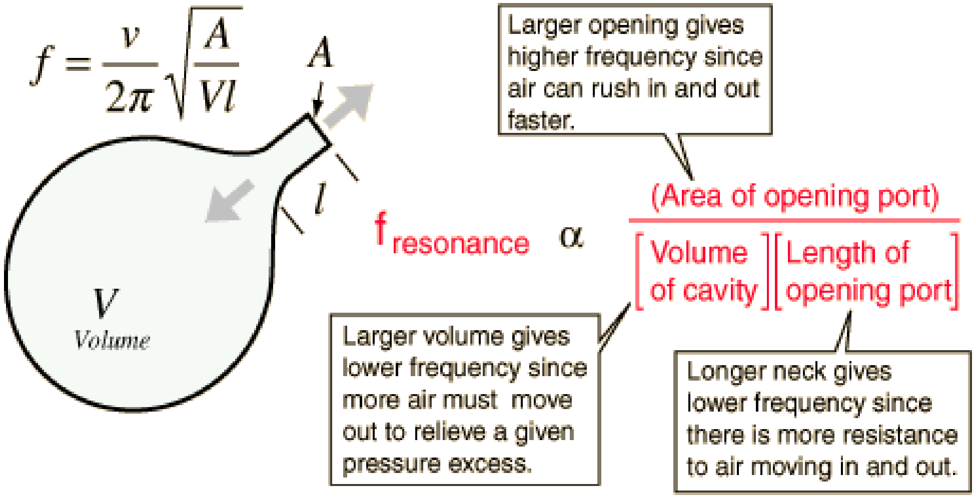
SDOF – Linear dynamic response to harmonic excitation





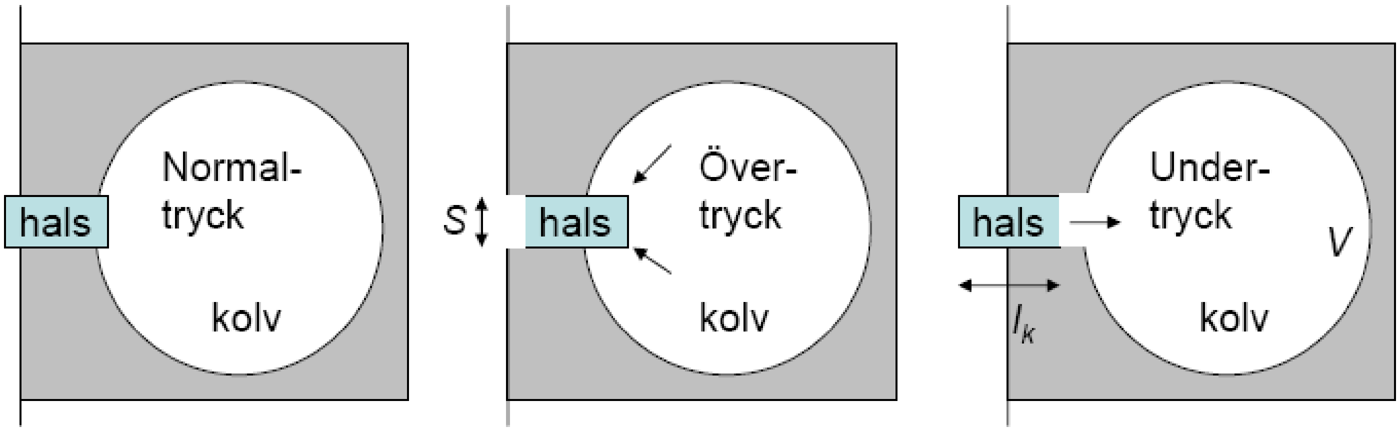
Helmholtz resonator

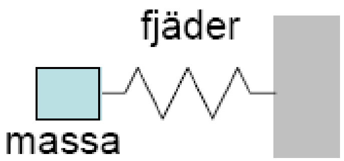


Source: hyperphysics

https://youtu.be/VAySXYqbc8M

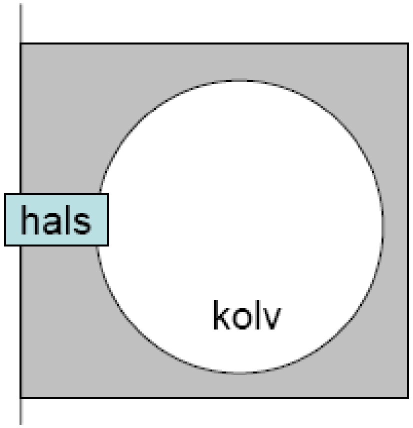
Helmholtz resonator



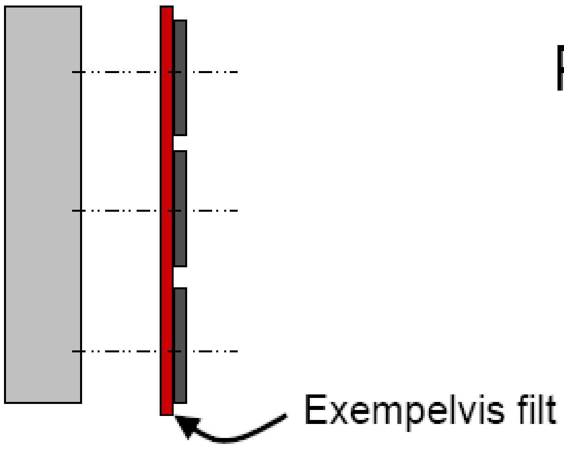


$$f_r = \frac{c}{2\pi}\sqrt{\frac{S}{l_k V}}$$

Helmholtz resonator (III)

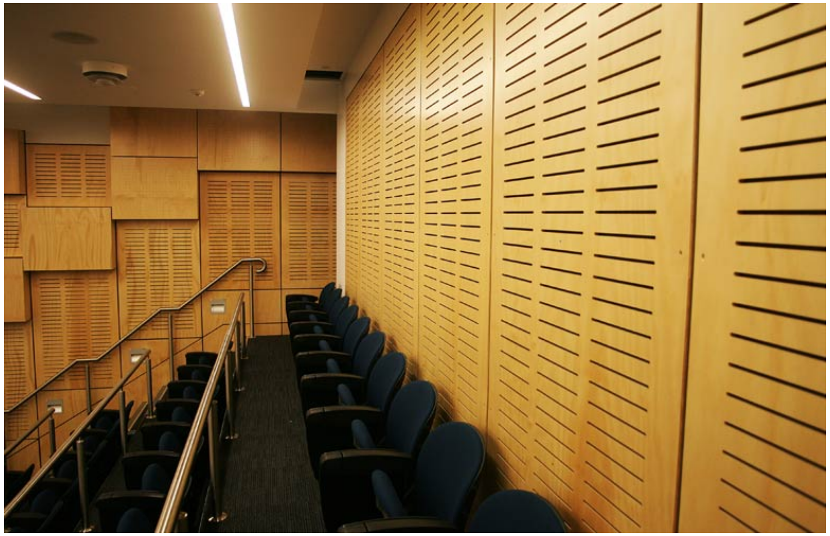


Där rörelsen är som störst.



Perforerad absorbent.

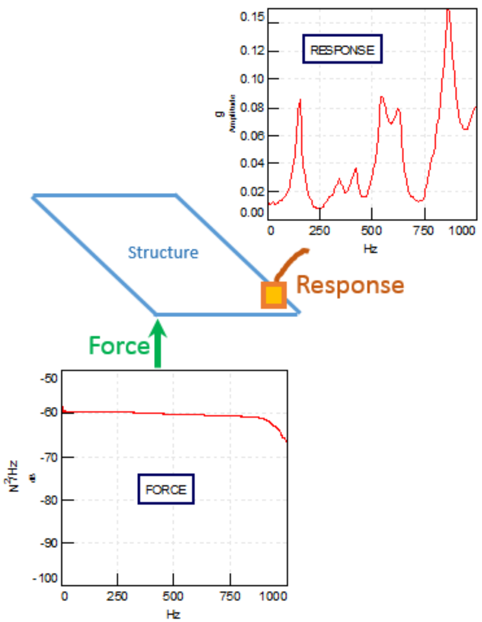
Helmholtz resonator (IV)

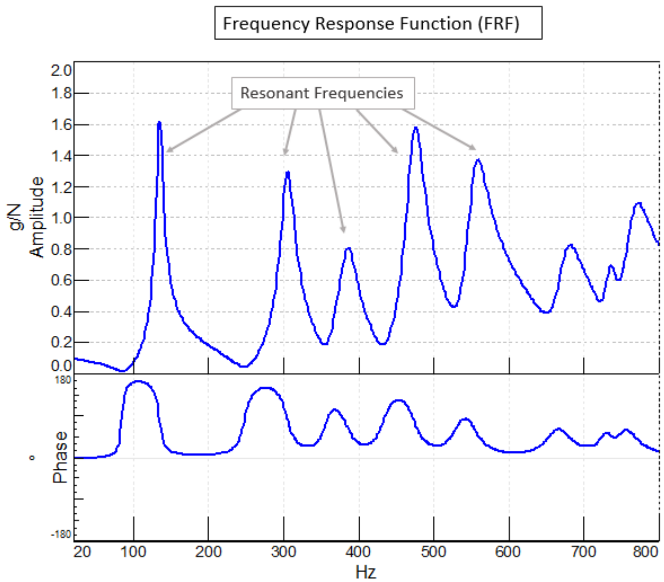


FRF of a complex system

- What happens if we evaluate FRF of a complex (linear) system such as a plate?
    - We gain useful info on it!

$$H_{ij}(\omega) = \frac{\widetilde{s_i}(\omega)}{\widetilde{s_j}(\omega)} = \frac{\text{output}}{\text{input}}$$





Source: https://community.sw.siemens.com/s/article/what-is-a-frequency-response-function-frf

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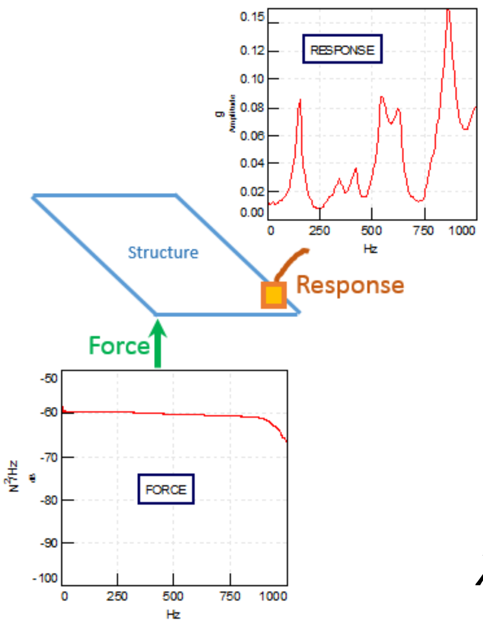
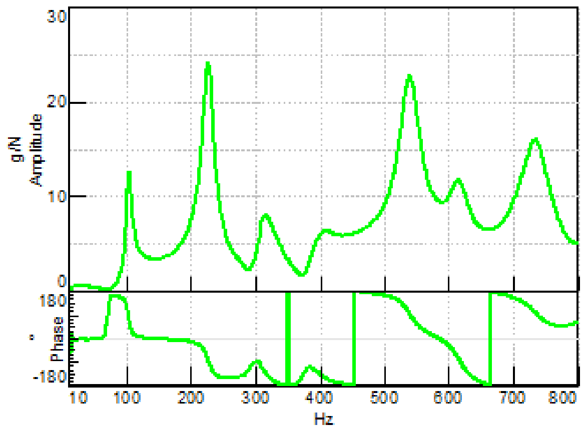
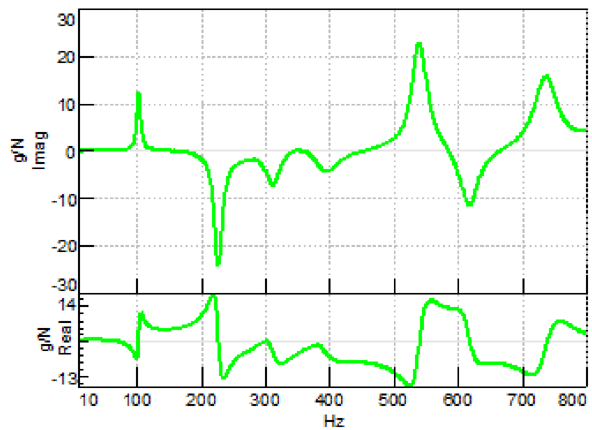
FRF of a complex system

- FRFs are complex
  - Amplitude/Phase
  - Real / imaginary part

$$H_{ij}(\omega) = \frac{\tilde{s}_i(\omega)}{\tilde{s}_j(\omega)} = \frac{\text{output}}{\text{input}}$$





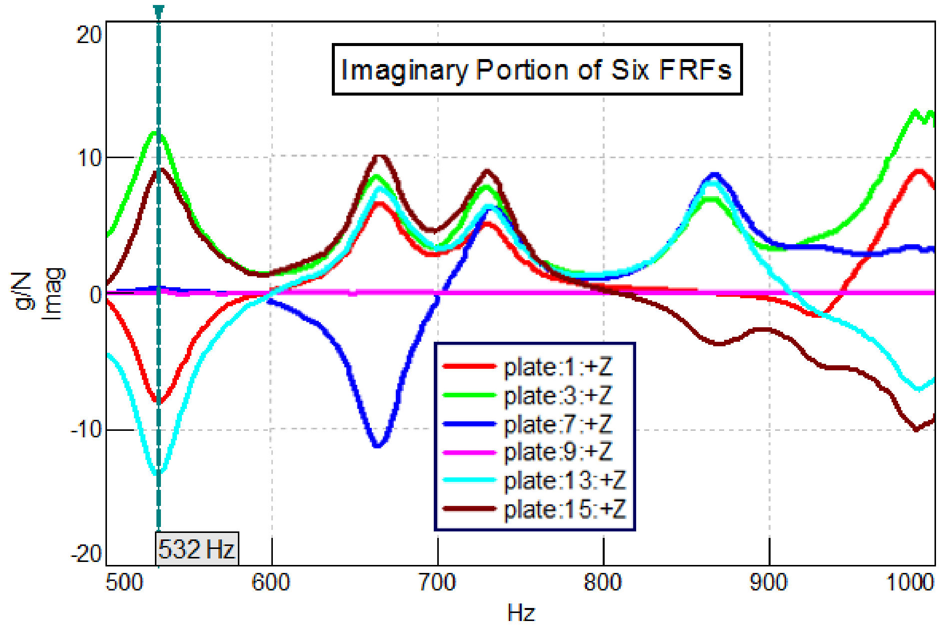


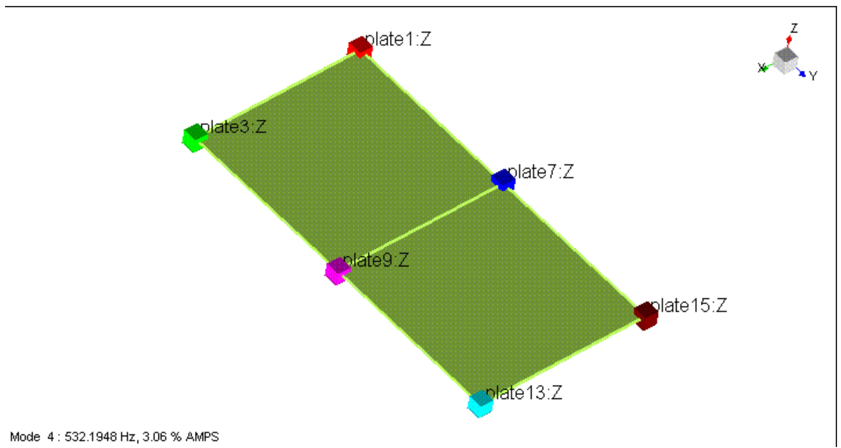
$$\lambda_1 = -\mathrm{R} + \mathrm{i}\omega_R = \mathrm{i}\omega_0 \; ; \lambda_2 = -\mathrm{R} - \mathrm{i}\omega_R \; ; \omega_R = \sqrt{\omega_0^2 - R^2}.$$

Source: https://community.sw.siemens.com/s/article/what-is-a-frequency-response-function-frf

FRF of a complex system

- Real and imaginary parts – the imaginary part has interesting information





Source: https://community.sw.siemens.com/s/article/what-is-a-frequency-response-function-frf

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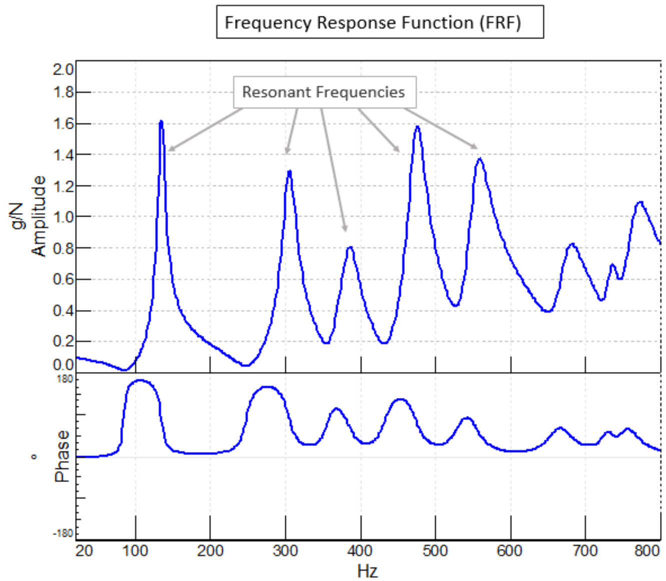
FRF of a complex system

- Each peak is showing a natural frequency
    - Each peak is a mass-spring-damper SDOF system?!



Source: https://community.sw.siemens.com/s/article/what-is-a-frequency-response-function-frf

Thank you for your attention!

mathias.barbagallo@construction.lth.se

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