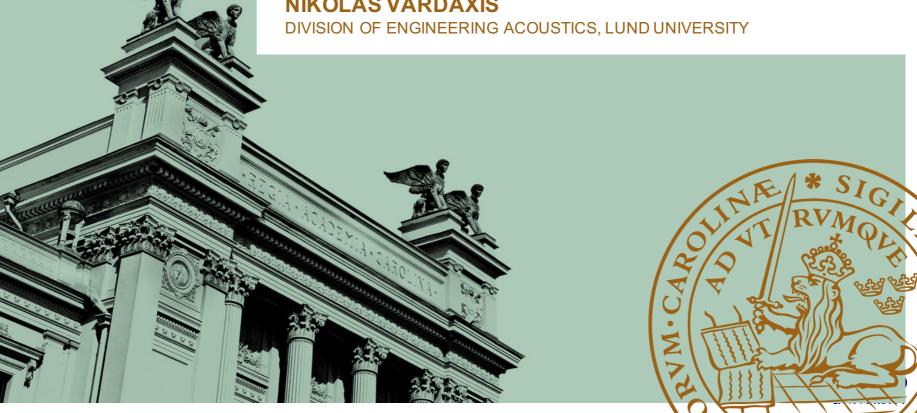


Acoustics VTAN01 13. RECAP

NIKOLAS VARDAXIS



Learning outcomes

- Definition of sound
- Harmonic oscillations and complex notation
- Acoustic variables and levels
- Addition of correlated and uncorrelated sources
- Frequency domain representation

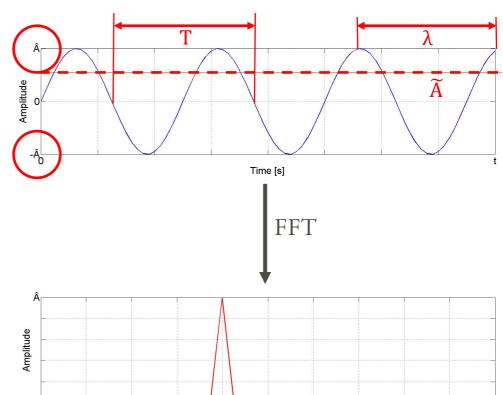


Time & frequency domains (I)

Harmonic signal: $y(t) = \widehat{A} \sin(\omega t) = \widehat{A} \cos(\omega t + \varphi) = \widehat{A} \sin(2\pi f \cdot t)$

- Amplitude: Â
- Period [s]: $T = \frac{1}{f}$
- Frequency [Hz]: $f = \frac{1}{T}$
- Wavelength [m]: $\lambda = cT = c/f$
- Propagation Speed [m/s]: c=f λ
 NOTE: c ≠ v
- Effective value (RMS):

$$A_{RMS} = \widetilde{A} = \sqrt{\frac{1}{\Delta t} \int_{t_0}^{t_0 + \Delta t} y^2(t) dt}, \quad \widetilde{A}_{\substack{\text{harmonic} \\ \text{signal}}} = \widehat{A} / \sqrt{2}$$



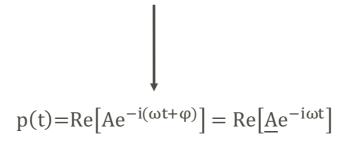
Frequency [Hz]

Frequency domain



Complex notation

• Equivalent description: $p(t) = \widehat{A} \cos(\omega t + \varphi)$



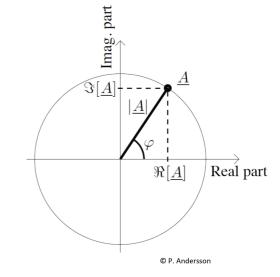
where the complex amplitude is defined as: $\underline{A} = Ae^{i\phi}$

and
$$e^{i\phi} = \cos(\phi) + i\sin(\phi)$$

The peak value and initial phase are

$$A = |\underline{A}|$$

$$\tan (\varphi) = \frac{\text{Im}[\underline{A}]}{\text{Re}[\underline{A}]}$$



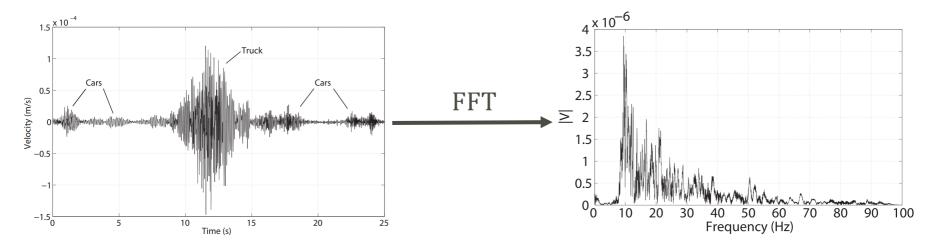


 $\underline{\text{NOTE 1:}}$ The complex number "i" is sometimes also expressed as "j"

NOTE 2: $\varphi_{cos} = \varphi_{sin} - \frac{\pi}{2}$

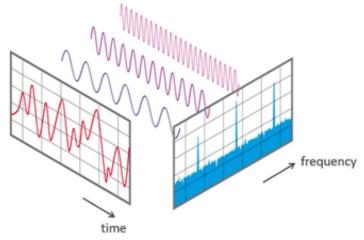
Time & frequency domains (II)

• A more complex time signal (traffic load)



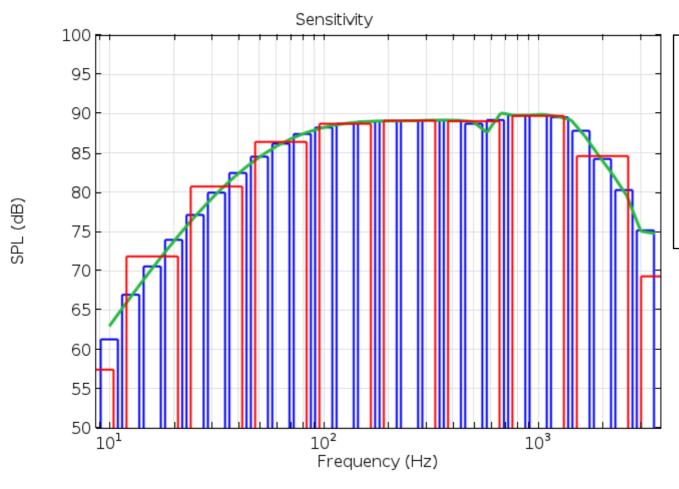
- Narrow band analyses
 - Impractical, time-consuming
 - Octave & 1/3 octave bands

NOTE: Spectrum (any magnitude plotted against frequency)





Time & frequency domains (II)



Frequency responses:

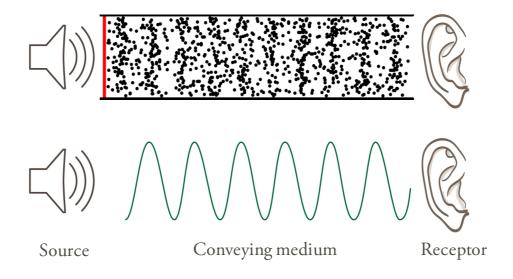
Narrow band

Octave bands

1/3 octave bands

Hearing process

- Pressure waves
- For a sound to be perceived
 - Frequency: 20 Hz 20 kHz
 - Sound pressure level (SPL): frequency dependent
- Inner ear detects: $\Delta p \in [20 \, \mu Pa, 200 \, Pa] \rightarrow$ wide range
 - Use of logarithmic scale (in decibels)





The decibel (dB) & SPL

- Logarithmic way of describing a ratio
 - Ratio: velocity, voltage, acceleration...
 - Need of a reference
- Sound pressure level (SPL / L_p)

$$L_{p} = 10 \log \left(\frac{\tilde{p}^{2}}{p_{ref}^{2}} \right) = 20 \log \left(\frac{\tilde{p}}{p_{ref}} \right)$$

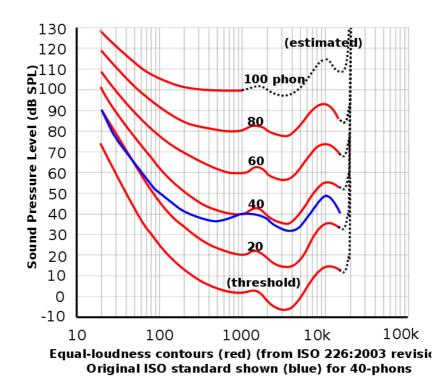
$$\tilde{p} = \tilde{p}(f) \equiv RMS \text{ pressure}$$
 $p_{ref} = 2 \cdot 10^{-5} \text{ Pa} = 20 \text{ }\mu\text{Pa}$
 $p_{atm} = 101 \text{ }300 \text{ Pa}$
 $p_{tot}(t) = p_{atm} \pm p(t)$

- \tilde{p} measured with microphones
- Frequency response of human hearing changes with amplitude



Frequency weightings

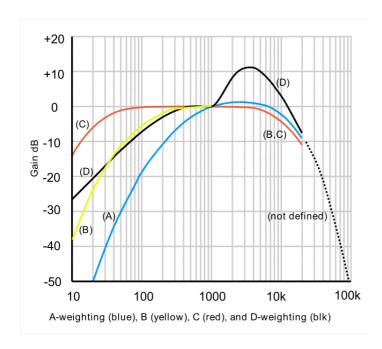
- Frequency response of human hearing changes with amplitude
- How to relate the objective measure to the subjective experience of sound?





Frequency weightings

Filters and calculation



$$L_{\text{weighted}} = 10 \log \left(\sum_{n=1}^{\infty} 10^{\frac{(L_n + \text{weighting})}{10}} \right)$$

Frekvens	A-filter	B-filter	C-filter
[Hz]	[dB]	[dB]	[dB]
10	-70.4	-38.2	-14.3
12.5	-63.4	-33.2	-11.2
16	-56.7	-28.5	-8.5
20	-50.5	-24.2	-6.2
25	-44.7	-20.4	-4.4
31.5	-39.4	-17.1	-3.0
40	-34.6	-14.2	-2.0
50	-30.2	-11.6	-1.3
63	-26.2	-9.3	-0.8
80	-22.5	-7.4	-0.5
100	-19.1	-5.6	-0.3
125	-16.1	-4.2	-0.2
160	-13.4	-3.0	-0.1
200	-10.9	-2.0	0
250	-8.6	-1.3	0
315	-6.6	-0.8	0
400	-4.8	-0.5	0
500	-3.2	-0.3	0
630	-1.9	-0.1	0
800	-0.8	0	0
1000	0	0	0
1250	0.6	0	0
1600	1.0	0	-0.1
2000	1.2	-0.1	-0.2
2500	1.3	-0.2	-0.3
3150	1.2	-0.4	-0.5
4000	1.0	-0.7	-0.8
5000	0.5	-1.2	-1.3
6300	-0.1	-1.9	-2.0
8000	-1.1	-2.9	-3.0
10000	-2.5	-4.3	-4.4
12500	-4.3	-6.1	-6.2
16000	-6.6	-8.4	-8.5
20000	-9.3	-11.1	-11.2



Frequency weightings (I)

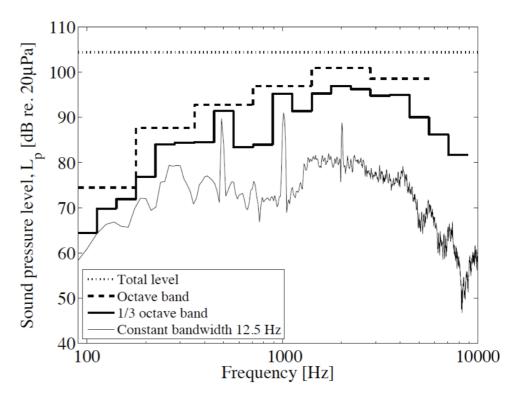
- Correlate objective sound measurements with subjective human response
 - A-weighting [dB(A)/dBA]: designed to reflect the response of how the human ear perceives noise, i.e. 20 Hz-20 kHz
 - » Only really accurate for relatively quiet sounds and pure tones?
 - » Low frequency noise is suppressed (wind turbine noise?)
 - C-weighting [dB(C)/dBC]: developed for high level aircraft noise
 - Z-weighting: zero frequency weighting (un-weighted values)
 - B-weighting: covers the mid-range between the A- and C-weighting
 - D-weighting: designed for use when measuring high level aircraft noise

Fallen into disuse



Frequency bands

- A sound in the frequency domain may be looked at in several ways.
 - Narrow bands;
 - Third-octave bands;
 - Octave bands;
 - Total value.



© P. Andersson



Octave and 1/3-octave bands

If f_n is the cut-off lower frequency and f_{n+1} the upper one, the ratio of the band limits is given by:

$$\frac{f_{n+1}}{f_n} = 2^k$$

where k=1 for full octave and k=1/3 for onethird octave band

ISO 266 Standard Frequencies for Acoustic Measurements			
ISO Band numbers	Octave band center frequency	One-third octave band center frequencies	
1	1.25 Hz		
2, 3, 4	2 Hz	1.6 Hz, 2 Hz , 2.5 Hz	
5, 6, 7	4 Hz	3.15 Hz, 4 Hz , 5 Hz	
8, 9, 10	8 Hz	6.3 Hz, 8 Hz , 10 Hz	
11, 12, 13	16 Hz	12.5 Hz, 16 Hz , 20 Hz	
14, 15, 16	31.5 Hz	25 Hz, 31.5 Hz , 40 Hz	
17, 18, 19	63 Hz	50 Hz, 63 Hz , 80 Hz	
20, 21, 22	125 Hz	100 Hz, 125 Hz , 160 Hz	
23, 24, 25	250 Hz	200 Hz, 250 Hz , 315 Hz	
26, 27, 28	500 Hz	400 Hz, 500 Hz , 630 Hz	
29, 30, 31	1000 Hz	800 Hz, 1000 Hz , 1250 Hz	
32, 33, 34	2000 Hz	1600 Hz, 2000 Hz , 2500 Hz	
35, 36, 37	4000 Hz	3150 Hz, 4000 Hz , 5000 Hz	
38, 39, 40	8000 Hz	6300 Hz, 8000 Hz , 10000 Hz	
41, 42, 43	16000 Hz	12500 Hz, 1 6000 Hz , 20000 Hz	

NOTE 1: Convert octave band to 1/3-octave band level reduction of -4.771dB for each 1/3 octave band:

$$L_{p} = 10 \log \left(\frac{1}{3}\right)$$

NOTE 2: Octave band level of three 1/3-octave band levels:

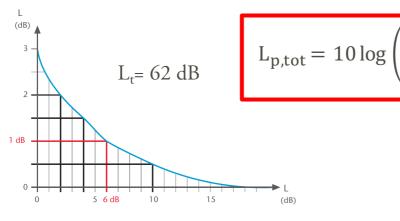
$$L_{\text{oct}} = 10 \log \left(\sum_{i=1}^{3} 10^{\frac{L_{p,i}}{10}} \right)$$



Summation of noise

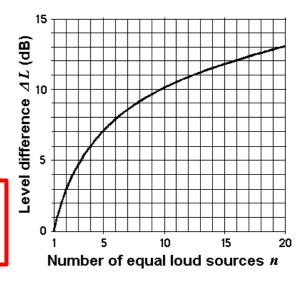
- Graphical methods
 - Adding equally loud incoherent sources
 - Adding two different sources

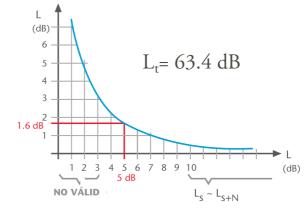
$$\sim$$
 e.g. L_1 =61 dB / L_2 =55 dB





$$\sim$$
 e.g. L_{S+N} =65 dB / L_{N} =60 dB







Summation of noise (I)

- Types of sources
 - Correlated (or coherent)
 - » Constant phase difference, same frequency
 - » Interferences (constructive/destructive)



$$L_{p,tot} = 20 \log \left(\sum_{n=1}^{N} 10^{\frac{L_{p,n}}{20}} \right)$$

Uncorrelated (or incoherent)

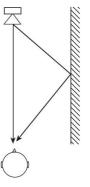


$$L_{p,tot} = 10 \log \left(\sum_{n=1}^{N} 10^{\frac{L_{p,n}}{10}} \right)$$

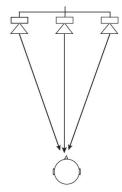


$$\tilde{p}_{\text{tot}}^2 = \tilde{p}_1^2 + \tilde{p}_2^2 + \frac{2}{\Delta t} \int_{t_0}^{t_0 + \Delta t} p_1(t) p_2(t) dt$$

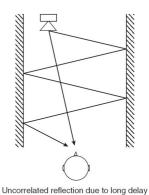
For uncorrelated sources, the 3rd term vanishes

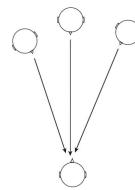


Correlation due to reflection



Correlation due to multiple sources





Uncorrelated multiple sources



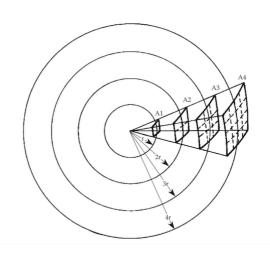
Sound (acoustic) intensity

- Sound power per unit area [W/m²]
 - Vector quantity: energy flow and direction

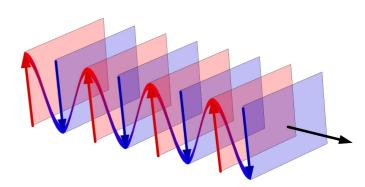
$$I = \langle pv \rangle = \frac{1}{\Delta t} \int_{0}^{T} p(t)v(t)dt$$

$$- \text{ In a free field:} \quad I = \frac{\widetilde{p^{2}}}{0C}; \quad I \propto p^{2}$$

- In a free field:
$$I = \frac{p^2}{\rho c}$$
; $I \propto p^2$



- Types of propagation
 - Plane: $I \equiv constant$;
 - Cylindrical: $I(r) \propto \frac{1}{r}$
 - Spherical: $I(r) \propto \frac{1}{r^2}$; $I(r) = \frac{\prod}{4\pi r^2}$

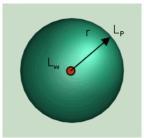


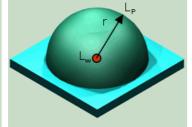
• In decibels...
$$L_I = 10 \log \left(\frac{I}{I_0}\right); \qquad I_0 = 10^{-12} \text{W}/\text{m}^2$$

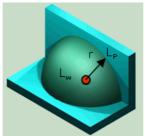


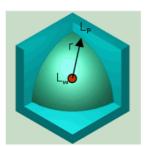
Notes / Definitions (I)

- Sound emission
 - Sound power continuously emitted from a sound source
- Sound power level (SWL / L_W / L_Π) or acoustic power
 - Total sound energy emitted by a source per unit time
 - » Constant regardless of the room
 - » Independent of the distance from the sound source
 - » Theoretical value
 - Units: Watts [W] or decibels [dB] (re: 10⁻¹² W)









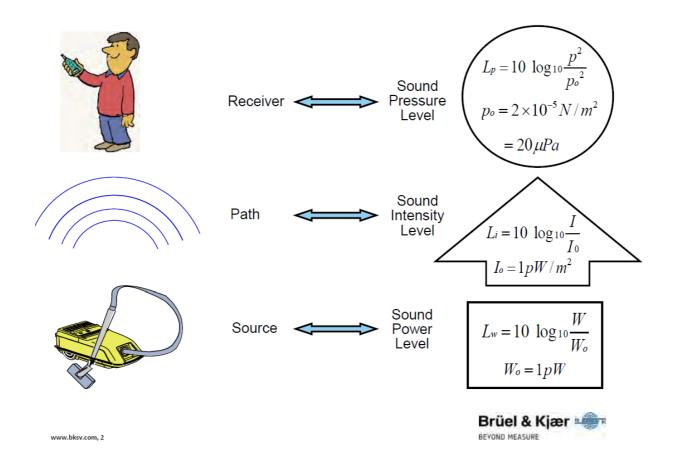
$$L_{W} = L_{p} + \left| 10 \log \left(\frac{Q}{4\pi r^{2}} \right) \right|$$

- Q=1: Full sphere
- Q=2: Half sphere
- Q=3: Quarter sphere
- Q=4: Eighth sphere



Source: www.sengpielaudio.com

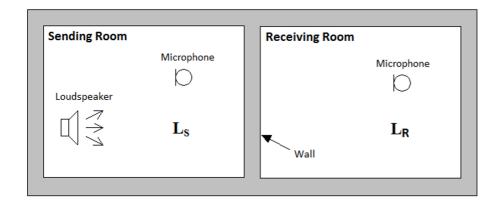
Do not mix up concepts (III)...





Building acoustics measurements

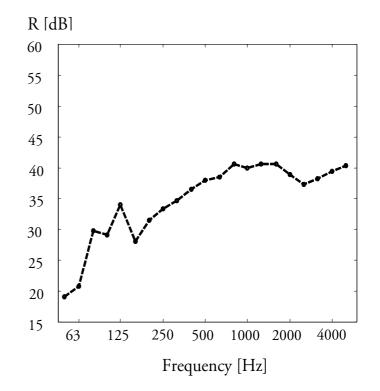
• Airborne sound insulation measurements (ISO standards)



$$R(f) = L_S(f) - L_R(f) + 10\log\left(\frac{S}{A(f)}\right)$$

Statement of results:

- $R'_w(C_{50-3150}; C_{tr})$
- $R_w(C_{50-3150}; C_{tr})$

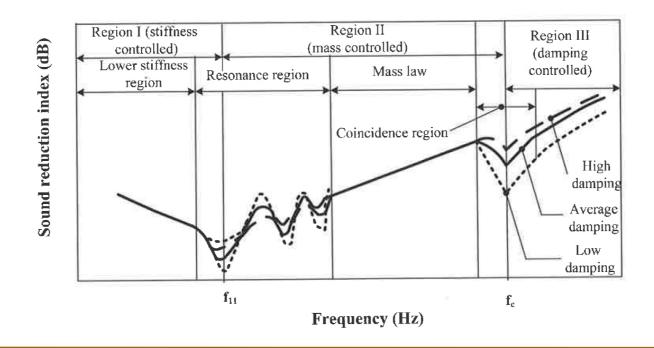




Sound reduction index of single-leaf partitions

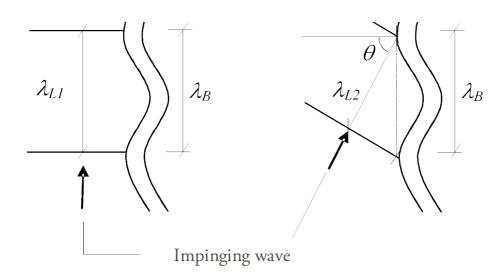
Exact method

- Region I: Stiffness-controlled region ($f < f_{11}$)
- Region II: Mass-controlled region $(f_{11} < f < f_c)$
- Region III: Damping-controlled region (f_c < f)





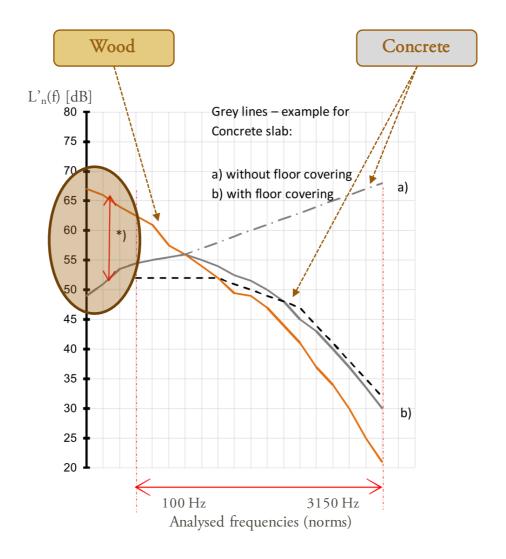
Reminder: Coincidence – critical frequency



- The wavelength of a bending wave λ_B is dependent on frequency, bending stiffness and mass density
- When the wavelength of sound in air coincides with the structural wavelength → Coincidence phenomena
 - Radiation efficiency becomes very high
 - Insulation inefficiencies



Impact sound "problems": SNQs





Thank you for your attention!

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