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Acoustics (VTAN01)
– Analytical models for sound reduction

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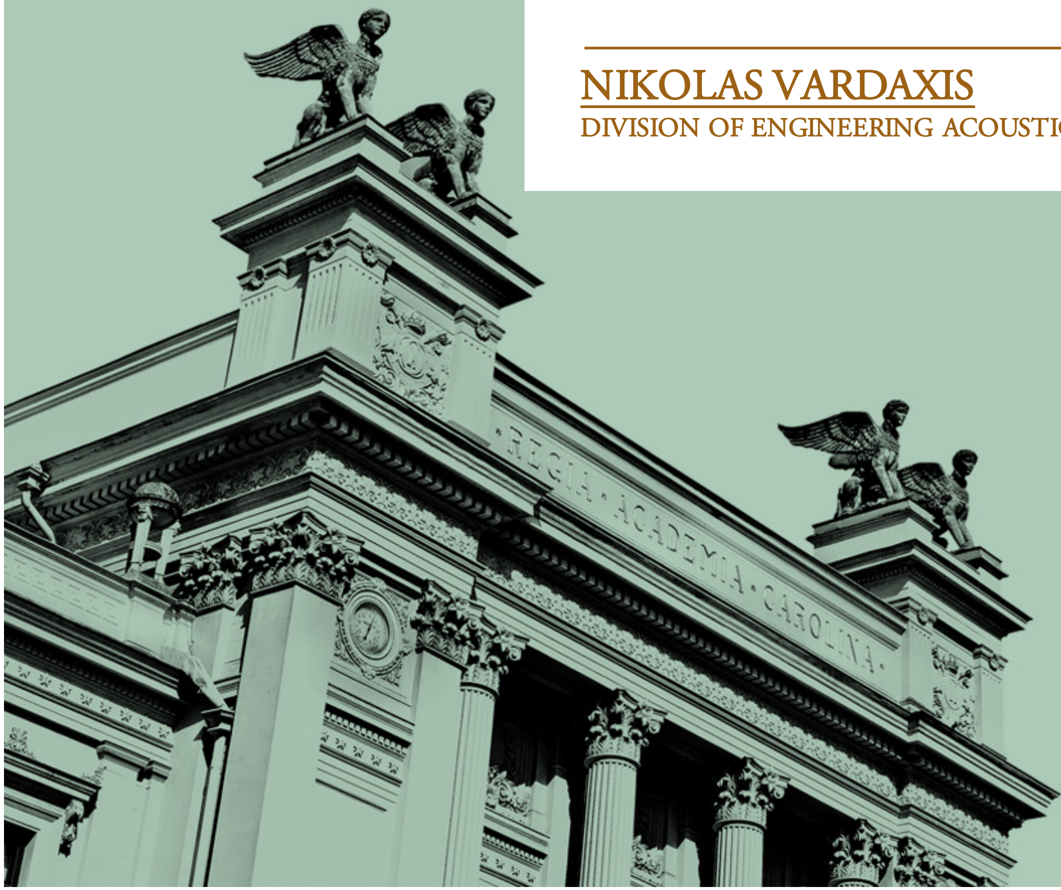


Outline

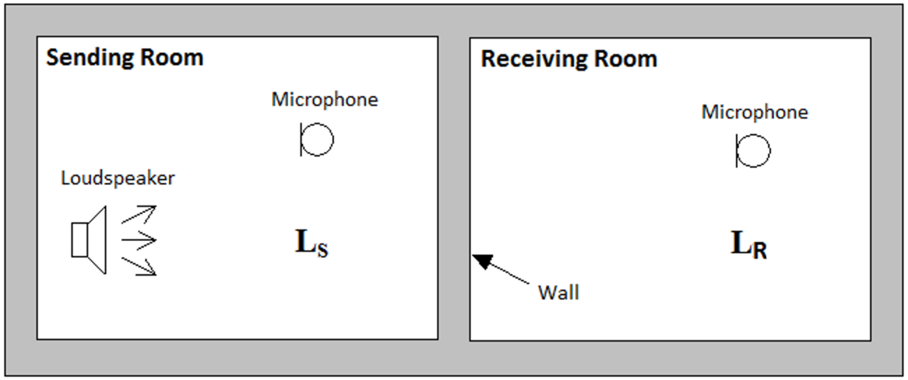
Introduction

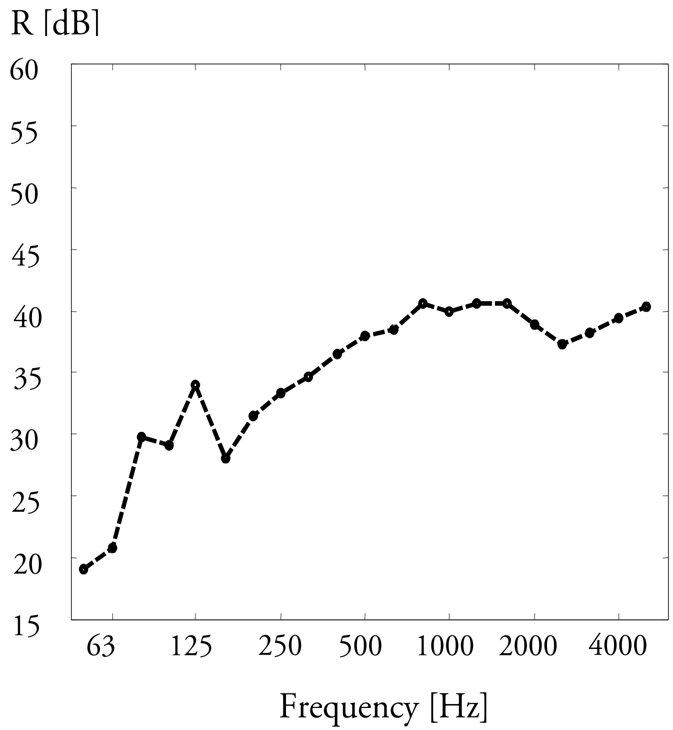
Analytical models for calculation of *R(f)* in single and double-leaf walls

Summary

… recap (I)

- Airborne sound insulation measurements (ISO standards)





$$R(f) = L_S(f) - L_R(f) + 10\log\left(\frac{S}{A(f)}\right)$$

Statement of results:

- $R'_w\,(C_{50\text{-}3150}\,;\,C_{tr})$
- $R_w\,(C_{50\text{-}3150}\,;\,C_{tr})$

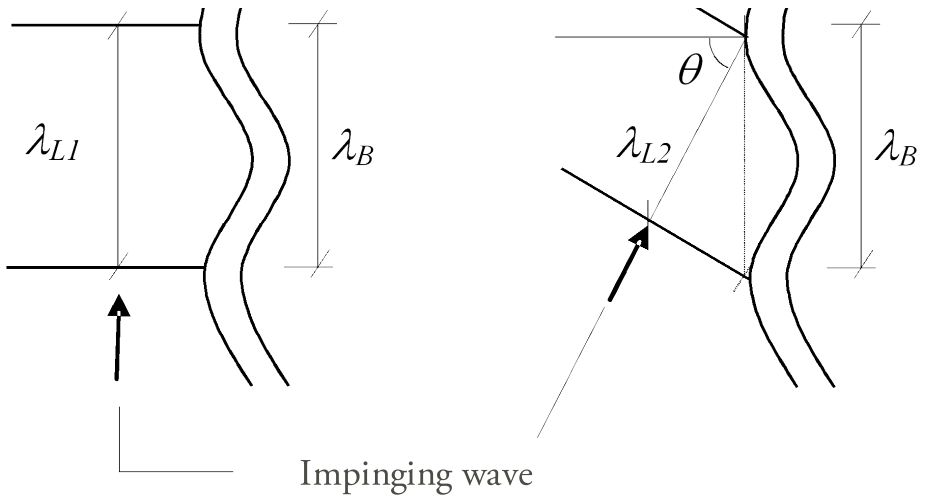
Outline

Introduction

Analytical models for calculation of *R(f)* in single and double-leaf walls **(just direct transmission)**

Summary

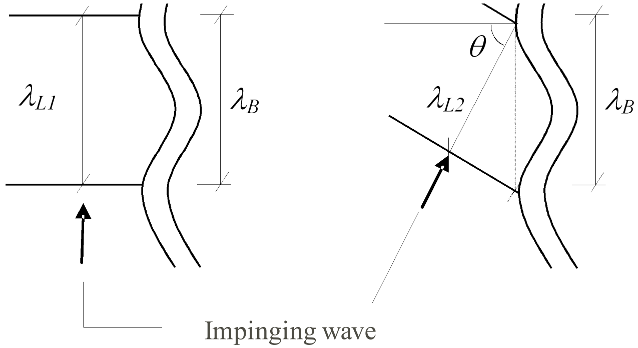
DEF: Coincidence – critical frequency (I)



- The wavelength of a bending wave λ_B is dependent on frequency, bending stiffness and mass density

- When the wavelength of sound in air coincides with the structural wavelength → Coincidence phenomena
  - Radiation efficiency becomes very high
  - Poor insulation

DEF: Coincidence – critical frequency (II)



- Bending wave velocity in a plate

$$c_B = \sqrt{2\pi f}\sqrt[4]{\frac{B}{m''}}$$

- If $f = f_c$ thus $c_B = c_o = 340$ m/s (f_c = critical frequency)

$$f_c = \frac{c_0^2}{2\pi}\sqrt{\frac{m''}{B}}$$

- Or expressed as a function of the coincidence number

$$f_c = \frac{K}{h}$$

NOTE: The condition for coincidence is that $\lambda_B = \lambda\sin(\phi)$. Therefore, if the incidence angle ϕ decreases, the coincidence frequency f_c increases according to $f_c(\phi) = f_c/\sin^2(\phi)$. The lowest frequency at which coincidence occur (critical frequency) occurs at the incidence angle $\phi = 90°$.

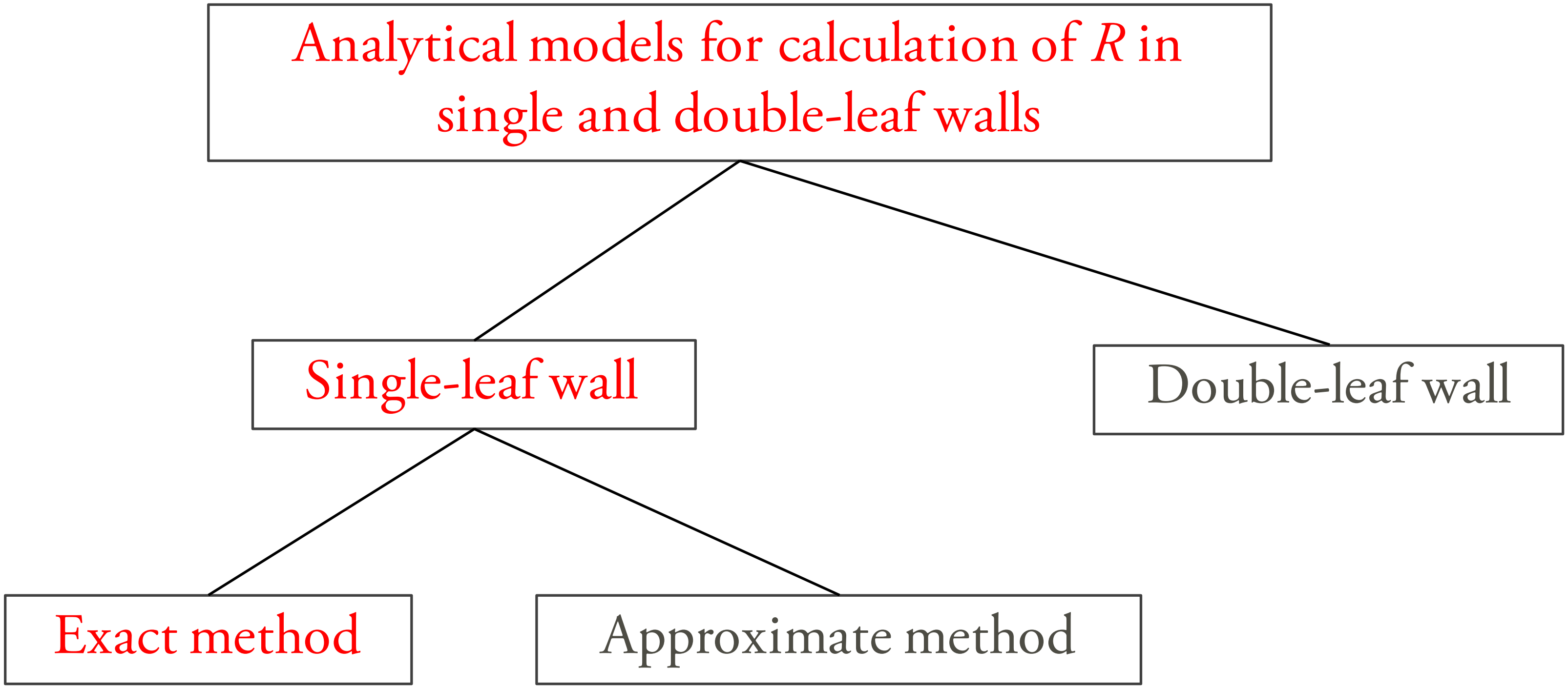
Critical frequency for common materials

- For a homogeneous isotropic plate of uniform thickness, the coincidence number is:

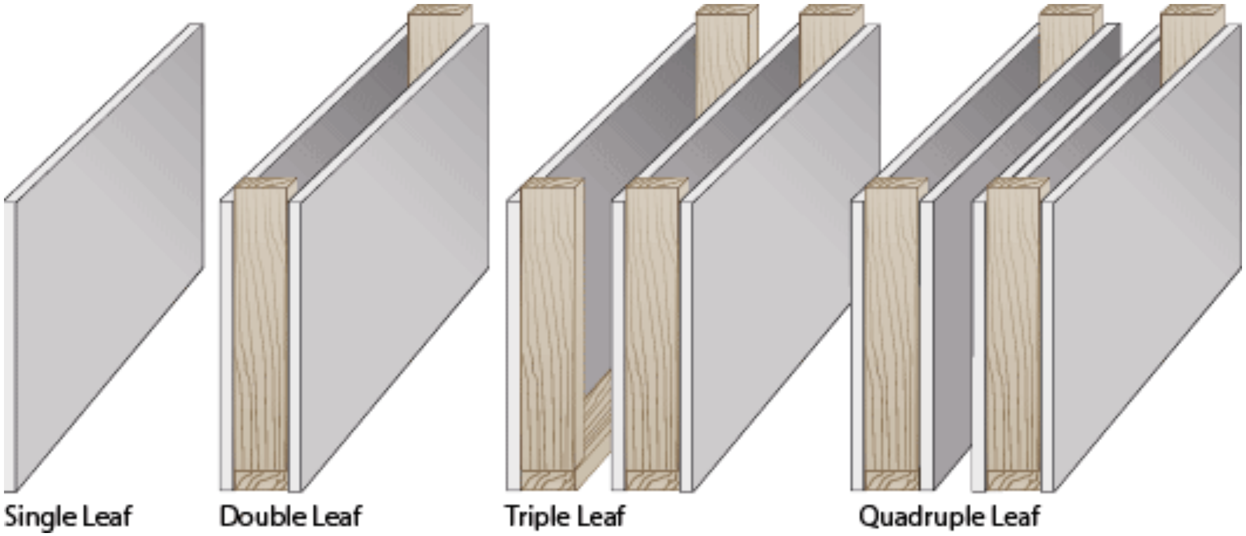
$$K = 60000\sqrt{\frac{\rho}{E}}$$

Material	Coincidence number (K)	Thickness [m]	f_c [Hz]
Concrete	18	160	110
Light concrete	38	70	540
Gypsum	32	10	3200
Steel	12-13	1	12000
Glass	18	3	6000

Outline

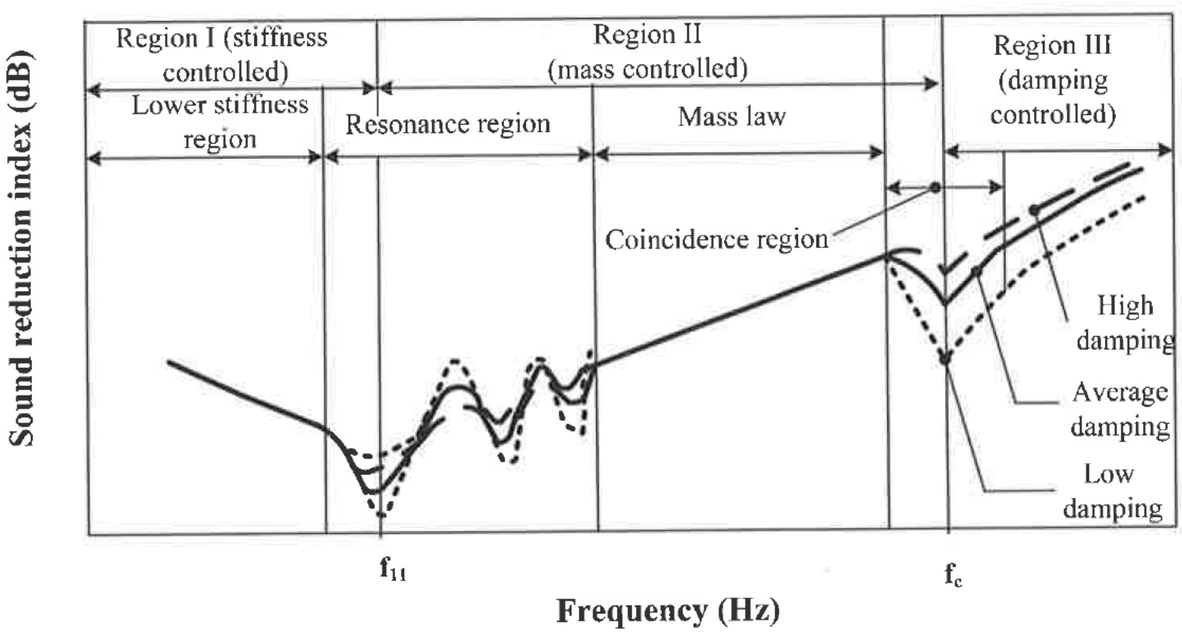

Analytical models for calculation of *R* in single and double-leaf walls
Single-leaf wall
Double-leaf wall
Exact method
Approximate method

Wall types



Sound reduction index of single-leaf partitions (I)

- **Exact method**
  - Region I: Stiffness-controlled region ($f < f_{11}$)
  - Region II: Mass-controlled region ($f_{11} < f < f_c$)
  - Region III: Damping-controlled region ($f_c < f$)



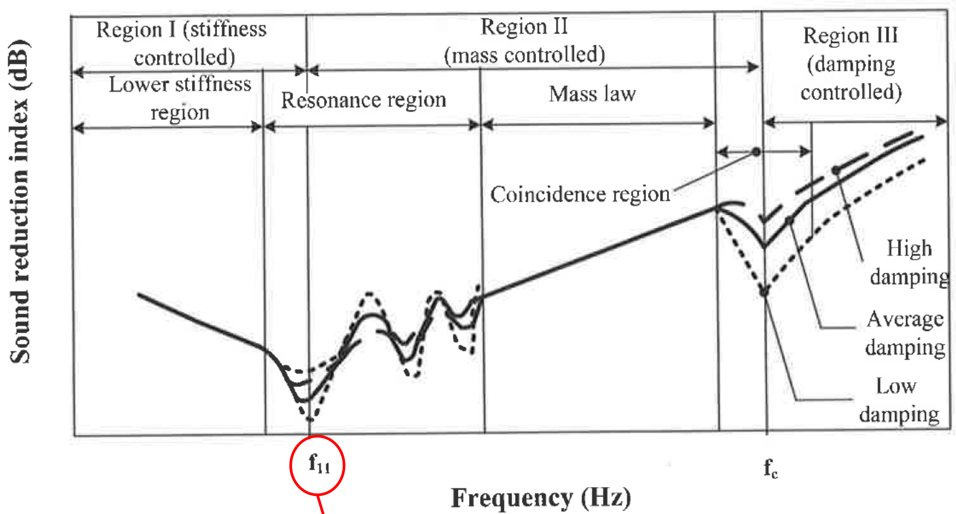
Sound reduction index of single-leaf partitions (II)

- Region I: Stiffness-controlled region ($f < f_{11}$)
  - Panel vibrates as a whole (considered thin)

$$R(f) = 10\log\left(\frac{1}{K_S^2}\right) - 10\log\left(\ln\left(1 + K_S^{-2}\right)\right)$$

$$K_S(f) = 4\pi f \rho_F c_F C_s$$

$$C_s = \frac{768(1-\upsilon^2)}{\pi^8 E h^3 \left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2}$$



$$f_{11} = \frac{\pi}{4\sqrt{3}} c_{Lplate} h \left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

For a simply supported plate

C_s: Mechanical compliance for a rectangular plate
E: Young's modulus of the material the wall is made of
h: wall thickness
a, b: plate dimensions
υ: Poisson's ratio of the wall
ρ_F: Density of the surrounding fluid (F), i.e. air
c_F: wave propagation speed in the fluid (F), i.e. air
c_{Lplate} wave propagation speed in the plate (longitudinal wave)

Sound reduction index of single-leaf partitions (III)

- Region II: Mass-controlled region ($f_{11} < f < f_c$)
  - Transmission loss independent of stiffness (controlled by mass inertia)
  - Some energy transmitted and part reflected at panel surface

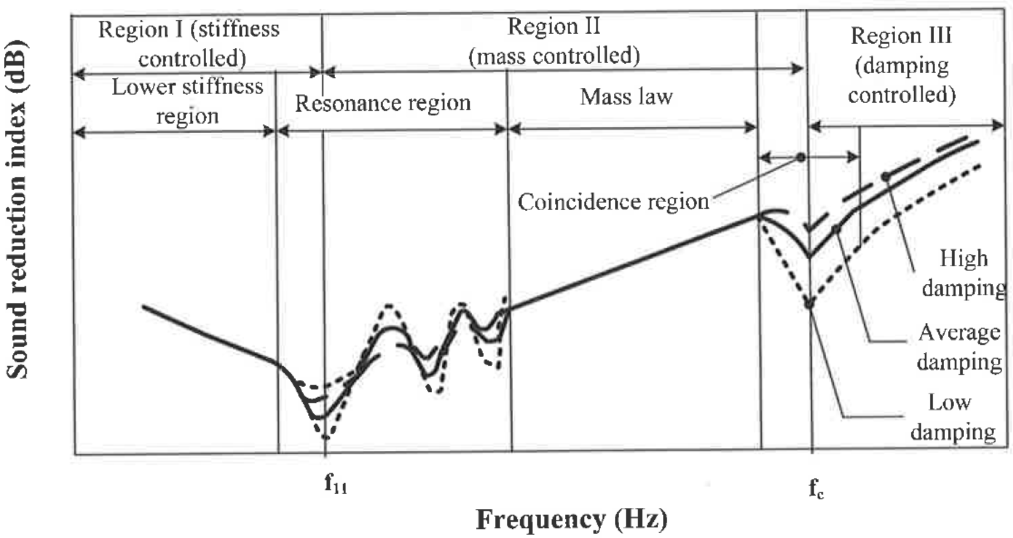
$$R(f) = 10\log\left(1+\left(\frac{\pi f m''}{\rho_F c_F}\right)^2\right) - 5dB$$

Random incidence correction

Mass law

>>1

m''= ρh is the surface mass of the panel



NOTE: Although the above equation is valid for frequencies up to f_c, it yields only accurate results for f ≤ 0.5f_c . The mathematical expresion around f_c is mathematically cumbersome and rarely used, its being the reason why approximate methods were developed.

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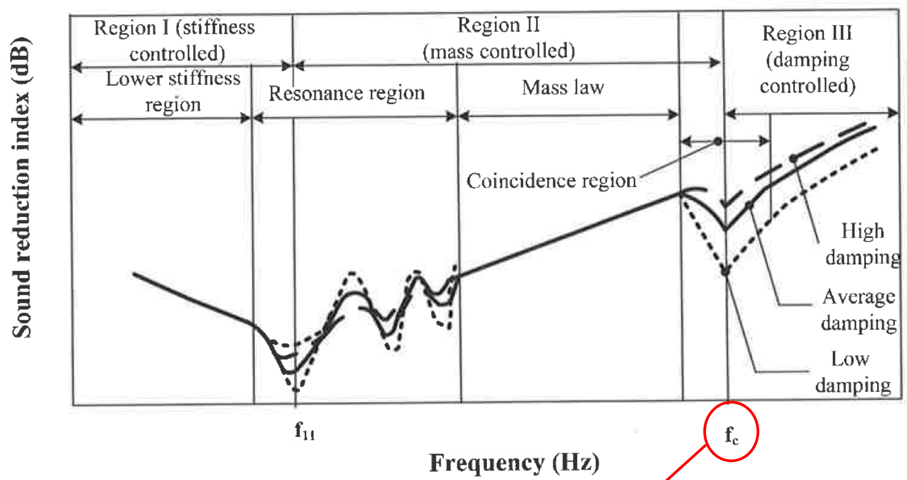
Sound reduction index of single-leaf partitions (IV)

- Region III: Damping-controlled region (f_c< f)
  - Curve "dip" controlled by internal material damping
  - Important for design (low insulation)

$$R(f) = R(f_c) + 10\log(\eta) + 33.22\log\left(\frac{f}{f_c}\right) - 5.7dB$$

$$R(f_c) = 10\log\left(1 + \left(\frac{\pi f_c m''}{\rho_F c_F}\right)^2\right)$$

η is the total loss factor or damping of the panel



$$f_c = \frac{c_F^2\sqrt{3}}{\pi h c_{Lplate}}$$

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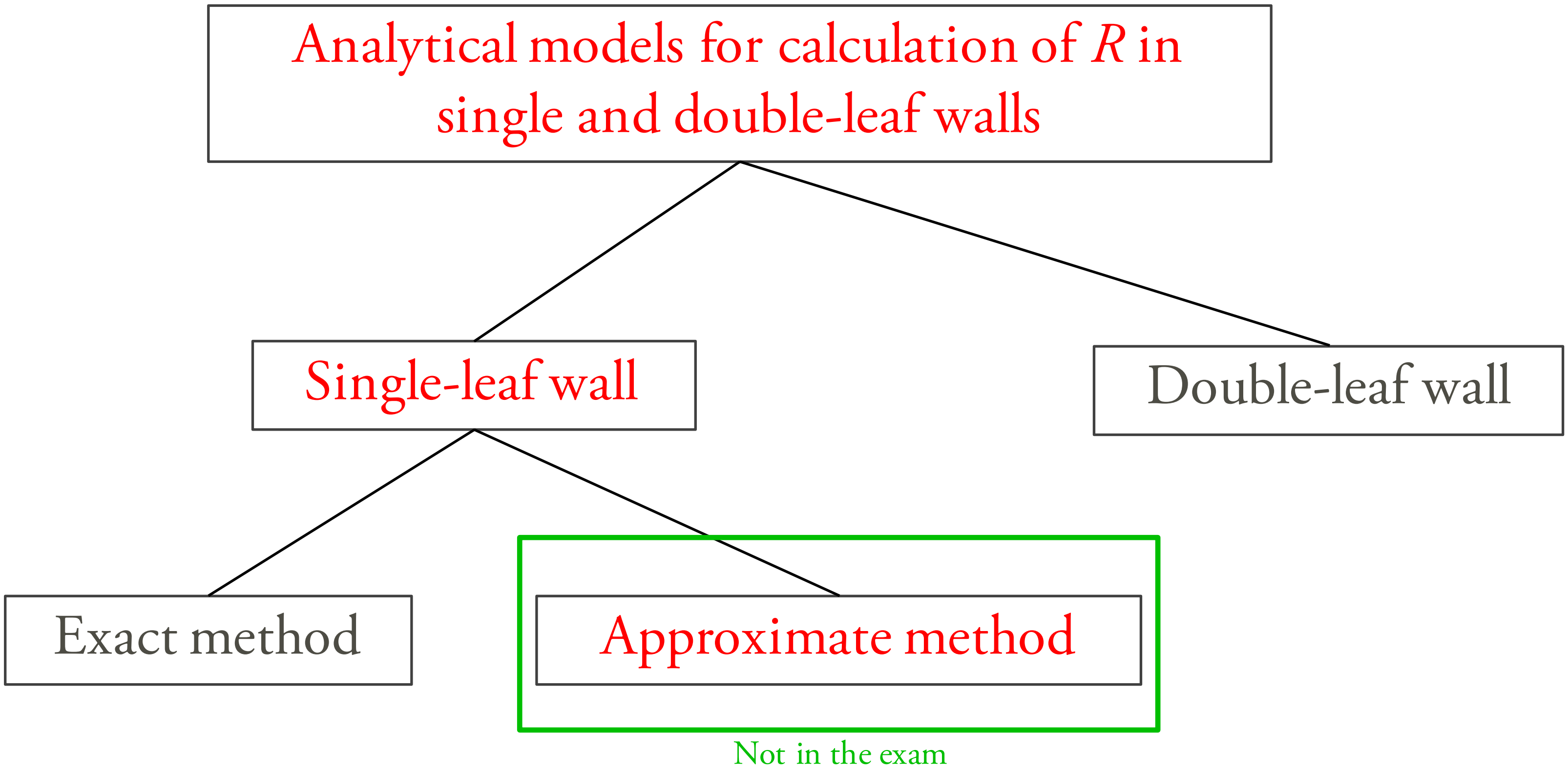
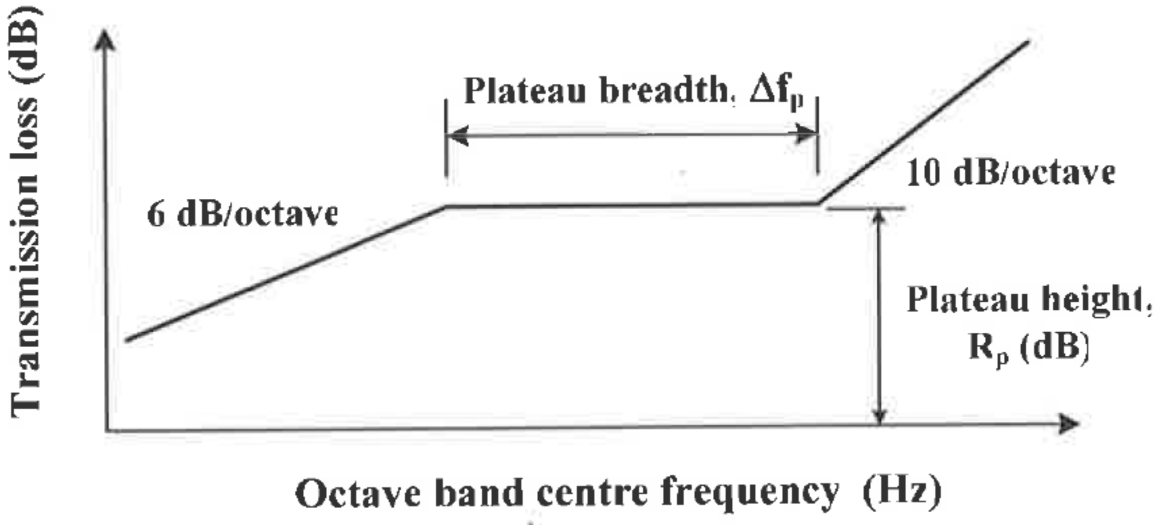
Outline


Analytical models for calculation of *R* in single and double-leaf walls
Single-leaf wall
Double-leaf wall
Exact method
Approximate method
Not in the exam

Sound reduction index of single-leaf partitions (I)

- **Approximate method**
  - Region I: Mass-controlled region ($f < f_1$)
  - Region II: "Plateau" ($f_1 < f < f_2$)
  - Region III: Stiffness-controlled region ($f_2 < f$)



Hyphotesis: Infinite panel and diffuse field excitation

NOTE: f_1 and f_2 are not the resonance and coincident frequency explained in the exact method (see next slides)

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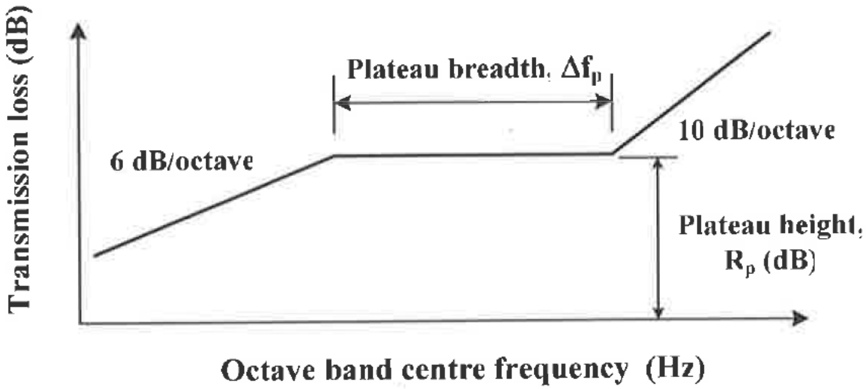
Sound reduction index of single-leaf partitions (II)

- Region I: Mass-controlled region ($f < f_1$)
  - Transmission independent of panel stiffness

$$R(f) = 20\log(m'') + 20\log(f) - 20\log\left(\frac{\rho_F c_F}{\pi}\right) - 5dB$$



Sound reduction index of single-leaf partitions (III)

- Region II: "Plateau" ($f_1 < f < f_2$)
  - Governed by internal damping
  - Height of the plateau depends on material
  - f_1 and f_2 are the lower and upper limits of the plateau
    - » Calculated with expresions of adjoining regions

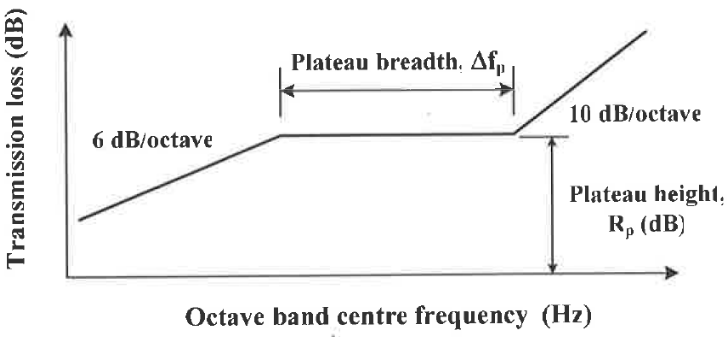


Table 4.2 Values of the plateau height (R_P) and plateau width (Δf_P) for the approximate method of calculation of the transmission loss for panels (partially after Watters, 1959).

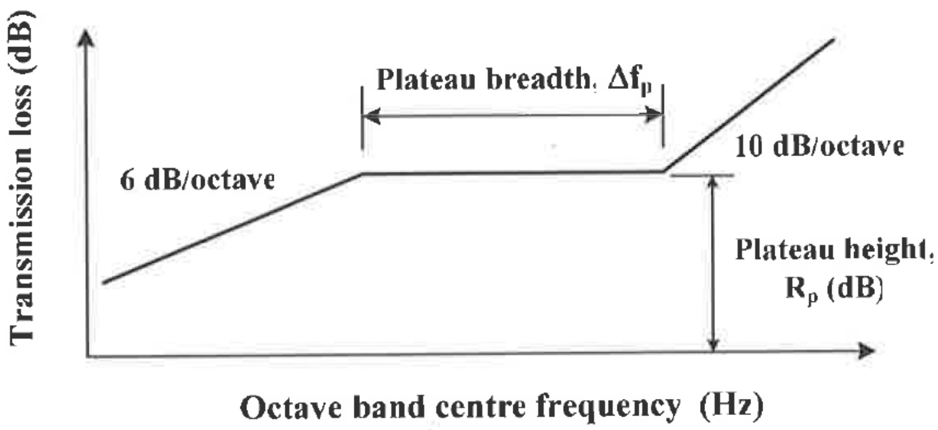
Material	Specific surface density (kg/m^2 per cm)	Plateau height, R_P (dB)	$\Delta f_P = f_2 - f_1$ (octave)	Plateau breadth, frequency ratio, f_2/f_1
Aluminum	26.6	29	3.5	11*
Brick	21	37	2.2	4.5
Concrete, dense	22.8	38	2.2	4.5
Glass	24.7	27	3.3	10
Lead	112	56	2.0	4
Masonry block				
Cinder**	11.4	30	2.7	6.5
Dense		32	3.0	8
Plywood, fir	5.7	19	2.7	6.5
Plaster, sand	17.1	30	3.0	8
Steel	76	40	3.5	11*

* These materials have, in general, very low damping. The numbers are for a typical panel in place
** Hollow block. The values are determined for 6-in (150 mm) plastered block.

Sound reduction index of single-leaf partitions (IV)

- Region III: Mass-controlled region ($f_2 < f$)
  - Governed by stiffness of the panel

$$R(f) = R(f_2) + 33.22\log\left(\frac{f}{f_2}\right)$$



NOTE: The slope of the expression (10 dB/octave) should just be used only for the 2 octaves above f_2. For the following octaves, one should use a slope equal to 6 dB/octave, i.e. "$20\log(f/f_{2oct})$" instead of "$33.22\log(f/f_2)$", where f_{2oct} is the frequency where the 3rd octave above f_2 starts.

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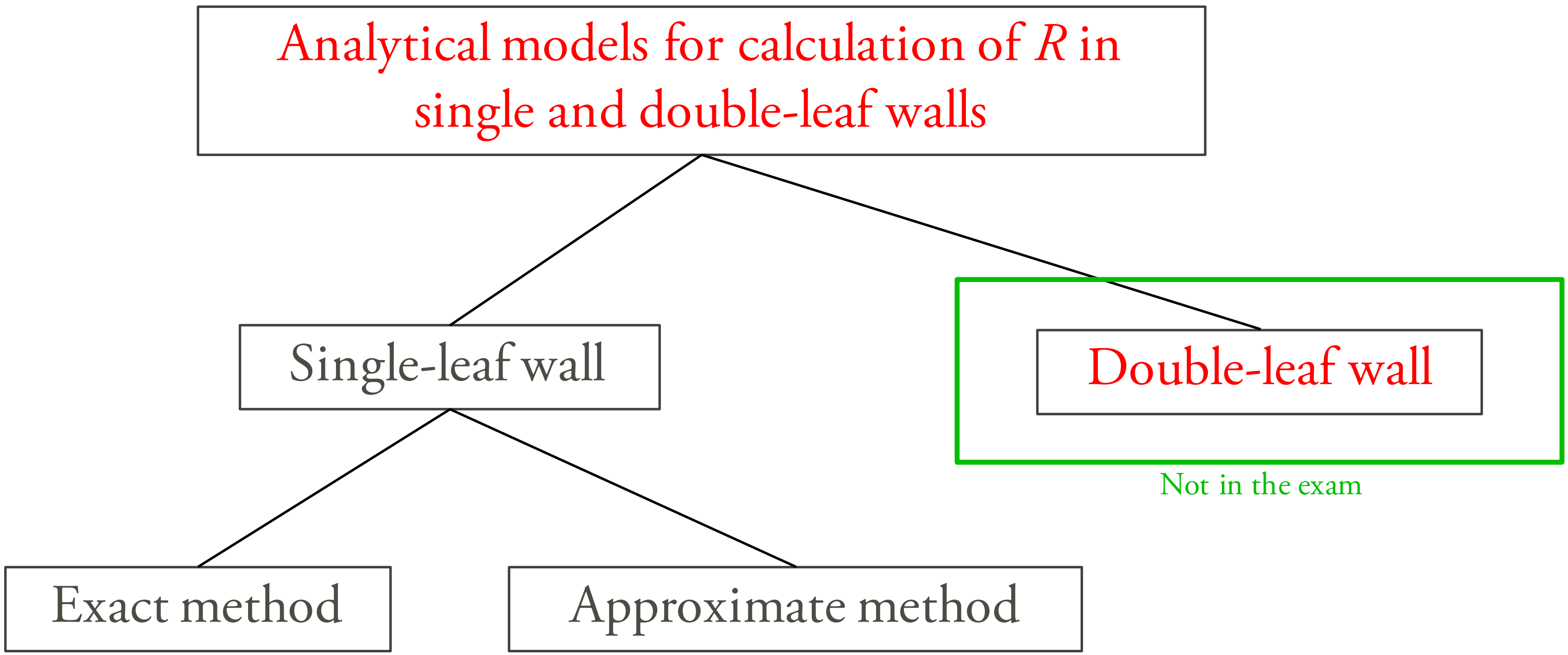
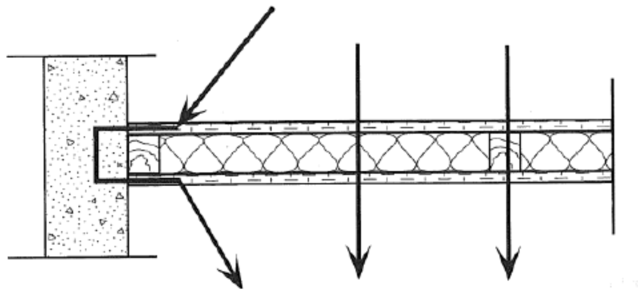
Outline


Analytical models for calculation of *R* in single and double-leaf walls
Single-leaf wall
Double-leaf wall
Not in the exam
Exact method
Approximate method

Introduction

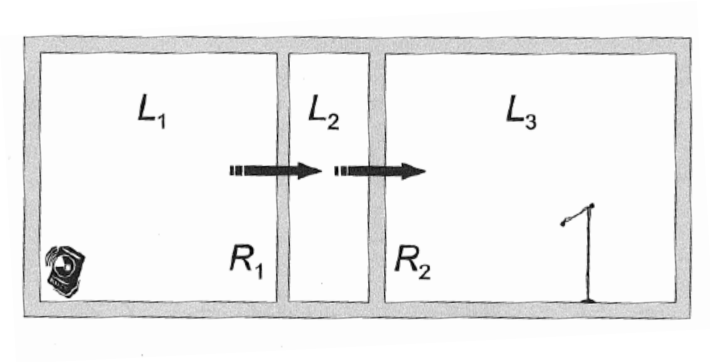
- Double-leaf wall literature → rather extensive
  - Theoretical analysis, less developed due to complexity



  - Analyses often carried out using FEM, SEA.

- Several theoretical derivations of sound transmission
  - Double-leaf wall without mechanical coupling
  - Double walls with structural connections
  - …

Sound reduction index of double-leaf walls



$$\left.\begin{array}{l} R_1 = L_1 - L_2 + 10\log\left(\dfrac{S}{A_2}\right) \\ R_2 = L_2 - L_3 + 10\log\left(\dfrac{S}{A_3}\right) \end{array}\right\} \Rightarrow R_{DoubleWall} = L_1 - L_3 + 10\log\left(\frac{S}{A_3}\right) \Rightarrow$$

$$\Rightarrow R_{DoubleWall} = R_1 + R_2 + 10\log\left(\frac{A_2}{S}\right)$$

- Approximate empirical model for a double leaf wall without structural connections, with cavity filled with porous absorber (Sharp 1978)

$$R = \begin{cases} R_M & ; f < f_0 \\ R_1 + R_2 + 20\log(f \cdot d) - 29dB & ; f_0 < f < f_d \\ R_1 + R_2 + 6dB & ; f > f_d \end{cases}$$

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{\rho_F}{d}\left(\frac{1}{m_1''} + \frac{1}{m_2''}\right)}$$

$$f_d = \frac{55}{d}$$

R_M denotes the mass law with $M = m_1 + m_2$

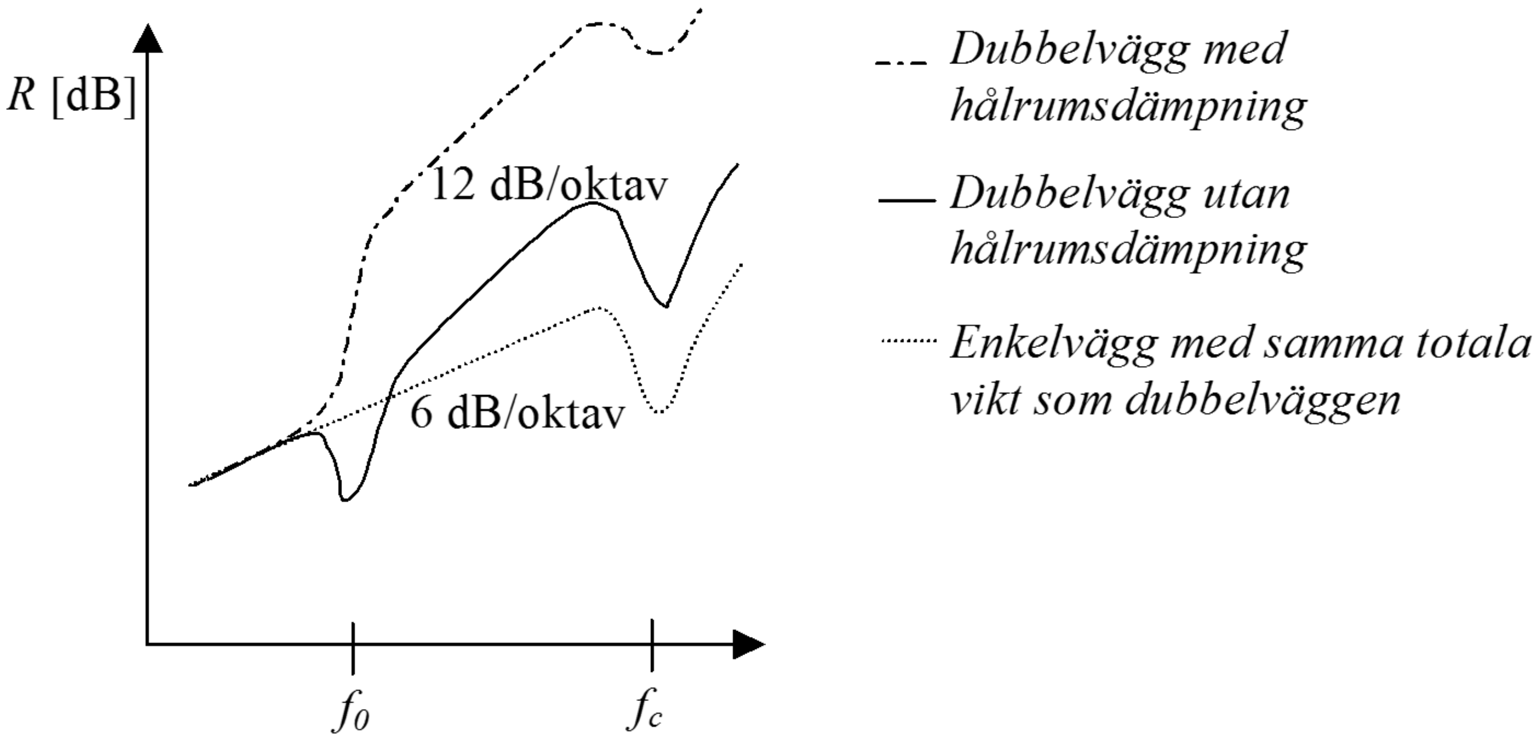
R_1 and R_2 denote the individual sound reduction index for each leaf

d: distance between the two leaves i.e. (cavity thickness)

NOTE: Diffuse field assumed in both rooms

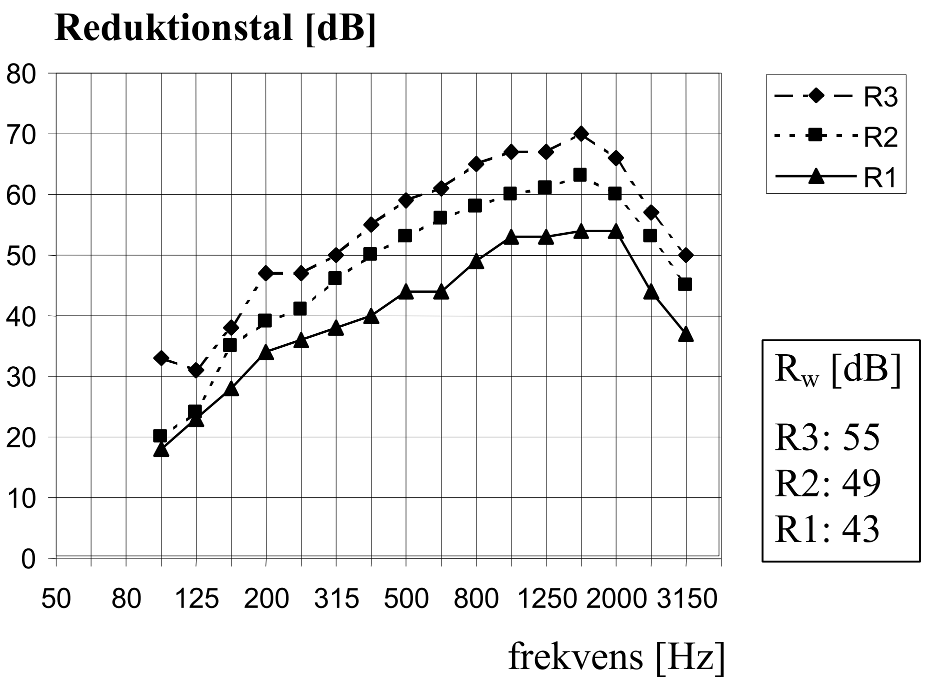
Examples (I)

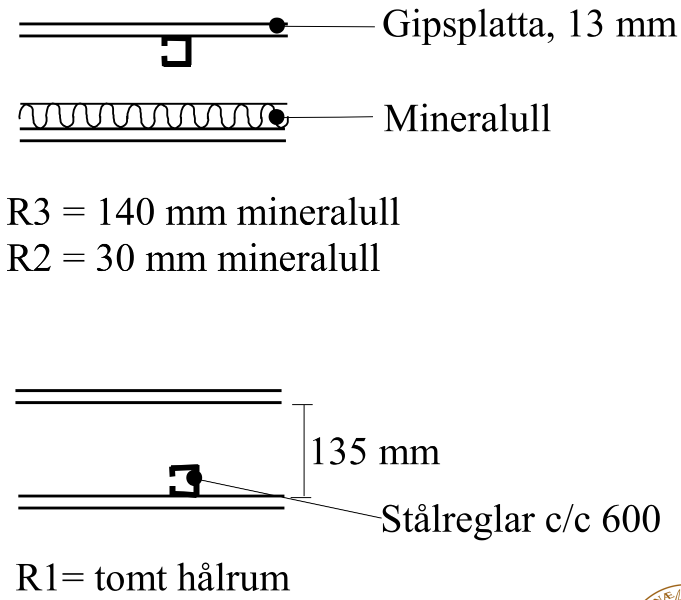
- Improvement in the sound reduction index of a double-leaf wall respect to a single wall, and also when including insulation in the cavity.



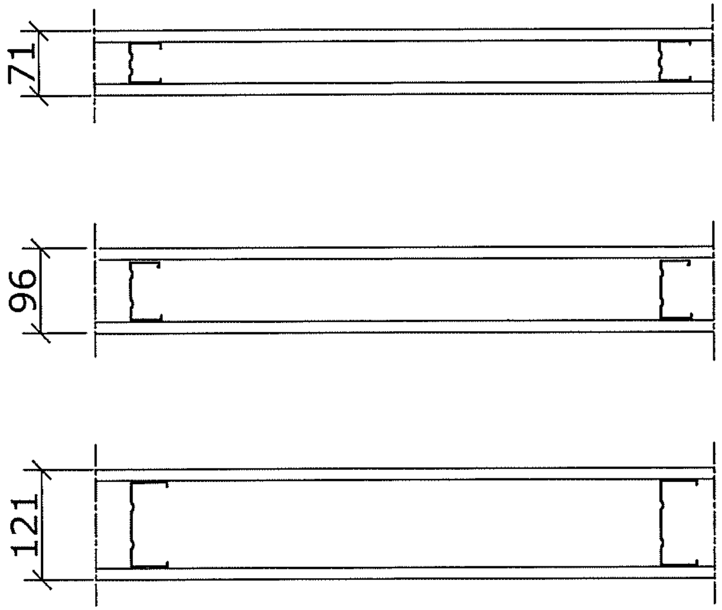
Examples (II)

- Variation in the sound reduction index of a double-leaf wall when varying parameters in the cavity (inclusion of insulation and its thickness).





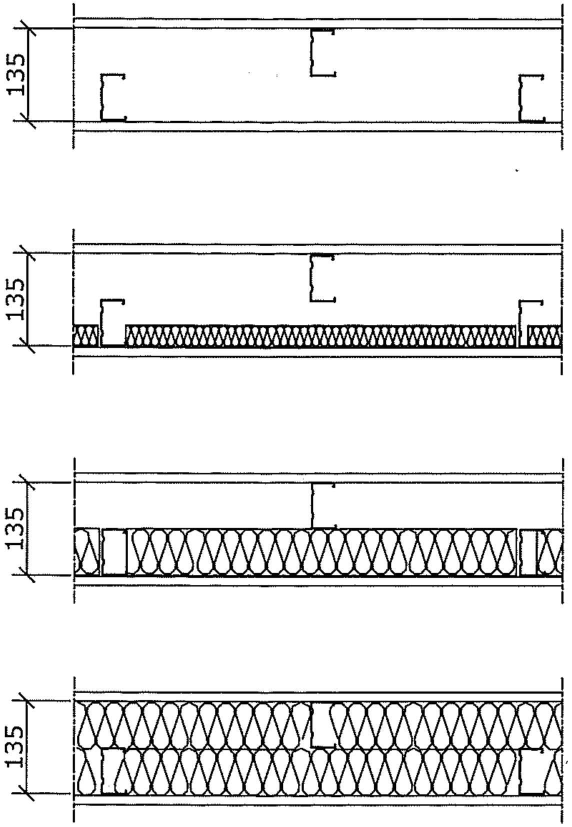
Examples (III)



	$R_{w\ lab}$
13 mm gips 45 mm regel 13 mm gips	33 dB
13 mm gips 70 mm regel 13 mm gips	36 dB
13 mm gips 95 mm regel 13 mm gips	37 dB

Figur 4:24. Exempel på inverkan på det vägda reduktionstalet av avståndet mellan dubbelväggarna. Laboratoriemätresultat.

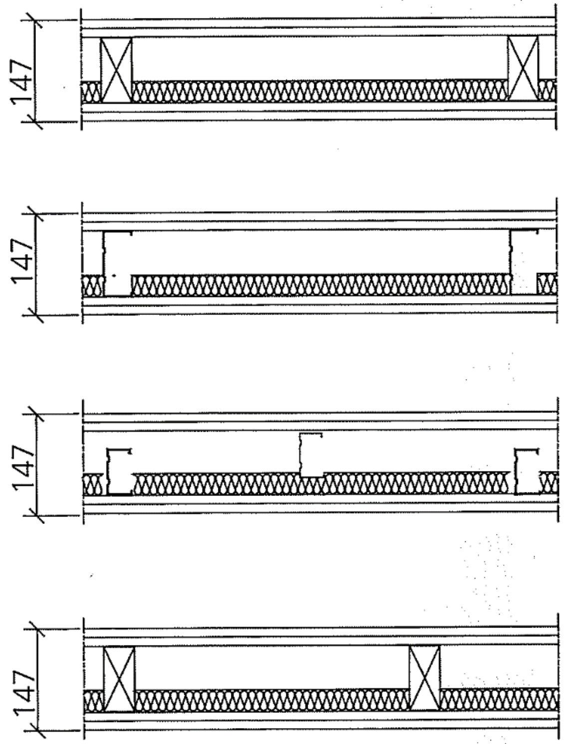
Examples (IV)



	R_w dB
13 mm gips utan absorbent Stålreglar c 600 mm 13 mm gips	41
13 mm gips 30 mm mineralull Stålreglar c 600 mm 13 mm gips	49
13 mm gips 70 mm mineralull Stålreglar c 600 mm 13 mm gips	50
13 mm gips 2x70 mm mineralull Stålreglar c 600 mm 13 mm gips	54

Figur 4:25. Exempel på inverkan på det vägda reduktionstalet av absorbent i spalten på dubbelvägg med separata reglar. Laboratoriemätresultat.

Examples (V)



Construction	$R_{w\,lab}$
2x13 mm gips 95 mm träregel, c 600 30 mm mineralull 2x13 mm gips	48
2x13 mm gips 95 mm stålregel, c 600 30 mm mineralull 2x13 mm gips	52
2x13 mm gips 70 mm skilda stålreglar, c 600 30 mm mineralull 2x13 mm gips	55
2x13 mm gips 95 mm träregel, c 450 30 mm mineralull 2x13 mm gips	42

Figur 4:26. Exempel på inverkan på det vägda reduktionstalet av olika förbindningar, reglar, i en dubbelvägg. Laboratoriemätresultat.

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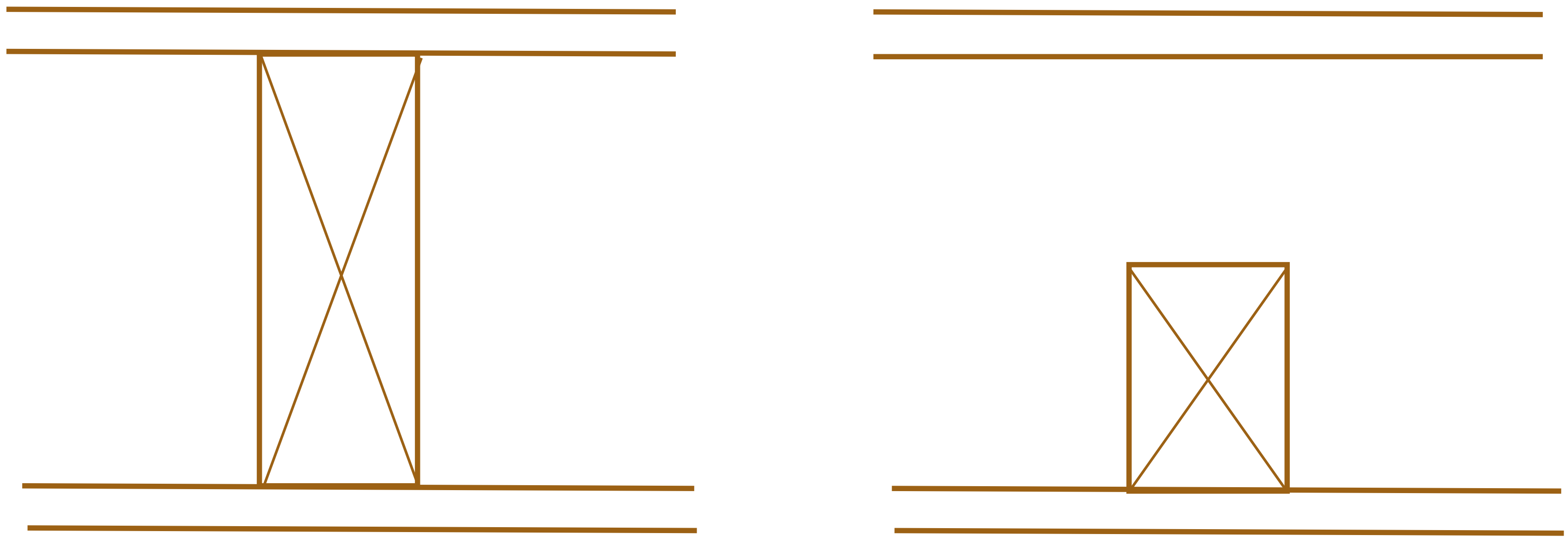
Examples (VI)

"Rule of thumb": decoupled structures perform much better → acoustic bridges eliminated



Outline

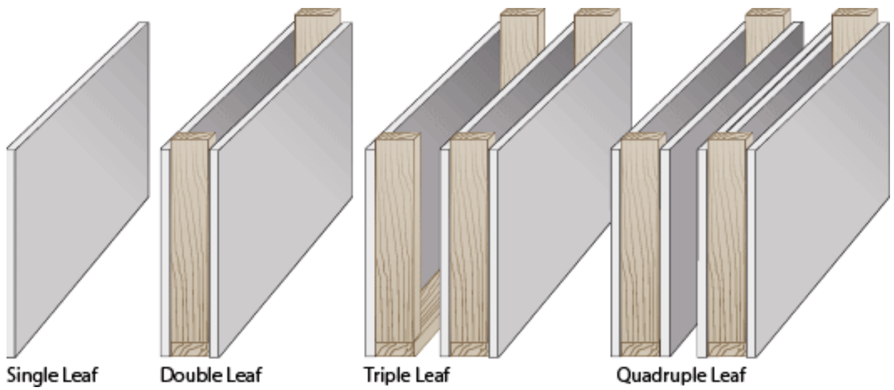
Introduction

Analytical models for calculation of *R(f)* in single and double-leaf walls

Summary

Summary (III)

- Analytical calculation methods of reduction sound index
  - Single-leaf wall
    - » Exact method
    - » Approximate method *(not in the exam)*
  - Double-leaf wall



Thank you for your attention!

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