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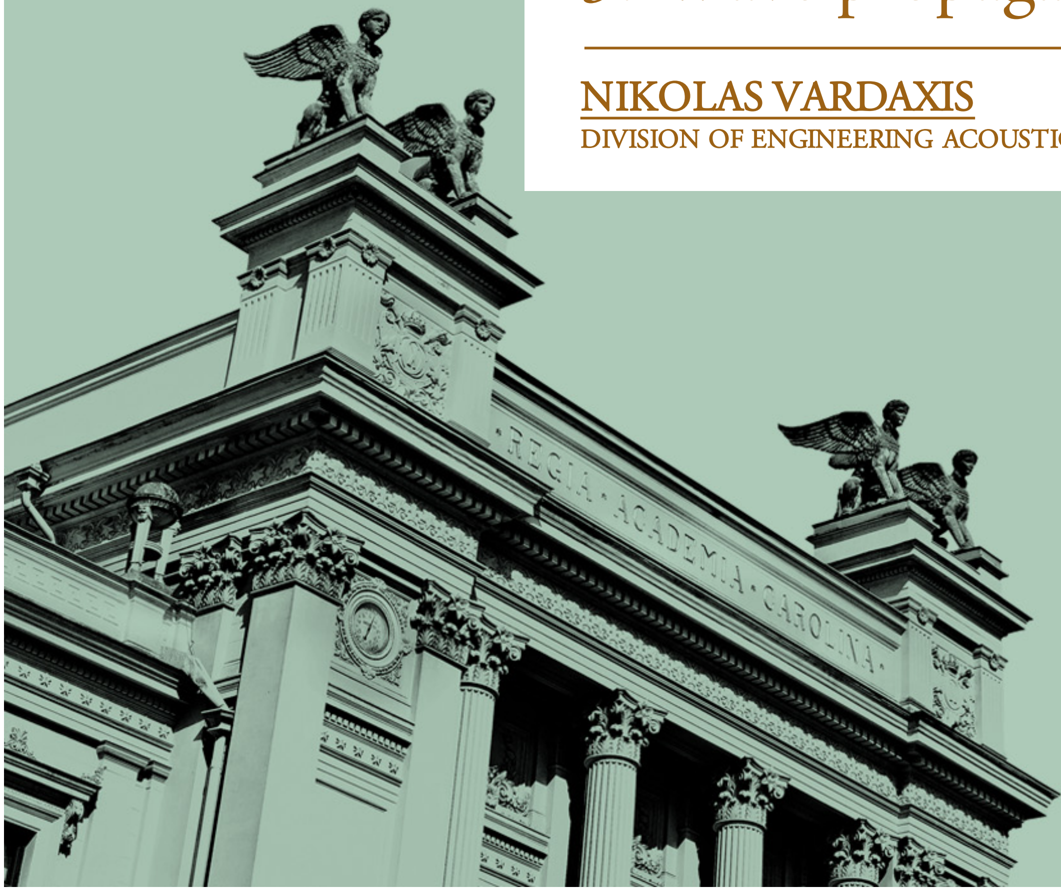
Acoustics (VTAN01)

3. Wave propagation

NIKOLAS VARDAXIS
DIVISION OF ENGINEERING ACOUSTICS, LUND UNIVERSITY



Outline

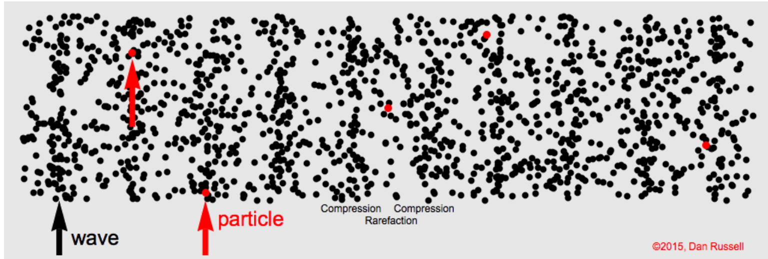
- Introduction
- Wave propagation in solid media
- Wave equation solution
- Wave propagation in fluids
- Wave phenomena
- Musical instruments
- Summary

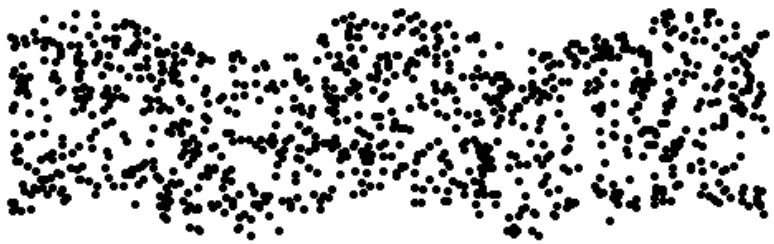
Learning outcomes

- Wave propagation in solid media
  - Longitudinal/quasi-longitudinal waves
  - Shear waves
  - Bending waves
- Wave equation solution
- Wave propagation in fluids
- Wave phenomena
- Musical acoustics

Types of waves – Classification

- Depending on propagation media
  - Mechanical waves (solids and fluids)
  - Electromagnetical waves (vacuum)
- Propagation direction
  - 1D, 2D and 3D
- Based on periodicity
  - Periodics and non-periodics
- Based on particles' movement in relation with propagation direction:
  - Longitudinal waves (solids and fluids)
  - Transverse waves (solids)
- …





<u>NOTE:</u> waves do not transport mass, just energy

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Types of waves in solid media

- Longitudinal waves (∞ medium ≈ beams)
  - Quasi-longitudunal waves (finite ≈ plates)

$$\frac{\partial^2 u_x}{\partial x^2} - \frac{\rho}{E'}\frac{\partial^2 u_x}{\partial t^2} = 0$$

$$c_L = \sqrt{\frac{E}{\rho}}$$

$$c_{qL} = \sqrt{\frac{E'}{\rho}} = \sqrt{\frac{E}{\rho(1-\upsilon^2)}}$$

- Shear waves

$$\frac{\partial^2 u_y}{\partial x^2} - \frac{\rho}{G}\frac{\partial^2 u_y}{\partial t^2} = 0$$

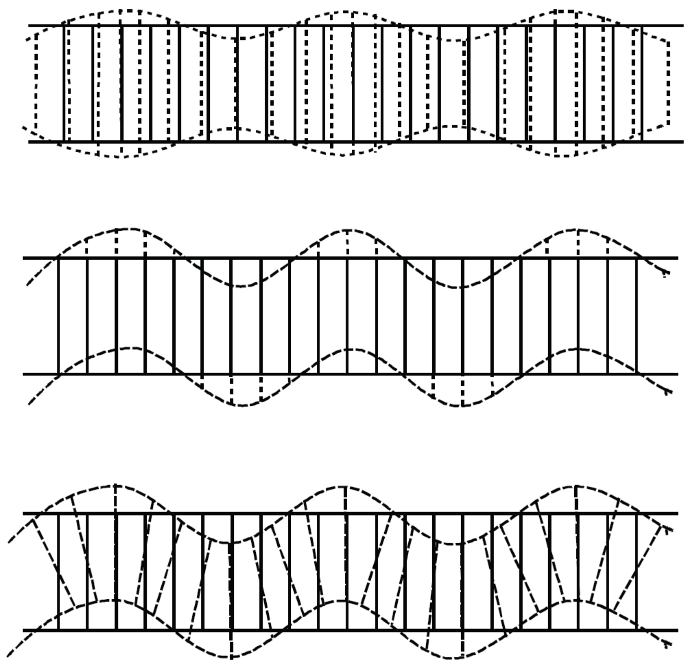
$$c_{sh} = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{2(1+\upsilon)\rho}}$$

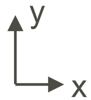
- Bending waves (dispersive)

$$B\frac{\partial^4 u_y}{\partial x^4} + m\frac{\partial^2 u_y}{\partial t^2} = 0$$

$$c_{B(\omega)} = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$$

Plate: E, G, ρ, υ, h





$$m = \rho h \qquad B_{beam} = E\frac{bh^3}{12}$$

$$B_{plate} = \frac{Eh^3}{12(1-\upsilon^2)}$$

<u>NOTE:</u> torsional waves (beams and columns) are not addressed here

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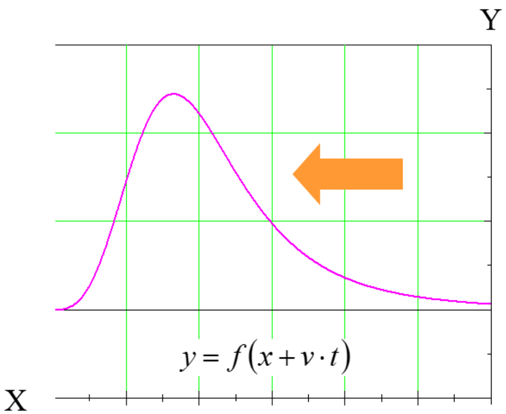
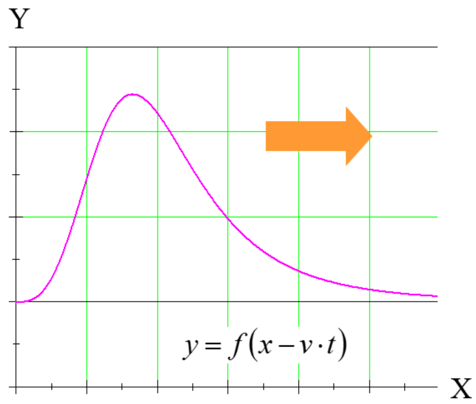
Wave equation solution (I)

- Travelling waves: $y = f(x \pm vt)$

Space — Sign — Propagation speed — Time

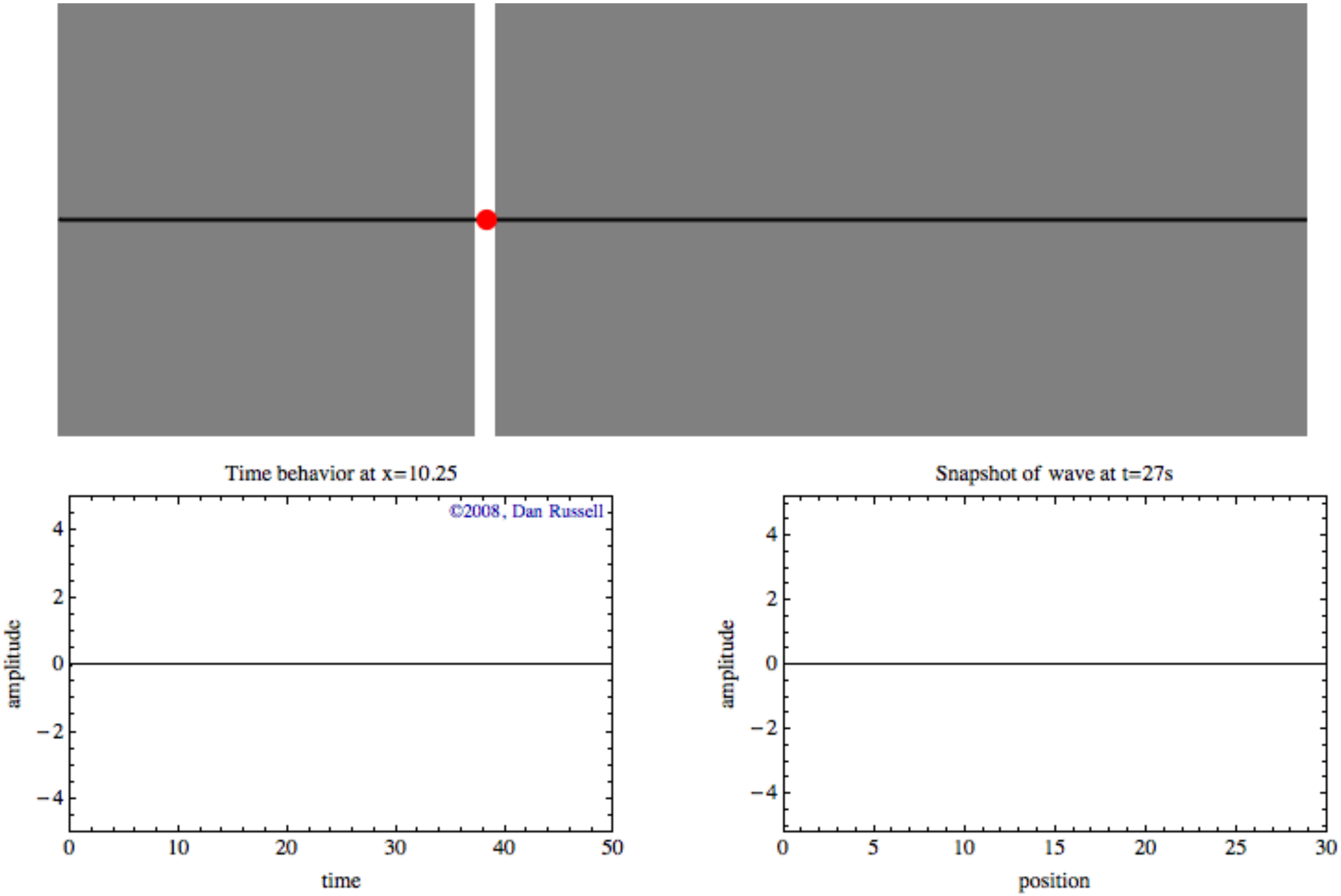




- Alternative forms: $y = f\left(x \pm \frac{\omega}{k} t\right) = f\left(\frac{kx \pm \omega t}{k}\right) = f(kx \pm \omega t)$
- Note:
  - Periodic functions: $f(x \pm vt) = f(x \pm vt + T)$
  - Harmonic functions: f is a sinus or cosinus

Wave equation solution (II)

Time and position dependency: $u(x,t) = \widehat{u_+}\cos(\omega t - kx) = \widehat{u_+}e^{-i(\omega t - kx)}$



Wave equation solution (II)

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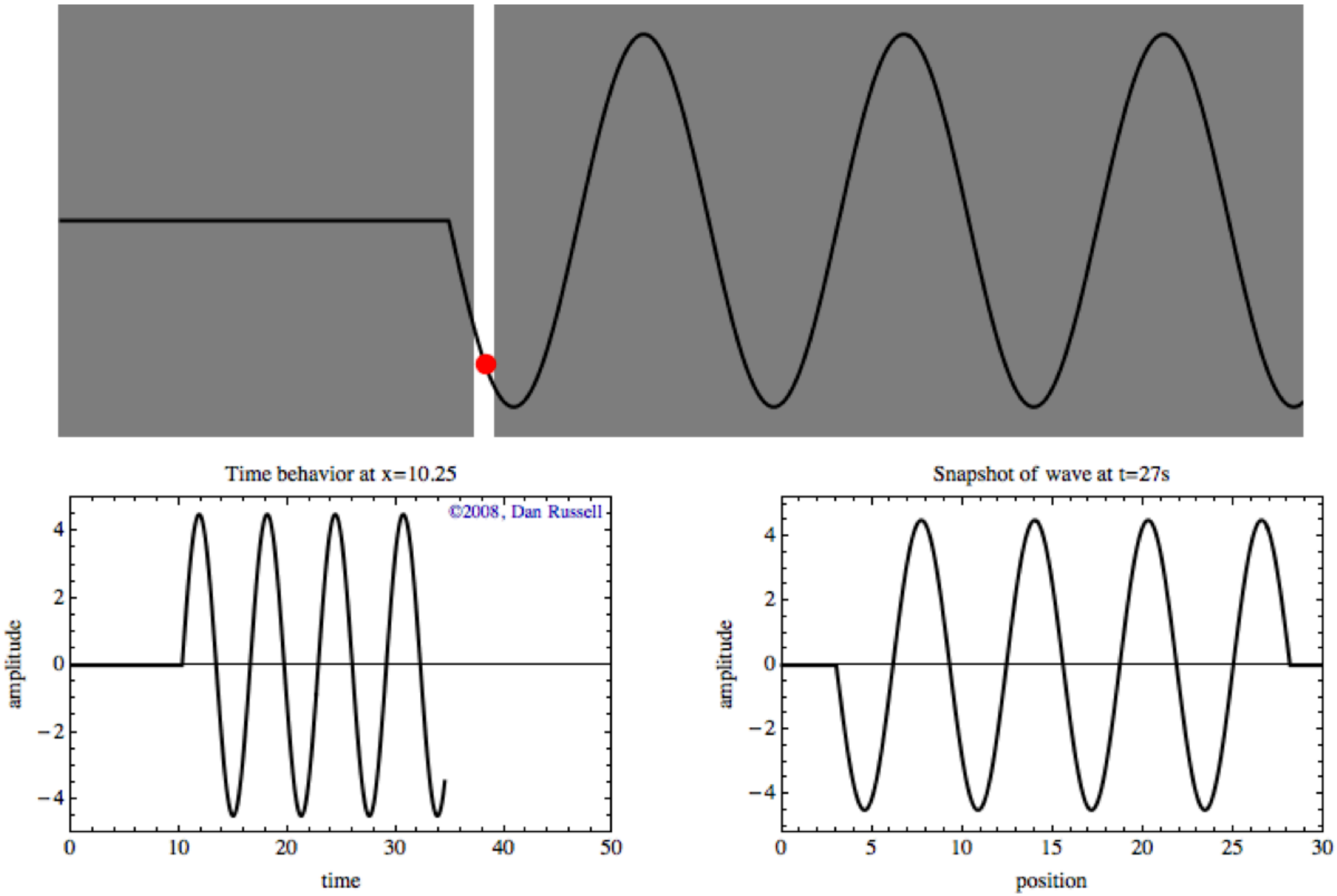

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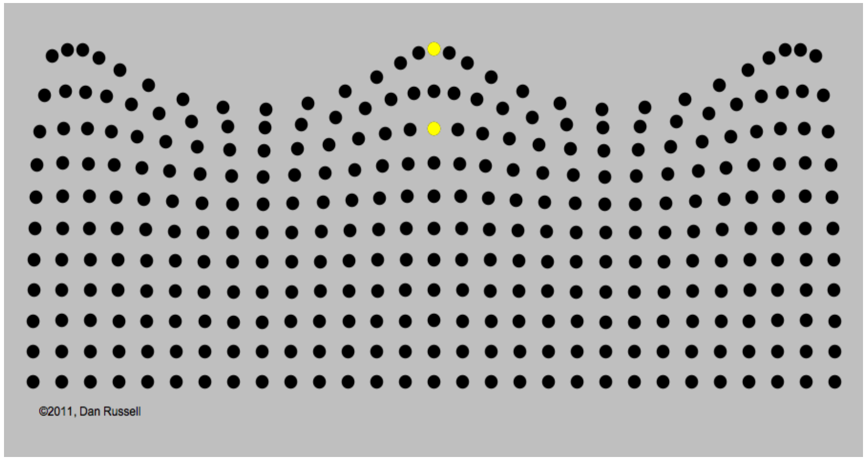
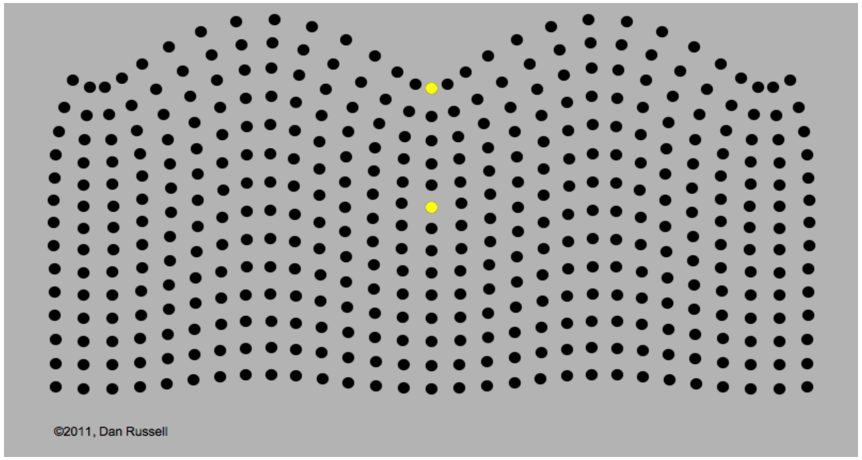
Other types…

- In reality, combinations of aforementioned waves can exist, e.g.





- Surface waves

Water waves

(long+transverse waves)

Particles in *clockwise circles*. The radius of the circles decreases increasing depth

Pure shear waves don't exist in fluids

- Body waves

Rayleigh waves

(long+transverse waves)

Particles in elliptical *paths*. Ellipses width decreases with increasing depth

Change from depth>1/5 of λ

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Musical instruments

Summary

Waves in fluid media

- Sound waves: longitudinal waves
  - Pressure as field variable

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \longrightarrow p(x,t) = \widehat{p_\pm}\cos(\omega t \pm kx) = \widehat{p_\pm} e^{-i(\omega t \pm kx)}$$

  - Velocity as field variable

$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2} \longrightarrow v(x,t) = \frac{1}{\rho c}\widehat{p_\pm}\, e^{-i(\omega t \pm kx)}$$

Comparing both equations: $Z \equiv \frac{p_\pm}{v_\pm} = \pm\rho c$ (acoustic impedance)

$$c_{medium} = \sqrt{\frac{D}{\rho}}, \quad c_{air} = \sqrt{\frac{\gamma P_0}{\rho(T = 0°C)}\left(1 + \frac{T}{2 \cdot 273}\right)} = 331.4\left(1 + \frac{T}{2 \cdot 273}\right), \quad k = \frac{2\pi}{\lambda}$$

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SPL & SIL & SWL

- Sound pressure level (SPL / L_p)

$$L_p = 10\log\left(\frac{\tilde{p}^2}{p_{ref}^2}\right) = 20\log\left(\frac{\tilde{p}}{p_{ref}}\right)$$

$\tilde{p} = \tilde{p}(f) \equiv$ RMS pressure
$p_{ref} = 2{\cdot}10^{-5}$ Pa = 20 μPa
$p_{atm} = 101\,300$ Pa
$p_{tot}(t) = p_{atm} \pm p(t)$

- Sound intensity
  - Sound power (i.e. rate of energy) per unit area [W/m^2]
    - » Instantaneous value: $\vec{I}(t) = p(t)\vec{v}(t)$
    - » Vector quantity: energy flow and direction: $\bar{\vec{I}} = \langle pv \rangle = \frac{1}{T}\int_0^T p(t)\vec{v}(t)dt$
    - » In a free field: $\bar{I} = \frac{\widetilde{p^2}}{\rho c}$
  - In decibels (SIL)… $L_I = 10\log\left(\frac{\bar{I}}{I_{ref}}\right)$; $I_{ref} = 10^{-12}\,{}^{W}\!/_{m^2}$
- Sound power
  - Rate of energy transported through a surface [W=J/s]: $W(t) = \int_S I_n(\vec{x},t)dS$
  - In decibels (SWL / L_W / L_Π)… $L_W = 10\log\left(\frac{\overline{W}}{W_{ref}}\right)$; $W_{ref} = 10^{-12}$W

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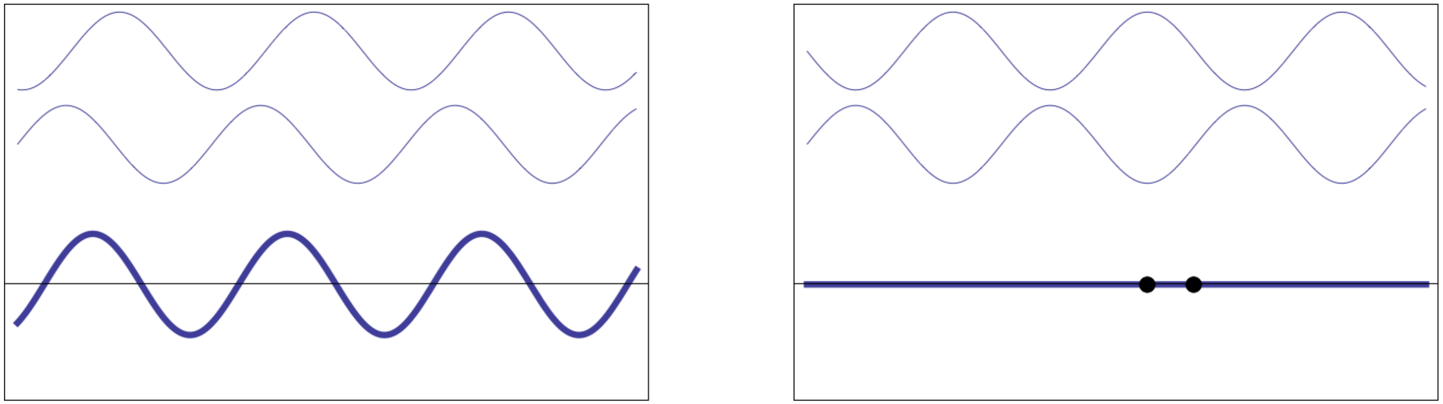
Wave phenomena

- Interferences: constructive / destructive

Constructive/destructive depending on Φ

$$\left.\begin{aligned} y_1(x,t) &= \hat{y}\cos(\omega t - kx) \\ y_2(x,t) &= \hat{y}\cos(\omega t - kx + \theta) \end{aligned}\right| \quad y(x,t) = y_1(x,t) + y_2(x,t) = 2\hat{y}\boxed{\cos\left(\frac{\theta}{2}\right)}\sin(\omega t - kx + \theta)$$



Source: Dan Russell

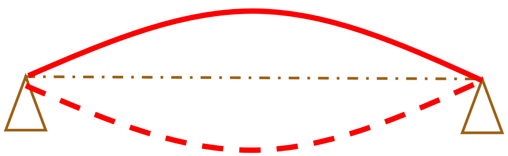
- Standing waves (coherent source)

$$\boxed{\begin{aligned} y_-(x,t) &= \hat{y}\cos(\omega t - kx) \\ y_+(x,t) &= \hat{y}\cos(\omega t + kx) \end{aligned}} \quad \Bigg| \quad y(x,t) = y_-(x,t) + y_+(x,t) = \boxed{2\hat{y}\sin(kx)}\cos(\omega t)$$

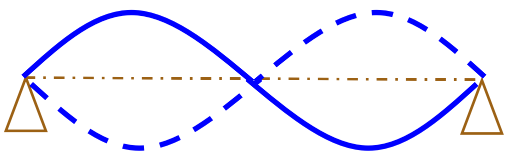
Two travelling waves of same frequency, type and fixed phase relation propagating in opposite directions

Position-dependent amplitude oscillating according to cos(ωt)

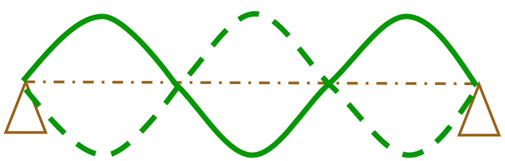
Standing waves in a string: resonances & eigenmodes



$\lambda=2L$
$f_1=v/2L$
Fundamental eigenfrequency / 1st harmonic



$\lambda=L$
$f_2=2f_1$
Second eigenfrequency / 2nd harmonic



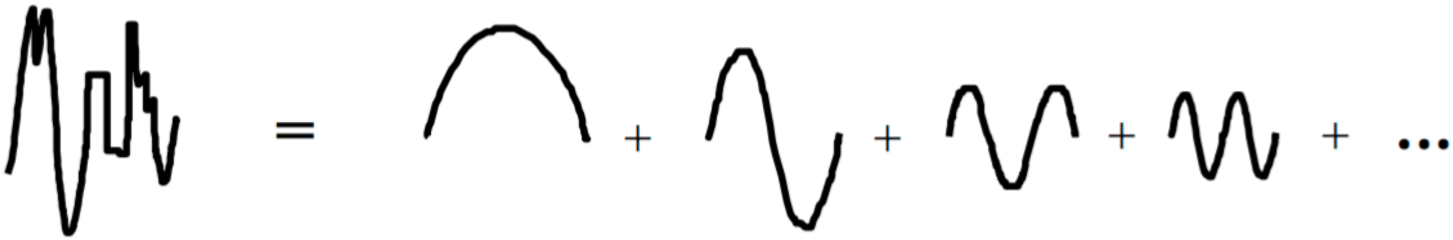
$\lambda=(2/3)L$
$f_3=3f_1$
Third eigenfrequency / 3rd harmonic

In general:

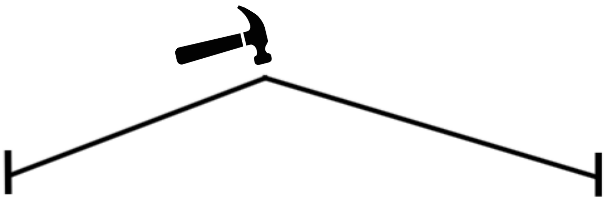
$\lambda=2L/n$
$f_n=n\cdot v/2L$

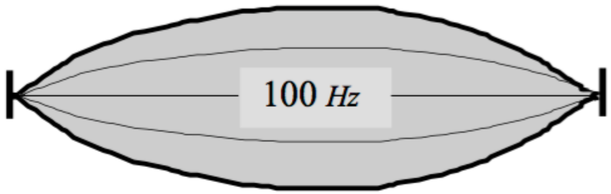
Eigenmode: different ways a string (structure in general) can vibrate generating standing waves

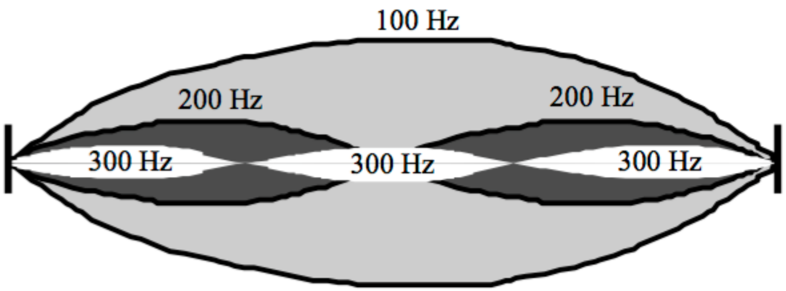
Standing waves and higher harmonics (I)

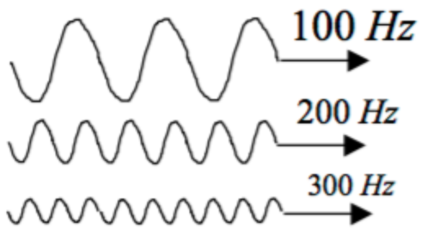


Any motion = sum of motion of all the harmonics





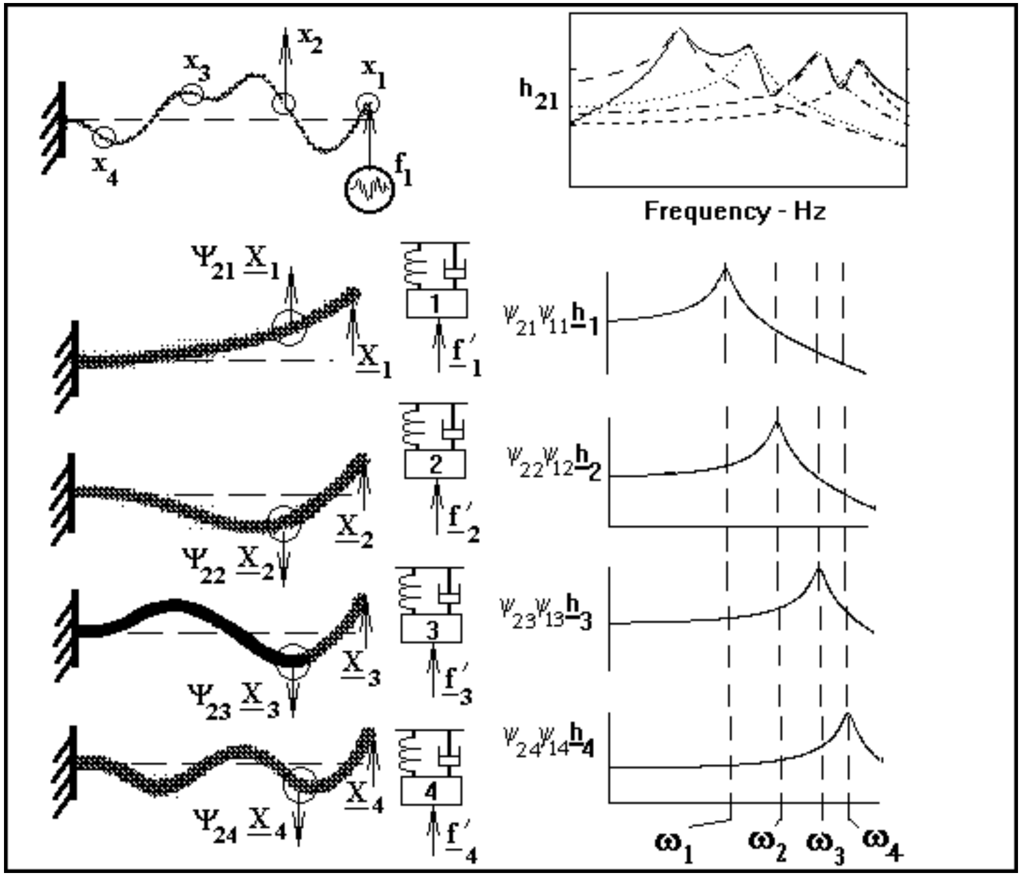




Source: J. Hetricks

Standing waves and higher harmonics (II)

Any motion = sum of motion of all the harmonics



Source: http://signalysis.com

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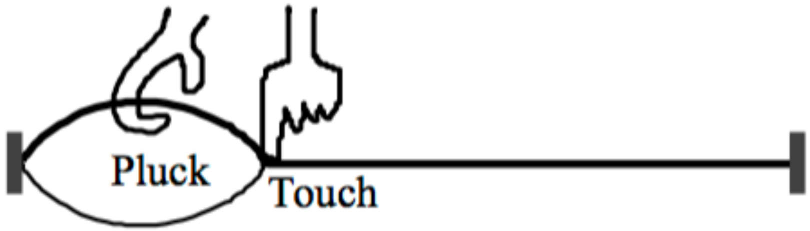
Music instruments: string (e.g. violin)

$\lambda = 2L/n$

$f_n = n \cdot v / 2L$

$v = (\text{tension}/\text{mass-length})^{1/2}$

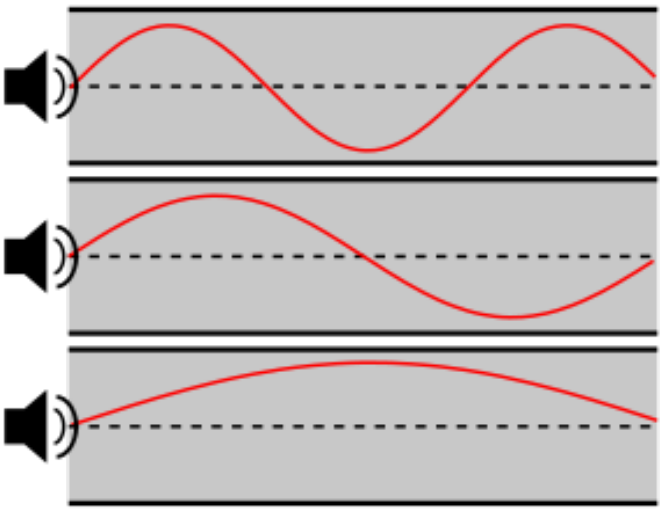


When ones plays → Changes L

Knobs (tune) → Vary tension

To discuss: Piano?

Music instruments: wood-wind



Change of v (molecular weight)

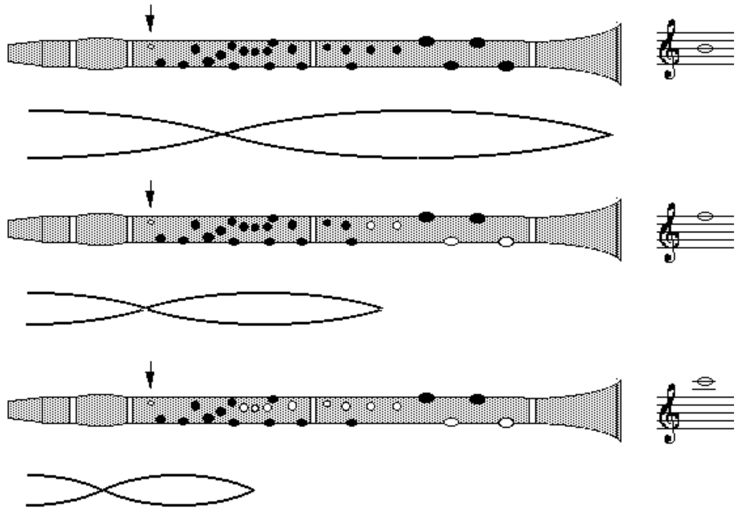
Open-open / Closed-closed:

$\lambda = 2L/n$

$f_n = n \cdot v/2L$

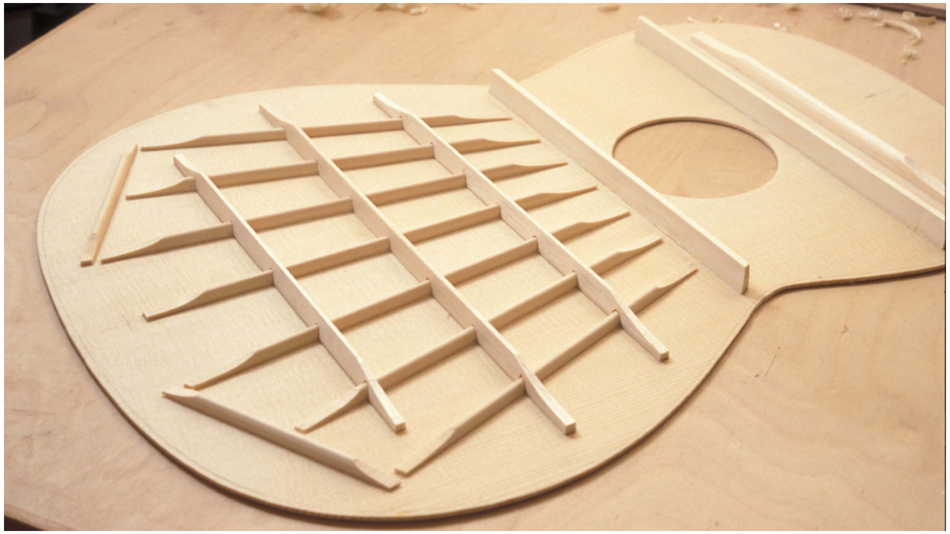
NOTE: Open-closed vary

$v = (\text{temperature}/\text{molecular weight})^{1/2}$



Cover holes → Vary L

Music instruments: soundboards

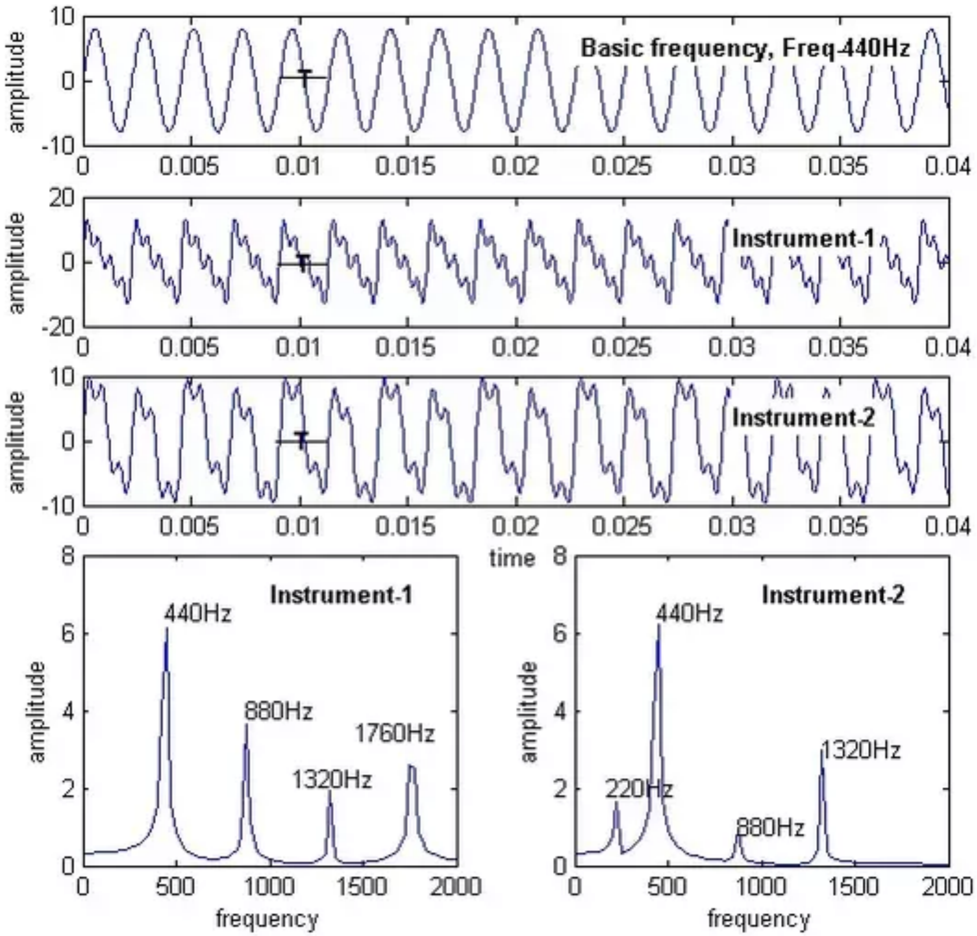


Standing waves and timbre (I)

- Characteristics of sound:
  - Loudness (amplitude)
  - Pitch (frequency)
  - Quality or Timbre
    - » "Cocktail" characteristic of every instrument/source
    - » Different combination of higher harmonics
    - » What makes us distinguish one instrument from another

- To discuss: helium & voice
  - Change of molecular weight (i.e. v) → natural frequency goes up

Standing waves and timbre (II)



Source: Quora

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Summary

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- Wave equation solution
- Wave propagation in fluid media
- Wave phenomena
  - Interference (constructive/destructive)
  - Standing waves
    - » Resonances
    - » Eigenmodes
- How musical instruments work

REFERENCES: Animations retrieved from Dan Russell's website

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Thank you for your attention!

nikolas.vardaxis@construction.lth.se

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