Exercises week 2 – Wave propagation & Measurement techniques

1. Show that the complex displacement function $p(x,t) = \hat{p}e^{i(\omega t - kx)}$ is a solution of the wave equation $\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$.

   Hint 1: If $p(x,t)$ is a solution then it must satisfy the equality stated by the wave equation

   Hint 2: The relation between the angular velocity and wave number is $c = \frac{\omega}{k}$

2. A wave that propagates in a particular rope is given by $y(x,t) = 0.06\sin(0.4\pi x + 50\pi t)$. Calculate:
   a. Its frequency, period, wavelength and propagation speed.
   b. The transversal velocity in any point of the rope.
   c. Let the wave propagate from both ends of the rope at the same time. Give the wave equation of the standing wave which originates after the interference of both waves.
   d. Calculate the distance between two consecutive peaks.

   NOTE: $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

   $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

3. The equation of the second harmonic of a stationary wave on a 10 m long string subjected to a force of 50 N is given by: $y(x,t) = 8\sin(0.2\pi x)\sin(20\pi t)$, x in metres, y in centimetres and t in seconds.
   a. Determine the frequency and velocity of propagation of the travelling waves whose interference produces the standing wave in this string and calculate its linear mass density.
   b. Write the wave equation of the fundamental term (corresponding to the first harmonic). Find the maximum vibration velocity of a point of the string in this mode, assuming that the maximum amplitude is the same as that of the second harmonic.
   c. Determine the positions of the nodes of the fourth harmonic.

4. A tuning fork mounted on a soundboard is struck by a hammer emitting a 612 Hz sound wave that spreads at 340 m/s and reaches a receiver. Considering plane wave propagation, calculate:
   a. If the maximum overpressure produced by the sound wave at the receiver is equal to $p_0 = 2\cdot10^{-4}$ Pa, give the equation of the travelling wave, explaining the choice made for the initial phase, and calculate its wavelength.
   b. Calculate the intensity of the sound perceived by the receiver.
   c. Using the intensity reference $I_0=10^{-12}$ W/m^2, calculate the current level in dB.
   d. In a second experiment, the tuning fork is hit again, the intensity level at the receiver being now 20 dB higher than before. What is the intensity that reaches the receiver in this case?
   NOTE: The density of the air in the conditions of the experiment is 1.22 kg/m^3

5. A transversal wave of 1 cm of amplitude and 100 Hz of frequency propagates along the x-axis with a velocity of 20 m/s. Write the expression of the elongation, velocity and acceleration of a particle located 10 cm from the focus/source. In which instant does the particle reach the maximum values of the previous expressions?

6. A wave propagating in a string is described by the equation (in S.I. units): $y(x,t) = 0.003\sin(80t - 6x)$ If the string has a fixed end on a wall, write the equation of the reflected wave.

7. Two sound waves described by the equation: $y(x,t) = 1.2\cos 2\pi(170t - 0.5x)$ are emitted from two different coherent sources and interfere at a point P located 20 m from one source and 25 m from the other. Determine the perturbation that each of the sources originate at point P, at time t=1 s. Calculate the phase difference of the waves at the point in question and determine the amplitude of the total disturbance at that point.

8. Two people separated by a distance of 4 cm toss each a stone into a lake. When they hit the surface of the water they generate coherent waves with a frequency of 24 Hz, which propagate with a speed of 12 cm/s. Determine the type of disturbance that will exist at a point A that is located 10 cm from one person and 12 cm from the other, as well as at another point B which is located 8 cm and 9.75 cm from both persons, respectively.

9. A guitar string of 1 m long fixed at both ends vibrates with a 4-node-pattern. The center points of the rope have a maximum displacement of 4 mm. If the waves speed in the string is 660 m/s, find the frequency with which the string vibrates and the expression of the resulting standing wave.

10. Consider the following signals: $x_1(t) = \cos 2\pi(10)t$ and $x_2(t) = \cos 2\pi(50)t$ which are sampled at 40 Hz. Is the sampling frequency correctly chosen so as to avoid aliasing?

11. Let an analog signal be $x_a(t) = 3\cos(100\pi t)$
    - **a.** Determine the minimum sampling rate required to avoid aliasing.
    - **b.** Suppose that the signal is sampled at the rate of f_s=200 Hz; what is the discrete-time signal obtained after sampling?
    - **c.** Suppose that the signal is sampled at the rate of f_s=75 Hz, what is the discrete-time signal obtained after sampling?

12. Consider the analog signal $x_a(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$. What is the Nyquist frequency for this signal? Is there any problem sampling x_a if the Nyquist frequency is employed?

Answers:

1. Theoretical
2. a) f=25 Hz / T=0.04 s / λ=5 m / c=125 m/s

b) $v(x,t) = 3\pi \cos(0.4\pi x + 50\pi t)$

c) $y^*(x,t) = 0.12 \cos(0.4\pi x) \sin(50\pi t)$

d) Δx=2.5 m

3. a) f_2=10 Hz / c_2=100 m/s / $\mu=5\cdot10^{-3}$ kg/m

b) $y = 8\sin(0.1\pi x)\sin(10\pi t)$ / $v_{max}=80\pi$ cm/s

c) x_1=0, x_2=2.5, x_3=4, x_4=7.5, x_5=10 [m]

4. a) $p(x,t) = 2\cdot10^{-4}\cos(3.6\pi x - 1224\pi t)$ / λ =0.555 m

b) $I=4.82\cdot10^{-11}$ W/m^2

c) L_I=17 dB

d) $I=5\cdot10^{-9}$ W/m^2

5. a) $t_y=5\cdot10^{-3}$ s

b) $t_v=1.25\cdot10^{-2}$ s

c) $t_a=10^{-2}$ s

6. Theoretical

7. y(20,1)= 1.2 Pa / y(25,1)=-1.2 Pa (destructive int.) / $\Delta\varphi=2\cdot2\pi+\pi$ rad

8. A≡constructive / C≡destructive

9. f=990 Hz / $y^*(x,t) = 8\cdot10^{-3} \sin(3\pi x)\sin(1980\pi t)$

10. Theoretical

11. a) f_s=100 Hz

b) $x(n) = 3\cos(\pi/2)n$

c) $x(n) = 3\cos\left(\frac{2\pi}{3}\right)n$

12. f_N=300 Hz / theoretical